Comments and Addenda

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Is There Spontaneous Symmetry Breaking in Gauge Theories with the Higgs-Kibble Mechanism?*

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It is argued that gauge theories with the Higgs-Kibble mechanism do not involve the spontaneous breaking of physical symmetries. The relationships between masses and coupling constants, usually considered a manifestation of the remnant of the symmetry, can be obtained, at least in the Abelian-Higgs model, directly from renormalizability and unitarity requirements.

Gauge theories with the Higgs-Kibble mechanism are commonly said to involve spontaneous breaking of symmetries (SBS). The usual view is that the problem with Goldstone bosons associated with SBS is evaded because in the Coulomb gauge the theory is neither manifestly covariant nor manifestly local and the Goldstone theorem does not apply, while in the Lorentz gauge the Goldstone bosons decouple from the physical space.

Nevertheless one can ask if there is any SBS involving *physical symmetries* in theories of this type. A symmetry is usually understood to be a transformation of the fields appearing in the Lagrangian, which leaves the action invariant. In formulations that involve an indefinite-metric space, we shall assume the existence of physical fields with the properties that (i) no physical field takes a physical state to an unphysical one, that (ii) all physical Heisenberg fields are local relative to one another with the normal connection between spin and statistics, and that (iii) there are enough of them so that essentially all physical states are obtainable by letting smeared physical fields successively act on the vacuum.[2] A *physical symmetry* will be defined to be a symmetry which does not transform physical fields to unphysical ones. It is spontaneously broken if the symmetry cannot be globally implemented by unitary transformations. The question we address is whether there is spontaneous breaking of any nontrivial physical symmetry in gauge theories with the Higgs-Kibble mechanism in the final form of such theories—not just in a "first stage" before the gauge vector mesons are added; the introduction of such stages does not seem to have any physical significance.

It has already been suggested[3] that one way to reconcile Goldstone's theorem with the absence of zero-mass particles in the physical space is for the symmetry to act trivially, i.e., as the identity, on the physical space, or, equivalently, to have the "charge" generating the symmetry vanish on the physical space. However, in indefinite-metric space formulations there actually can be zero-mass zero-norm states in the subspace of states satisfying subsidiary conditions, as we will discuss below. We will formulate a sufficient condition under which one can show, in spite of the presence of these zero-mass zero-norm states in this subspace, that there are no nontrivial physical symmetries that are spontaneously broken. In the Abelian-Higgs model we shall verify that this condition is met.

We consider gauge theories in manifestly covariant gauges. One must distinguish between those gauge conditions which destroy both local and global symmetry and gauge conditions which break only local gauge invariance but maintain global symmetry. Examples of the first type are the generalized Stueckelberg gauges with $\xi \neq \infty$, in the notation of Fujikawa *et al.*[4]; these include the unitarity gauge ($\xi = 0$) in which the unphysical fields are completely decoupled. In these gauges the Lagrangian is not invariant under the group transformations. Since there is no symmetry to be broken, it is not surprising that there need be no Goldstone bosons. In contrast, in the second type of gauge (e.g., the analog of Lorentz gauges), the Lagrangian is invariant under global transformations, which are formally associated with conserved Noether currents j_μ^α. As far as we know, one has to deal with an indefinite-metric space in the second type of gauge. The subspace of states satisfying certain subsidiary conditions will be denoted by $\mathcal{H}_1$, and the metric on this subspace $\mathcal{H}_1$ is presumably positive semidefinite. If there are no zero-norm states in $\mathcal{H}_1$, one can identify the physical space with $\mathcal{H}_1$. If there are zero-norm states forming a subspace $\mathcal{H}_0$, one identifies the physical space, as usual, with the quotient space $\mathcal{H} = \mathcal{H}_1/\mathcal{H}_0$. The physical fields, according to our definition, map $\mathcal{H}_1$ into $\mathcal{H}_1$, and are relatively local with the normal connection between spin and statistics.[5] (Since the zero-norm states in $\mathcal{H}_1$ are orthogonal to any vector in $\mathcal{H}_1$, the matrix elements of a physical field in $\mathcal{H}_1$ depend only on equivalence classes; i.e., on vectors in $\mathcal{H}$.) The basic *assumption* one needs for the following conclusions is that the *Noether currents* j_μ^α *are physical fields.*

The usual statement in gauge theories with the Higgs-Kibble mechanism is that the Goldstone boson χ^α corresponding to each broken symmetry resides in the unphysical subspace in the second type of covariant gauge, and that the mass spectrum in the physical space can be made strictly positive if all the gauge symmetries are broken.[1] For simplicity, let us consider this situation; the case of some gauge symmetries remaining intact can be treated with certain modifications. We first note that under the assumption that the j_μ^α are physical fields, there would be zero-mass states in $\mathcal{H}_1$. If there is spontaneous symmetry breaking in the indefinite-metric space, with χ^α denoting the Goldstone boson with the same quantum numbers as j_0^α, one must have[6] $\langle \chi^\alpha | j_0^\alpha | 0\rangle \neq 0$. Four-momentum conservation then implies that $j_0^\alpha|0\rangle$ contains a massless state. From our definition of physical fields, this state is in $\mathcal{H}_1$. So the subspace $\mathcal{H}_1$ of states satisfying the subsidiary condition does not have a strictly positive-mass spectrum. To have a positive-mass spectrum in the physical space, presumably by virtue of the Higgs-Kibble mechanism, one must not identify the physical space with $\mathcal{H}_1$. Indeed, a positive-mass spectrum on the physical space is possible if the zero-mass states reside in $\mathcal{H}_0$, and the physical space is identified with $\mathcal{H} = \mathcal{H}_1/\mathcal{H}_0$.

We next consider the space integrals[7]

$$Q_r^\alpha = \int_{|\vec{x}|<r} d^3x\, j_0^\alpha(x)\,.$$

Although the limit of Q_r^α as $r\to\infty$ does not exist, there are no difficulties with the limit in

$$\int d^3x [j_0^\alpha(x), \phi],$$

where ϕ is a local physical operator. Because of the positive-mass spectrum in $\mathcal{H}$, the $r=\infty$ limit of Q_r^α annihilates the vacuum[8] in the sense that $\lim_{r\to\infty}\langle\phi|Q_r^\alpha|0\rangle = 0$, where $|\phi\rangle$ is a local state in $\mathcal{H}$. (The states in this expression are understood to be elements of $\mathcal{H}$, and operators are also the induced ones on $\mathcal{H}$.) Furthermore, the matrix elements $\lim_{r\to\infty}\langle\phi|Q_r^\alpha|\psi\rangle$ are well defined for any $|\psi\rangle\in\mathcal{H}$, and are independent of x_0. So the limit of Q_r^α as $r\to\infty$ has the characteristics of an exact symmetry generator as far as the physical space $\mathcal{H}$ is concerned. This contradicts the original premise of such theories that the symmetry is broken in its physical consequences, unless the matrix elements $\lim_{r\to\infty}\langle\phi|Q_r^\alpha|\psi\rangle$ are actually all zero. However, the "symmetry" generated by Q_r^α on the physical space $\mathcal{H}$ is then just the identity. So we have deduced that if the Noether currents j_μ^α are physical fields, the corresponding symmetry on the physical space is the identity map, and obviously there can be no spontaneous breaking of this symmetry.

Of course, there can still be SBS in the indefinite-metric space, i.e., SBS with respect to transformations involving unphysical fields and unphysical states, but this appears to have little physical significance. What happens to unphysical fields and unphysical states is subject to a large degree of arbitrariness. For instance, we have already mentioned that the Goldstone bosons present in the unphysical space in the second type of gauge are absent in the first type of gauge.

We now examine the validity of the assumption that the j_μ^α are physical fields. In the case of the Abelian-Higgs model, the Noether current can be verified to be a physical field. The Lagrangian[9] is

$$L = L_0 + L_I\,,$$

$$\begin{aligned} L_0 = &-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}(gv)^2 U^\mu U_\mu + \tfrac{1}{2}\partial_\mu\psi\partial^\mu\psi \\ &- \frac{(2hv^2)}{2}\psi^2 + \tfrac{1}{2}\partial^\mu\chi\,\partial_\mu\chi \\ &- B\partial^\mu U_\mu + \tfrac{1}{2}\alpha B^2 - (gv)U^\mu\partial_\mu\chi\,, \end{aligned} \qquad (1)$$

$$\begin{aligned} L_I = &\frac{g^2}{2}U^\mu U_\mu(\psi^2+\chi^2) + g^2 v\psi U^\mu U_\mu + g\,U^\mu(\chi\partial_\mu\psi - \psi\partial_\mu\chi) \\ &- \tfrac{1}{4}h(\psi^4 + 4v\psi^3 + \chi^4 + 2\chi^2\psi^2 + 4v\psi\chi^2) \\ &- \frac{\delta m^2}{2}(\psi^2+\chi^2+2v\psi), \end{aligned}$$

and $F^{\mu\nu}=\partial^\mu U^\nu - \partial^\nu U^\mu$. The Lagrangian (1) is invariant under the substitutions:

$$U_\mu \to U_\mu;\;\; B\to B;\;\; \chi\to\chi+\epsilon+(\epsilon/v)\psi,\;\; \psi\to\psi-(\epsilon/v)\chi\,. \tag{2}$$

One notes that if the term $B\partial^\mu U_\mu$ in L_0 had been replaced by $B(\partial^\mu U_\mu + (1/\epsilon)\chi)$, corresponding to a generalized Stueckelberg gauge, the Lagrangian would not be invariant under (2), and there need be no Goldstone bosons at all as we have already remarked. We now consider the Lagrangian (1). The canonical quantization has been carried out,[10] and the subspace $\mathcal{H}_1$ is defined by $B^{(+)}|\text{phys}\rangle=0$. (Here $B=\alpha^{-1}\partial_\mu U^\mu$ is a massless free field even in the presence of interactions.) The transformations (2) are formally associated with the Noether current $j_\mu=\partial^\nu F_{\nu\mu}+\partial_\mu B$. Since $[j_\mu(x),B(y)]=0$ for all x and y, and $j_\mu(x)$ is local[10] relative to all Heisenberg fields in the theory, j_μ is a physical field. So the conclusion of the previous section is valid: There are zero-norm states in $\mathcal{H}_1$, (e.g., $B^{(-)}|0\rangle$ is a massless zero-norm state), and the physical symmetry must be trivial. As an illustration, ψ is not physical because it creates from the vacuum, e.g., unphysical states with a pair of massless χ quanta. However, $\psi_{\text{phys}}=\psi+(2v)^{-1}(\psi^2+\chi^2)$ is physical and is invariant under (2). Similarly, a physical vector field can also be constructed.

Because of the complex nature of subsidiary conditions in non-Abelian theories, we have not been able to ascertain whether the assumptions on physical fields and Noether currents hold. However, independently of this, formal considerations indicate in this case also the absence of a spontaneous breaking of physical symmetries.[11] From this viewpoint the situation is not analogous to that in chiral dynamics with massless pions[12] or that of ferromagnetism. Since in a formulation that deals only with physical fields there would be no SBS, one might regard the artifice of introducing unphysical states to be as much responsible for certain desirable features of gauge theories, such as possible renormalizability, as the spontaneous breaking of a symmetry involving unphysical fields.

This is not to say that the standard procedures in gauge theories with the Higgs-Kibble mechanism are not useful as convenient recipes for getting renormalizable vector-meson theories. However, it does suggest that perhaps a reverse procedure, namely, first introducing unphysical states to cancel divergences, and then fixing parameters to maintain the unitarity of the S matrix on the physical subspace, merits further investigation. If the parameters turn out to be fixed in a way that corresponds to a symmetry involving unphysical fields, the two procedures would be equivalent. But if there are additional solutions not corresponding to an unphysical symmetry, then the latter procedure opens new possibilities for model building.

Let us illustrate this procedure in the case of a neutral massive vector field. The first step is to find a starting Lagrangian L_0 for which the vector-meson propagator vanishes for large momentum like $1/k^2$ in momentum space. It has been shown by Nakanishi,[13] for instance, how this can be done with the introduction of an unphysical scalar field χ. L_0 is given by

$$L_0=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{m^2}{2}U_\mu U^\mu - B\partial_\mu U^\mu+\tfrac{1}{2}\alpha B^2 - mU^\mu\partial_\mu\chi+\tfrac{1}{2}\partial_\mu\chi\partial^\mu\chi\,.$$

The auxiliary field $B=(1/\alpha)\partial_\mu U^\mu$ satisfies a free equation, $\Box B=0$, and the physical subspace is specified by $B^{(+)}|\text{phys}\rangle=0$.

When interaction is added, to have renormalizability one may require the additional terms to be of renormalizable type in the sense of ordinary power counting, with the vector propagator counted as $1/k^2$ in momentum space. Then the only remaining problem is to ensure the unitarity of the S matrix in the physical subspace. A sufficient condition is to require that B still satisfy a free-field equation, so that physical states scatter into physical states.[14]

If the additional interaction is between the vector field and a scalar field ψ, one adds to L_0 the quantity $L_0'+L_I$, where $L_0'=(\mu^2/2)\psi^2+\frac{1}{2}\partial_\mu\psi\partial^\mu\psi$, $L_I=L_{\text{counter}}+L_1$, with $L_{\text{counter}}=a\psi+b\chi+c\psi^2+d\chi^2$, and the constants a and b are to be adjusted so that $\langle\psi\rangle_0$ and $\langle\chi\rangle_0$ vanish. Since

$$\partial_\mu F^{\mu\nu}-m^2U^\nu - m\partial^\nu B+\partial^\nu\chi = -\frac{\partial L_I}{\partial U_\nu}\,, \tag{3}$$

$$\Box\chi+m\partial_\mu U^\mu=\frac{\partial L_I}{\partial\chi}+\partial_\mu\frac{\partial L_I}{\partial(\partial_\mu\chi)}\,, \tag{4}$$

one finds that $\Box B=0$ if the following constraint is satisfied:

$$\partial_\mu\frac{\partial L_I}{\partial U_\mu}-m\frac{\partial L_I}{\partial\chi}+m\,\partial_\mu\frac{\partial L_I}{\partial(\partial_\mu\chi)}=0\,. \tag{5}$$

One can then write down the most general $L_1(U_\mu,\psi,\chi)$ containing terms with mass dimension greater than 2 and less than or equal to 4, with arbitrary coefficients. The requirement (5) fixes these coefficients to be exactly those of the Higgs Lagrangian (1).

Thus, in this case the reverse procedure recovers the Higgs model.[15] Whether, if one considers models with a richer spectrum, i.e., a larger number of fields, there exist additional solutions, not corresponding to gauge theories with SBS of unphysical symmetries, is an open question.[16]

Note added in proof. Dr. Swieca has informed us that if $j_\mu = \partial^\nu F_{\mu\nu}$ on the physical space and if $F_{\mu\nu}$ is local relative to itself, the absence of massless particles in the physical spectrum and asymptotic completeness would imply a vanishing charge on the physical space without assuming the existence of charged local fields, so that our conditions can be weakened.

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[1]P. W. Higgs, Phys. Rev. Lett. 12, 132 (1964); T. W. B. Kibble, Phys. Rev. 155, 1554 (1967).

[2]These assumptions are made only for the case when all gauge symmetries are eventually broken. To include an exact gauge symmetry such as that of quantum electrodynamics, modifications to this formulation will be needed.

[3]See D. Kastler, in *Proceedings of the 1967 International Conference on Particles and Fields,* edited by C. Hagen, G. Guralnik, and V. Mathew (Interscience, New York, 1967). For the Abelian model, a heuristic argument that the "charge" vanishes on the physical space has been given by B. Zumino, in Lectures at the 1972 Cargese Institute. Kibble (Ref. 1) has noted the absence of symmetry when suitable variables are used; see also Y. S. Kim and F. L. Markley, Nuovo Cimento 63A, 60 (1969).

[4]K. Fujikawa, B. W. Lee, and A. I. Sanda, Phys. Rev. D 6, 2923 (1972); Y.-P. Yao, *ibid.* 7, 1647 (1973).

[5]In contrast, an unphysical integer-spin field may anticommute with itself in some manifestly covariant and local formulations; see H. P. Dürr and E. Rudolph, Max Planck Institute report, 1972 (unpublished).

[6]See H. Reeh, Nuovo Cimento 51A, 638 (1967); H. Ezawa and J. A. Swieca, Commun. Math. Phys. 5, 330 (1967). References to the original papers may be found there. For extension to the case of an indefinite metric, see Ref. 8.

[7]For a more precise definition, see, e.g., Ref. 6.

[8]See Ref. 6, and also H. Reeh, Commun. Math. Phys. 14, 315 (1969), and his lecture at Haifa (1971) for a justification of this statement and the following one.

[9]P. Higgs, Phys. Rev. 145, 1156 (1966) and Ref. 1; B. W. Lee, Phys. Rev. D 5, 823 (1972).

[10]See, e.g., N. Nakanishi, Prog. Theor. Phys. 49, 640 (1973).

[11]To this end one may use the polar decomposition of field variables; see Kibble, Ref. 1, and Kim and Markley, Ref. 3.

[12]Furthermore, one sees that starting with a local gauge theory with only SU(3) symmetry, use of the Higgs-Kibble mechanism does not automatically lead to a description of the approximate SU(3) invariance of strong interactions. This explains why larger initial groups have been used in this connection [see, e.g., P. W. Higgs, Phys. Rev. 145, 1156 (1966); K. Bardakci and M. B. Halpern, Phys. Rev. D 6, 696 (1972)].

[13]N. Nakanishi, Phys. Rev. D 5, 1324 (1972).

[14]In other formulations the condition that $B(x)$ is a free field is not necessary and the procedure for ensuring unitarity must be generalized.

[15]Note that one could have added an additional mass term $M^2U^2/2$ to L_0, and still arrive at the same L_1 (with B satisfying a free Klein-Gordon equation instead). In this case one can recover ordinary massive quantum electrodynamics by setting $m = 0$ at the end.

[16]After completing this work we received a report [Phys. Rev. Lett. 30, 1268 (1973)] by J. M. Cornwall, D. N. Levin, and G. Tiktopoulos in which a somewhat different reverse procedure is used to recover a class of non-Abelian models.