$\bar{p}p$ Elastic Scattering at 2.33 GeV/c

H. B. Crawley, W. J. Kernan, and F. Ogino
Ames Laboratory-USAEC and Department of Physics, Iowa State University, Ames, Iowa 50010
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The differential scattering cross section is presented for $\bar{p}p$ elastic scattering at an incident laboratory momentum of 2.33 GeV/c based upon 11 758 events. The experiment was performed at the Brookhaven National Laboratory using the 31-inch hydrogen bubble chamber and an electrostatically separated beam. The attempts to fit limited regions of the data are presented for different parametrizations. The parametrization which corresponds to two coherent interfering exponentials successfully reproduces a very large t region for the scattering.

I. INTRODUCTION

In two recent publications[1,2] it has been shown that two coherent interfering exponentials can successfully reproduce a very large t region in the elastic scattering of $\bar{p}p$ at quite different incident laboratory momenta. We have completed analysis of this elastic scattering reaction at a laboratory momentum of 2.33 GeV/c. These data provide an additional test of the usefulness of this parametrization.

II. EXPERIMENTAL DETAILS

Approximately 47 000 frames from the 31-inch hydrogen bubble chamber were scanned for all two-pronged events. All frames were triple-scanned to determine scanning efficiencies and to evaluate possible scanning biases. Out of 25 400 two-pronged events found in the scanning, 24 507 events were measured. The measuring was performed with a system of computer-controlled manually operated measuring machines. The events were processed through an on-line geometry system for simultaneous reconstruction of the tracks, thus allowing extensive quality control checks and immediate remeasuring on questionable tracks.

After later processing through kinematical fitting routines and after checks for ionization consistency were performed, events corresponding to possible "lost" elastic scattering events were remeasured. Approximately 13 400 elastic scattering events were identified. After fiducial volume restrictions were made and events with a high χ^2 were eliminated, 11 758 were left on which the analysis was based. The differential cross-section data are presented in Table I.

III. THE DIFFRACTION REGION

The differential cross section for the region $-t \leq 0.4$ (GeV/c)^2 is shown in Fig. 1. This region is often called the diffraction region and has usually been interpreted using a limited t range and the simple exponential form

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt}\right)_0 e^{bt}. \quad (1)$$

Alternative forms of analysis have occasionally been used for the diffraction region; for example, the inclusion of a curvature term in the exponential[3] which would correspond to the equation

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt}\right)_0 e^{bt+ct^2}. \quad (2)$$

Another method of analysis that has been used is to include a larger region in t and to parametrize the data using two coherent interfering exponentials.[1,2] This would correspond to the equation

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt}\right)_0 \left|\frac{e^{b_1 t/2} + |A| e^{i\varphi} e^{b_2 t/2}}{1 + |A| e^{i\varphi}}\right|^2. \quad (3)$$

It has recently been shown[2] at an incident laboratory momentum of 2.85 GeV/c that Eqs. (1) and (2) provide a poor representation of even a limited t range of the data. Not only are the fits at this energy poor in the sense of χ^2 probability, but also there is a significant change in the resulting parameters as the t range is varied. The present experiment has more limited statistics and hence does not result in as strong a conclusion, but it does independently verify the earlier results of Ref. 2 in the same energy region.

We have attempted to fit the data of this experiment from $t = -0.04$ (GeV/c)^2 to $t' = t_{\max}$ to both Eqs. (1) and (2). The maximum t used in each fit was varied from 0.210 (GeV/c)^2 to 0.350 (GeV/c)^2.

The parameters that result from the fitting procedure varied in an almost continuous fashion as the t range of the data used was varied. The results of the fits are presented in Table II for both parametrizations for the extreme values of t'; that is, $t' = -0.210$ (GeV/c)^2 and $t' = -0.350$ (GeV/c)^2.

TABLE I. The values of $d\sigma/dt$ measured in this experiment. The errors listed are the point-to-point errors and do not include the uncertainty in normalization which is 2.7%.

$-t_{min}$ $(\text{GeV}/c)^2$	$-t_{max}$ $(\text{GeV}/c)^2$	$d\sigma/dt$ $(\text{mb}/(\text{GeV}/c)^2)$	Error	$-t_{min}$ $(\text{GeV}/c)^2$	$-t_{max}$ $(\text{GeV}/c)^2$	$d\sigma/dt$ $(\text{mb}/(\text{GeV}/c)^2)$	Error
0.040	0.045	242.6	10.2	0.230	0.240	13.700	1.700
0.045	0.050	205.9	9.4	0.240	0.250	12.900	1.700
0.050	0.055	171.8	8.6	0.250	0.260	8.580	1.360
0.055	0.060	172.5	8.6	0.260	0.270	6.220	1.160
0.060	0.065	170.5	8.6	0.270	0.280	6.440	1.170
0.065	0.070	164.7	8.4	0.280	0.290	5.790	1.110
0.070	0.075	152.5	8.1	0.290	0.300	3.650	0.880
0.075	0.080	139.9	7.7	0.300	0.325	2.660	0.480
0.080	0.085	127.8	7.4	0.325	0.350	2.230	0.440
0.085	0.090	122.3	7.2	0.350	0.375	1.290	0.330
0.090	0.095	112.0	6.9	0.375	0.400	1.030	0.300
0.095	0.100	106.8	6.8	0.400	0.425	1.030	0.300
0.100	0.105	98.7	6.5	0.425	0.450	0.430	0.190
0.105	0.110	87.5	6.1	0.450	0.475	0.860	0.270
0.110	0.115	82.4	5.9	0.475	0.500	1.290	0.330
0.115	0.120	82.4	5.9	0.500	0.550	1.420	0.250
0.120	0.125	67.8	5.4	0.550	0.600	1.370	0.240
0.125	0.130	68.6	5.4	0.600	0.650	1.760	0.270
0.130	0.135	68.2	5.4	0.650	0.700	1.890	0.290
0.135	0.140	61.8	5.1	0.700	0.800	1.650	0.190
0.140	0.145	56.6	4.9	0.800	0.900	1.420	0.170
0.145	0.150	51.9	4.7	0.900	1.000	1.110	0.150
0.150	0.155	44.2	4.4	1.000	1.200	0.761	0.090
0.155	0.160	47.2	4.5	1.200	1.400	0.375	0.063
0.160	0.165	42.5	4.3	1.400	1.600	0.300	0.057
0.165	0.170	31.7	3.7	1.600	1.800	0.247	0.051
0.170	0.175	34.3	3.8	1.800	2.000	0.193	0.045
0.175	0.180	32.2	3.7	2.000	2.200	0.161	0.042
0.180	0.190	36.7	2.8	2.200	2.400	0.150	0.040
0.190	0.200	31.1	2.6	2.400	2.600	0.107	0.034
0.200	0.210	25.1	2.3	2.600	2.800	0.086	0.030
0.210	0.220	14.8	1.8	2.800	3.000	0.075	0.028
0.220	0.230	16.7	1.9	3.000	3.200	0.011	0.011

TABLE II. Results of fitting Eqs. (1), (2), and (3) to the data of this experiment.

Equation used	(1)	(1)	(2)	(2)	(3) (our data)	(3) (ANL data)
t range	0.04 to 0.21	0.04 to 0.35	0.04 to 0.21	0.04 to 0.35	0.04 to 1.60	~0.04 to ~1.50
$\left(\frac{d\sigma}{dt}\right)_0$	402.5 ± 11.2	430.1 ± 10.3	404.3 ± 27.0	359.8 ± 16.7	375.6 ± 9.3	356.2 ± 5.0
b or b_1	13.82 ± 0.26	14.60 ± 0.20	13.92 ± 1.32	11.32 ± 0.76	10.21 ± 0.46	10.04 ± 0.20
c or b_2	···	···	0.42 ± 5.64	−11.54 ± 2.57	3.57 ± 0.35	3.44 ± 0.13
χ^2	39.9	78.8	40.1	48.2	54.3	64.8
χ^2/ν	1.33	1.97	1.38	1.24	1.02	1.16
Probability	~10%	< 0.1%	~8%	~15%	42.5%	~20%
$\lvert A\rvert$	···	···	···	···	0.268 ± 0.042	0.269 ± 0.017
$\lvert\varphi\rvert$	···	···	···	···	164.9 ± 2.7°	163.8 ± 1.1°

As is obvious from Table II the parameters are a reasonably strong function of the t range used in the fitting attempt. In particular, depending upon the parametrization used and the t range of the data fitted, the estimates of the differential cross section at $t=0$ can vary from approximately 360 ± 17 mb/(GeV/c)^2 to 430 ± 10 mb/(GeV/c)^2. From the results presented in Table II there would be a tendency to conclude that a value of $(d\sigma/dt)_0$ of approximately 405 mb/(GeV/c)^2 would be consistent with any parametrization. It will turn out, however, that this value is not consistent with the value based on fitting to Eq. (3).

IV. TWO COHERENT INTERFERING EXPONENTIALS

The data for the t range from 0.04 (GeV/c)^2 to 1.60 (GeV/c)^2 has been used to determine the five parameters of Eq. (3). An excellent fit is achieved with a χ^2 of 54.3 for 53 degrees of freedom. This corresponds to a χ^2 probability of 42.5%. The parameters for this fit are given in Table II. The results of this fit are shown along with the data in Fig. 2. The log-log scale has been chosen simply to compress the plot for reasonable presentation.

It is not surprising that the parameters $(d\sigma/dt)_0$ and b_1 which are equivalent in meaning to parameters in Eqs. (1) and (2) also have somewhat different values when the estimate is based upon fitting to Eq. (3). However, it is worth emphasizing that small changes in the estimate of $(d\sigma/dt)_0$ can change the estimate of the real to imaginary ratio of the amplitudes in a significant fashion. For example, at 2.33-GeV/c incident momentum the optical point corresponds to a differential cross section at $t=0$ of 364.5 mb/(GeV/c)^2. If one used the estimated value for this based on Eq. (1) and the limited t range of -0.04 to -0.210 (GeV/c)^2 of 402.5 ± 11.2 mb/(GeV/c)^2, then the estimated value exceeds the optical point by 38.0 ± 11.2 mb/(GeV/c)^2. Using the parameters estimated from Eq. (3) this excess decreases to 11.1 ± 9.3 mb/(GeV/c)^2.

In a similar fashion changes in the estimate of the "dominant slope" b could change the theoretical ideas as to the antishrinkage of the Regge trajectories. This could come about if a new analysis of all the $\bar{p}p$ elastic scattering data, based upon Eq. (3) instead of Eq. (1), failed to show that b decreases as the center-of-mass energy increases.

The point of these comments is to indicate that many existing ideas in the literature, for example, real to imaginary ratios of the amplitudes and the antishrinkage of the slope in $\bar{p}p$ scattering, may not be really valid if the parametrization on which the estimates have been based is not correct. We have certainly shown that the estimates will change in a significant fashion if the parametrization is changed.

V. COMPARISON WITH OTHER DATA

After our analysis of our data was completed, additional data[4] at almost the same incident mo-

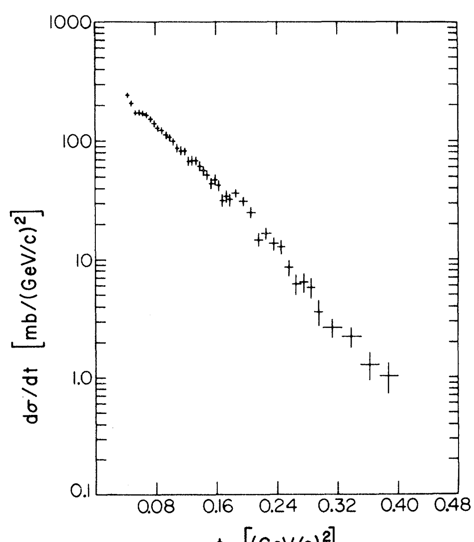

FIG. 1. The differential cross section in the range $|t|<0.4$ (GeV/c)^2 for $\bar{p}p$ elastic scattering at an incident laboratory momentum of 2.33 GeV/c.

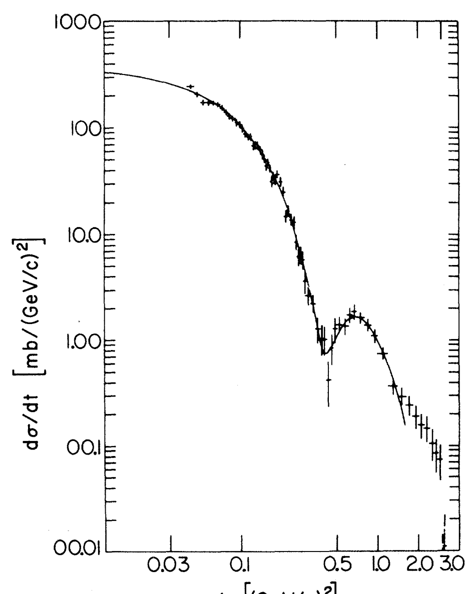

FIG. 2. The differential cross section plotted versus t. The solid line is the result of fitting Eq. (3) to the data.

mentum was published. An excellent check on the analysis of $\bar{p}p$ elastic scattering using Eq. (3) is possible by fitting this high-statistics experiment of the Argonne group with this equation and comparing the results of that fit with the fit to our own data. We have done this and the results are presented in Table II. The quoted uncertainties on the parameters which result from fitting the Argonne data should be taken as lower limits. We have used their data tables which allow us to tabualte only their statistical errors. In their own analysis, the quoted uncertainties are determined mostly from a 3% uncertainty in the correction for losses. Since we cannot be sure how this uncertainty influences the point-to-point errors, we have used the statistical errors only and the resulting uncertainties are lower limits.

Despite this difficulty the confidence level of 20% that we achieve in fitting their data with Eq. (3) represents a quite acceptable fit to their data. With the exception of $(d\sigma/dt)_0$, the two experiments show excellent agreement on all parameters.

If their normalization uncertainty of 4% is also included in the Argonne data, then the estimate of $(d\sigma/dt)_0$ for their data is 356 ± 15 mb/(GeV/c)^2. If the 2.7% uncertainty in our normalization is included, our estimate of $(d\sigma/dt)_0$ for our data would be approximately 376 ± 14 mb/(GeV/c)^2. Our conclusion from this is that all parameters are in excellent agreement for the two experiments analyzed using Eq. (3).

VI. CONCLUSION

We have shown that two coherent interfering exponentials can successfully represent our data over the t range $0.04 \leq -t \leq 1.60$ (GeV/c)^2. In this we support the conclusions of an earlier experiment[2] of our group at 2.85 GeV/c. When combined with the original use of this parametrization by Kalbfleisch *et al.*[1] there is a strong suggestion that this parametrization may work over a wide range of laboratory momenta as well as a wide t range at any particular momentum.

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