Quark-Parton Model Predictions for Hadronic Charge Ratios in Inclusive Lepton-Induced Reactions*

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We investigate the predictions of the quark-parton model for hadronic charge ratios in the current-fragmentation region in inclusive lepton-induced reactions. We use parton distribution functions given by McElhaney and Tuan from fits to single-arm inelastic electron scattering data, and we obtain relative parton fragmentation functions from a fit to the π^+/π^- ratio in electroproduction from a proton target. The electroproduced π^+/π^- ratio from a neutron target is predicted to be $\gtrsim 1.2$ at moderate ω and < 1 at small ω, providing a dramatic test of the model. The model gives a neutrino-(antineutrino-) produced π^+/π^- (π^-/π^+) ratio of 3.0 ± 0.6 from any target. Also the ratio of charged to neutral K production in e^+e^- annihilation should be 1.38 ± 0.10.

I. INTRODUCTION

The parton model[1-3] appears to provide a qualitative description of the main features of deep-inelastic lepton scattering. Since SU(3) is a well-established approximate hadronic symmetry, it is tempting to identify partons as quarks. However, at the present time, the experimental justification for doing this is weak. The primary evidence that partons have quark charges is the consistency of inelastic electron scattering data on the proton-neutron difference with the Gottfried sum rule. The current extrapolated experimental value is[4] "$0.27 \pm (?)$," and this may agree with the quark-parton-model prediction of 0.33.

Recently there have been several experiments which detected the hadronic final states produced in deep-inelastic electron or muon scattering.[5-10] Additional experiments of this type are currently in preparation, as are experiments on the final hadronic states produced in neutrino interactions and in electron-positron annihilation. In this paper we will investigate the features that can be expected in these reactions if they all are to be consistent with a reasonable quark-parton model. In particular we will be interested in aspects of the data that are sensitive to the relative charges of the quarks. To this end we will consider primarily the charge ratios of pions produced in the current fragmentation region. Experimentally, these ratios are relatively easy to measure and are to a great extent independent of normalization problems and radiative effects. Also, in electroproduction experiments, these charge ratios seem to provide the most striking evidence for a change in the dynamic mechanism as one goes from real to virtual photons.[5]

II. THE QUARK-PARTON MODEL

The formulation of the parton model that we will use is found in the works of Feynman[11] and Gronau, Ravndal, and Zarmi.[12] We follow the notation of the latter reference.

The model for inelastic lepton scattering is described in the current-parton Breit frame. As illustrated in Fig. 1(a), the nucleon, with (large) longitudinal momentum P, is regarded as a collection of independent pointlike constituents. The lepton current, with momentum $-2xP$, interacts incoherently with a parton of momentum xP, reversing its momentum. The struck parton then fragments into hadrons, a typical one of which, h, will possess a fraction z of the parton's momentum. The remaining partons, with momentum $(1-x)P$, also fragment into hadrons, as shown in Fig. 1(b). It is assumed that the fragmentation processes are independent of x, since for finite x the struck parton is separated by a large momentum difference from the nucleon fragments. It is also assumed that none of the fragments have appreciable momentum in the wrong direction, that is, the direction opposite that shown in Fig. 1(b). At the limited energies of present experiments, these assumptions can only be approximately valid. ($x = -q^2/2q\cdot p$, where q and p represent the four-momenta of the lepton current and the initial nucleon, respectively.)

The model is completely determined when two sets of functions, the parton distribution functions and the parton fragmentation functions, are specified. The former denote the average number of partons of a given type in an interval of x and are designated by the type of parton [e.g., $u(x)dx$, $\bar{s}(x)dx$, etc.]. Similarly, the fragmentation functions specify the average number of hadrons of a given type in an interval of z arising from the fragmentation of a given type of parton. They are designated in an obvious notation [e.g., $D_u^{\pi^+}(z)dz$, $D_{\bar{d}}^{K^0}(z)dz$, etc.]. The D functions are also functions of the (limited) transverse momentum of the hadrons, but we will integrate over this variable.

The structure functions for various reactions can now be constructed. For example, for the case of inclusive π^- production by electrons from a proton target, we have

$$L_{2,\pi^-}^{ep}(x,z) = x[\tfrac{4}{9}u(x)D_u^{\pi^-}(z) + \tfrac{4}{9}\bar{u}(x)D_{\bar{u}}^{\pi^-}(z) + \tfrac{1}{9}d(x)D_d^{\pi^-}(z) + \tfrac{1}{9}\bar{d}(x)D_{\bar{d}}^{\pi^-}(z) + \tfrac{1}{9}s(x)D_s^{\pi^-}(z) + \tfrac{1}{9}\bar{s}(x)D_{\bar{s}}^{\pi^-}(z)]\,. \quad (1)$$

III. THE DISTRIBUTION FUNCTIONS

The distribution functions are constrained by single-arm inelastic electron scattering measurements since

$$F_2^{ep}(x) = x\,\{\tfrac{4}{9}[u(x)+\bar{u}(x)] + \tfrac{1}{9}[d(x)+\bar{d}(x)] + \tfrac{1}{9}[s(x)+\bar{s}(x)]\} \quad (2)$$

and

$$F_2^{en}(x) = x\{\tfrac{1}{9}[u(x)+\bar{u}(x)] + \tfrac{4}{9}[d(x)+\bar{d}(x)] + \tfrac{1}{9}[s(x)+\bar{s}(x)]\}\,. \quad (3)$$

To reproduce the nucleon quantum numbers, they must also satisfy the following normalization conditions:

$$\int_0^1 [u(x)-\bar{u}(x)]\,dx = 2\,, \quad (4a)$$

$$\int_0^1 [d(x)-\bar{d}(x)]\,dx = 1\,, \quad (4b)$$

$$\int_0^1 [s(x)-\bar{s}(x)]\,dx = 0\,. \quad (4c)$$

Kuti and Weisskopf[13] constructed a model for these distribution functions based on reasonable physical assumptions and fits to the then-existing data. The nucleon was assumed to consist of three valence quarks plus a core of quark-antiquark pairs. The sea of core quarks was assumed to be composed of equal numbers of each type, all with the same longitudinal momentum distribution,

$$u(x) = u_v(x) + c(x)\,, \quad (5a)$$

$$d(x) = d_v(x) + c(x)\,, \quad (5b)$$

$$s(x) = \bar{s}(x) = \bar{u}(x) = \bar{d}(x) = c(x)\,, \quad (5c)$$

where $u_v(x)$ and $d_v(x)$ represent the distribution functions for valence quarks.

The core quark momentum distribution was as-

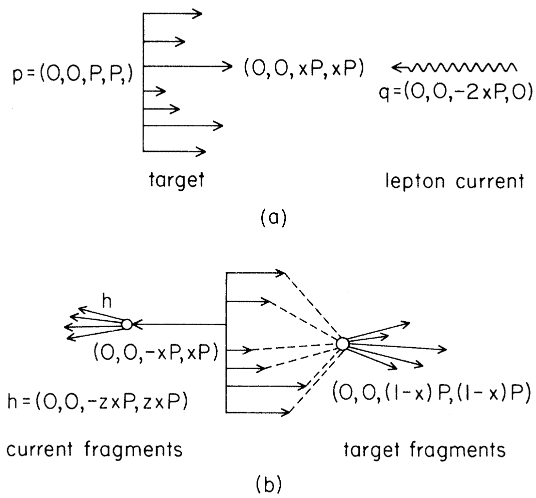

FIG. 1. Schematic diagram of inelastic lepton scattering in the parton model. See text for description.

sumed to be given by phase space, and the valence quark distributions by phase space and by Regge-theory considerations at small x. Because of the low value of the experimental mean charge squared per parton,[14] it was necessary to assume that some of the nucleon momentum is carried by neutral constituents.

Since u_v and d_v are proportional in the Kuti-Weisskopf model, the model cannot accommodate a neutron-proton ratio, F^{en}/F^{ep}, of less than 2/3, in contradiction to recent data.[15] McElhaney and Tuan have presented modified versions of the Kuti-Weisskopf distribution functions which remove this difficulty.[16] We will use one of their versions which is a four-parameter fit to the data and is based on the addition of a low-lying daughter Regge trajectory:

$$u_v(x) = 1.74\,x^{-1/2}(1-x)^3(1+2.3x)\,, \quad (6a)$$

$$d_v(x) = 1.11\,x^{-1/2}(1-x)^{3.1}\,, \quad (6b)$$

$$c(x) = 0.10\,x^{-1}(1-x)^{7/2}\,. \quad (6c)$$

These functions are certainly not unique, but they have the virtue of being an excellent fit to the data, and they will probably be adequate for our purposes. With the exception of the region around $x=1$, we doubt that it is possible to construct a reasonable theory which both fits the data and differs numerically from the above in any significant way. For example, the difference between the u_v and d_v functions is directly determined by the data,

$$F_2^{ep}(x) - F_2^{en}(x) = \tfrac{1}{3}x[u_v(x) - d_v(x)]\,. \quad (7)$$

IV. THE FRAGMENTATION FUNCTIONS

Isospin and charge-conjugation invariance reduces the number of independent D functions for pions to three:

$$D_u^{\pi^+} = D_d^{\pi^-} = D_{\bar{u}}^{\pi^-} = D_{\bar{d}}^{\pi^+} , \tag{8a}$$

$$D_d^{\pi^+} = D_u^{\pi^-} = D_{\bar{d}}^{\pi^-} = D_{\bar{u}}^{\pi^+} , \tag{8b}$$

$$D_s^{\pi^+} = D_s^{\pi^-} = D_{\bar{s}}^{\pi^+} = D_{\bar{s}}^{\pi^-} . \tag{8c}$$

We define the ratio

$$\eta(z) \equiv D_u^{\pi^+}(z)/D_d^{\pi^+}(z), \tag{9}$$

and we make the assumption that

$$D_s^{\pi^+}(z) = D_d^{\pi^+}(z). \tag{10}$$

This assumption is based on the physical idea that $D_s^{\pi^+}$ and $D_d^{\pi^+}$ are both unfavored with respect to $D_u^{\pi^+}$ since a π^+ can be formed by the addition of one (anti-) quark to the fragmenting u quark, but not to a fragmenting d or s quark. The validity of this assumption is not critical for our numerical results since even at quite small x (~ 0.01) strange quarks contribute only about 10% of the electron scattering cross section.

So that we can compare with data in a statistically significant way, we will consider the integrals of the D functions over the current fragmentation region. We take

$$D \to \int_{0.4}^{1} D(z)dz, \tag{11}$$

but we do not include the πN and $\pi\Delta$ final states in the integral. These states arise primarily from π exchange and do not survive in the Bjorken limit.[17] In practice this means that we terminate the integral at $z = 0.8$ for low-s (7 GeV^2) data.

In Sec. V we will obtain a numerical estimate of the average value of η in the sense of Eq. (11). This estimate will also be approximately valid for $\eta(z)$ for $z > 0.4$, at least to $z = 0.8$. This is because experimentally the π^+/π^- ratio in electroproduction from protons appears to be relatively constant in the region $z > 0.4$, and we will obtain information on the relative contribution of the various D functions from these data. The $\pi^+\pi^-$ ratio from two experiments[5,6] is shown in Fig. 2.[18] For $z < 0.4$ there is a tendency for the charge ratio to decrease.

V. ep CHARGE RATIOS

We are now in a position to determine η from measurements of pion charge ratios in the reaction

$$ep \to e\pi^{\pm} + \text{anything}. \tag{12}$$

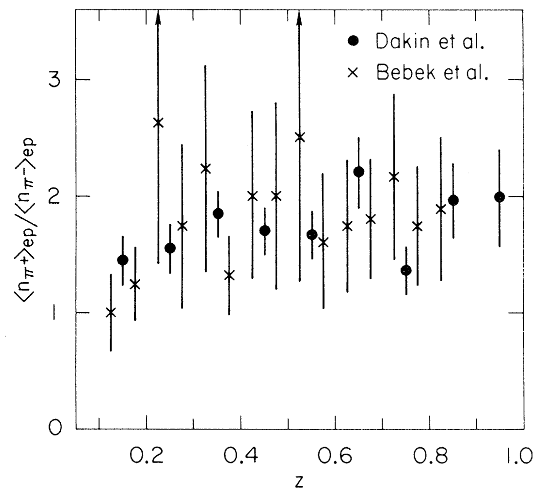

FIG. 2. π^+/π^- ratio in electroproduction from a proton target as a function of z. The data of Dakin *et al.* (Ref. 5) are in the kinematic range $-0.5 \geq q^2 \geq -2.5$ GeV^2 and $3 \leq \omega \leq 60$. The data of Bebek *et al.* (Ref. 6) are at the datum point $q^2 = -2.0$ GeV^2 and $\omega = 4$.

Figure 3 shows data from four experiments for the ratio of positive to negative pions in the current fragmentation region as a function of ω ($\equiv 1/x$). One experiment,[5] from SLAC, has data at relatively high center-of-mass energy squared ($12 \leq s$

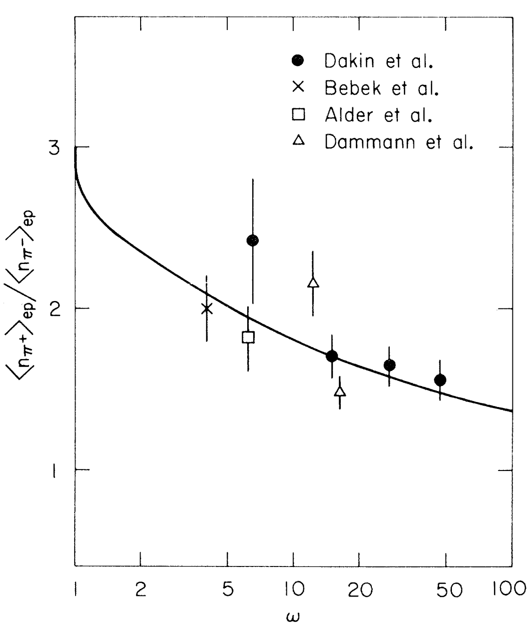

FIG. 3. π^+/π^- ratio in electroproduction from a proton target as a function of ω. The data of Dakin *et al.* (Ref. 5) are in the range $12.0 \leq s \leq 30.0$ GeV^2, and the other data (Refs. 6–8) have $s \approx 7$ GeV^2. The curve represents a one-parameter fit to the data, Eq. (13) with $\eta = 3.0$.

≤ 30 GeV^2), while the other three experiments from Cornell[6] and DESY[7,8] have data at $s \approx 7$ GeV^2. The SLAC experiment did not separate types of particles, so there may be some kaon contamination.

The curve on Fig. 3 is a fit to the quark-parton-model prediction,

$$\frac{\langle n_{\pi^+}\rangle_{ep}}{\langle n_{\pi^-}\rangle_{ep}} = \frac{4\eta u_v(x) + d_v(x) + (5\eta+7)c(x)}{4u_v(x) + \eta d_v(x) + (5\eta+7)c(x)}, \quad (13)$$

with η as a free parameter. The fit had a χ^2 of 11.4 for seven degrees of freedom. The fitted value of η was

$$\eta = 3.0, \quad (14)$$

and given the many uncertainties of this approach, we consider this value good to about 20%. Working from the data of Bebek *et al.*,[6] Cleymans and Rodenberg have obtained a somewhat smaller value of η; this is presumably due to their assumption that $c(x) = 0$.[19]

One obvious consequence of this model is that even as $x \to 1$, the charge ratio will not rise above η.[20]

VI. *en* AND νN CHARGE RATIOS

Having determined η from *ep* data, we can now predict the charge ratios for other reactions. For

$$en \to e\pi^{\pm} + \text{anything}, \quad (15)$$

we expect

$$\frac{\langle n_{\pi^+}\rangle_{en}}{\langle n_{\pi^-}\rangle_{en}} = \frac{u_v(x) + 4\eta d_v(x) + (5\eta+7)c(x)}{\eta u_v(x) + 4d_v(x) + (5\eta+7)c(x)}, \quad (16)$$

which is plotted in Fig. 4, with $\eta = 3.0$. This curve

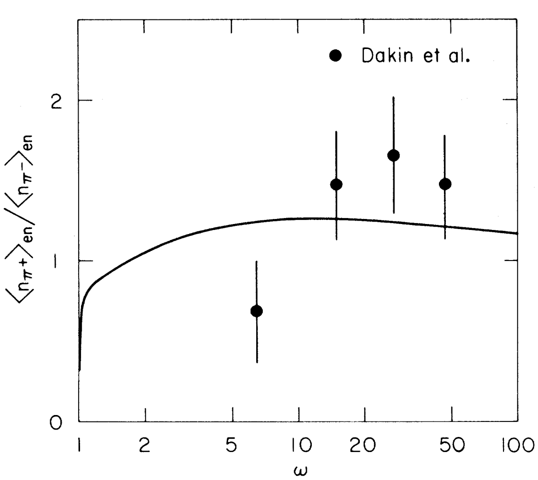

FIG. 4. π^+/π^- ratio in electroproduction from a neutron target as a function of ω. The curve is the prediction of the model, Eq. (16) with $\eta = 3.0$. The data (Ref. 5) are in the range $12.0 \leq s \leq 30.0$ GeV^2.

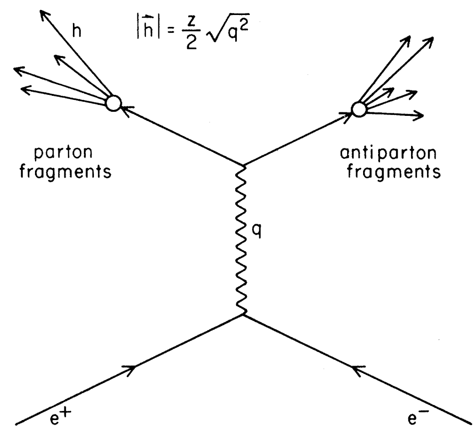

FIG. 5. Schematic diagram of e^+e^- annihilation in the parton model. See text for description.

provides a rather unique test of the model. The charge ratio should rise to a broad maximum of between 1.2 and 1.3 around $\omega = 12$, then fall to unity at $\omega = 1.7$ and continue to fall below unity as $\omega \to 1$. The positions of the maximum and of the $\langle n_{\pi^+}\rangle_{en} = \langle n_{\pi^-}\rangle_{en}$ point do not depend on η. The predicted π^+/π^- ratio of between 1.2 and 1.3 at moderate ω is in sharp contrast to the π^+/π^- ratio of about 0.8 observed in photoproduction in the photon fragmentation region.[21]

Data from a SLAC experiment[5] are also shown in Fig. 4. While not in disagreement with the predictions, these data are clearly not of sufficient precision to test them. More data on this reaction are expected in the next year from Cornell and SLAC experiments.

The predictions for pionic charge ratios from neutrino-induced reactions are rather simple. In the (very good) approximation that the square of the Cabbibo angle is zero, neutrinos interact only with d and $\bar{u}$ quarks, while antineutrinos interact only with u and $\bar{d}$ quarks. Therefore

$$\frac{\langle n_{\pi^+}\rangle_{\nu p}}{\langle n_{\pi^-}\rangle_{\nu p}} = \frac{\langle n_{\pi^+}\rangle_{\nu n}}{\langle n_{\pi^-}\rangle_{\nu n}} = \frac{\langle n_{\pi^-}\rangle_{\bar{\nu} p}}{\langle n_{\pi^+}\rangle_{\bar{\nu} p}} = \frac{\langle n_{\pi^-}\rangle_{\bar{\nu} n}}{\langle n_{\pi^+}\rangle_{\bar{\nu} n}} = \eta = 3.0 \pm 0.6, \quad (17)$$

for the region $z > 0.4$. Neutrino bubble chamber experiments at CERN and NAL should be able to test this prediction in the near future.

VII. e^+e^- CHARGE RATIOS

Electron-positron annihilation into hadrons is conventionally described in this model as the pair production of a parton-antiparton pair, each of which subsequently fragments into hadrons. This is shown schematically in Fig. 5. The fragmentation process is assumed to be identical to that which occurs in lepton scattering. Thus,

$$\frac{d\sigma}{dz}(e^+e^- \to h + \text{anything}) = \frac{4\pi\alpha^2}{3q^2}\sum_i Q_i{}^2[D_i^h(z) + D_{\bar{i}}^h(z)], \tag{18}$$

where the sum is over the type of parton and Q_i is the parton charge.

Obviously, charge-conjugation invariance prohibits us from gaining any information on relative parton charges by observing pions. However, the kaon system offers hope of a successful test.

Isospin invariance yields

$$D_u^{K^+} = D_d^{K^0}, \tag{19a}$$

$$D_{\bar{s}}^{K^+} = D_{\bar{s}}^{K^0}, \tag{19b}$$

$$D_d^{K^+} = D_u^{K^0}, \tag{19c}$$

$$D_{\bar{d}}^{K^+} = D_{\bar{u}}^{K^0}, \tag{19d}$$

$$D_{\bar{u}}^{K^+} = D_{\bar{d}}^{K^0}, \tag{19e}$$

and

$$D_s^{K^+} = D_s^{K^0}. \tag{19f}$$

In addition, SU(3) invariance gives

$$D_u^{K^+} = D_{\bar{s}}^{K^+} = D_u^{\pi^+}, \tag{20a}$$

$$D_d^{K^+} = D_{\bar{d}}^{K^+} = D_s^{\pi^+}, \tag{20b}$$

and

$$D_{\bar{u}}^{K^+} = D_s^{K^+} = D_d^{\pi^+}. \tag{20c}$$

Thus, for the ratio of charged to neutral kaon production we obtain

$$\frac{\langle n_{K^+}\rangle_{e^+e^-}}{\langle n_{K^0}\rangle_{e^+e^-}} = \frac{5\eta + 7}{2\eta + 10} = 1.38 \pm 0.10, \tag{21}$$

again in the region $z > 0.4$.

Although SU(3) invariance is a sufficient condition for Eq. (21) to hold in this model, it is hardly necessary. One might imagine that SU(3) breaking occurs in such a way as to inhibit all kaon production relative to pion production by a constant factor. In such a case, the prediction will still be valid.

This prediction should be tested at the SLAC storage ring SPEAR within the next year.

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Relation Between the Increasing Proton-Proton Total Cross Section and the Structure in Elastic Scattering at Small Momentum Transfers

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We show that the CERN Intersecting Storage Rings measurements of the increasing proton-proton total cross section and of the structure in elastic scattering at very small momentum transfers can be quantitatively represented and correlated by a simple phenomenological structure. This structure can be given a very simple geometric interpretation.

The proton-proton total cross section is increasing significantly at the highest energies at which measurements are being made at the CERN Intersecting Storage Rings[1-4] (ISR), that is, at total center-of-mass energies, $\sqrt{s}$, from about 53 GeV to about 62 GeV. An indication that the total inelastic cross section had begun to increase was obtained from an analysis[5] of the data from one of the early ISR experiments, which measured the inclusive cross section for $p+p\to p+(\cdots)$. A recent analysis[6] of cosmic-ray data suggests a proton-proton total cross section of about 60 mb at $\sqrt{s}\approx 238$ GeV. A further interesting observation[7,8] at the ISR, which is also especially marked at the highest energies, is the increasing effective slope of $(d\sigma_E/dt)\propto e^{(B_{\rm eff})t}$ for very small $|t| \lesssim 4m_\pi^2 \cong 0.076$ (GeV/c)^2. Since the total cross section $\sigma_T(s)$ is essentially determined by $d\sigma_E(s,t=0)/dt$ via the optical theorem,[9] it is very natural to attempt to *quantitatively* relate the increasing total cross section and the steepening diffraction pattern. We show that this can be achieved by a very simple phenomenological structure, which represents and correlates $d\sigma_E(s,t)/dt$ at small $|t|$, and $\sigma_T(s)$, over an enormous range of s.

At a given s the phenomenological structure for the diffractive amplitude is of the following form, where A, b, and N are constants:

$$F(s,t)=N\left\{\frac{A}{f(s)}+\frac{1}{[4m_\pi^2/g(s)-t]}\right\}e^{bt}\,. \qquad (1)$$

We have

$$\sigma_T(s)=4\pi F(s,0)\,, \qquad (2a)$$

$$d\sigma_E(s,t)/dt=\pi\,|F(s,t)|^2\,. \qquad (2b)$$

Since relatively strong variations occur around $s_0=(53\text{ GeV})^2=2810\text{ (GeV)}^2$, we choose this as our energy for normalizations

$$g(s_0)=f(s_0)=1\,, \qquad (3a)$$

thus

$$N=\sigma_T(s_0)/4\pi(A+1/4m_\pi^2)\,, \qquad (3b)$$

$$F(s_0,t)=N\{A+[1/(4m_\pi^2-t)]\}\,e^{bt}\,. \qquad (3c)$$

The first term in Eq. (3c) is the usual simple exponential parametrization which is arbitrary[10] but convenient, since the dominant feature of the data at small $|t|$ is the near-linear behavior of $\ln[d\sigma_E(t)/dt]$. The relatively small deviations from this behavior are represented by the second term in the curly brackets. This polelike structure, which roughly corresponds to a "long-range" contribution to the diffractive (imaginary) amplitude from the excitation and deexcitation of real intermediate states via exchange of a pion, has a general theoretical basis[11] in the contribution from the two-pion branch cut in the unitarity equations.[12] The parameter A, which controls its relative weight, and the usual slope parameter b, are determined by a fit to $d\sigma_E(t)/dt$ at $\sqrt{s}=53$ GeV. This is the upper curve in Fig. 1, which repre-