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2^+ Nonet as Gauge Particles for SL(6,C) Symmetry

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We construct an SL(6,C) gauge-invariant Lagrangian which describes a nonet of massive positive-energy 2^+ particles. Of importance for the model are the concepts of covariant constraint and spontaneous symmetry breaking. The distinguishing feature of the present theory is the set of conserved currents which generate the algebra of SL(6,C). We also present gauge-invariant and unitarity-preserving quark and meson (quark-antiquark composite) Lagrangians.

I. INTRODUCTION

The recognition of gauge symmetries of the second kind among physical theories and the association of gauge particles with them has alternated between *internal* and *spin-containing* symmetries. First in this context was Weyl's recognition of the electromagnetic vector potential (with its associated helicity-one photon) as the gauge field corresponding to an internal symmetry, U(1). Second was the recognition by Weyl and by Fock and Ivanenko that the *vierbein* field (helicity-two graviton) was the gauge field corresponding to the spin-containing symmetry SL(2,C). Third was the association by Yang and Mills and by Shaw[1] of the spin-one isotriplet field with the internal symmetry, SU(2).

Continuing the discussion initiated in a recent note,[2] this paper is concerned with the construction of a model whose Lagrangian is invariant under SL(6,C) transformations of the second kind. This symmetry incorporates both the internal SU(3) and the spin-containing SL(2,C). In common with other gauge theories, this model involves a set of gauge fields with universal coupling. A new feature is the central role played by spontaneous symmetry breaking. This is absolutely necessary if the ghosts or infinite-dimensional multiplets implicit in an unbroken noncompact symmetry are to be avoided. One consequence of the spontaneous symmetry breaking is that we shall not be troubled by the presence of massless gauge particles. *Indeed, this scheme is intended as a symmetry of strong interactions.*

The main problem is to set up a suitable Lagrangian for the gauge fields and to analyze their particle content. In this paper we shall exhibit an SL(6,C)-invariant Lagrangian whose structure is such that only 2^+ states (singlet and octet) are caused to propagate and thereby represent the gauge degrees of freedom.

The symmetry group which we are using has the structure of a semidirect product

$$(P \otimes SU(3)) \cdot SL(6,C),$$

where P denotes the Poincaré group, and SU(3) the internal symmetry according to which physical states are classified. These are unbroken.

The remaining factor, $SL(6, C)$, is a gauge symmetry of the second kind and it is going to be spontaneously broken.

The spin–unitary–spin–containing $SL(6, C)$ transformations are of two kinds: (a) pure gauge transformations which approach the identity asymptotically and so do not affect state vectors, and (b) asymptotically rigid (or first kind) transformations which do affect the state vectors and which are spontaneously broken, i.e., which fail to leave the ground state invariant. It is necessary to distinguish between these two kinds of transformation since in order to compute anything one must begin by choosing a gauge, i.e., by violating the invariance with respect to the transformations (a). The transformations (b), on the other hand, need not be violated by the gauge-choosing mechanism. Their violation by the spontaneous mechanism is therefore a meaningful effect; the two mechanisms are logically distinct.

In this paper we shall deal with the classical equations of motion and their interpretation. Quantum effects are not considered. Thus, we shall look for a $P \otimes SU(3)$-invariant solution of the classical equations to represent the vacuum state. This solution will not be invariant under the rigid $SL(6, C)$ transformations which are therefore to be thought of as spontaneously violated. We next consider the effect of small perturbations about the ground-state solution. The propagation of these excitations in the linear approximation determines the bare-particle content of the system. We shall require that these perturbations carry positive energy and propagate with finite (less than light) velocities. To this extent the ground state is stable. In principle it would be possible to test this stability more deeply at the classical level by performing a complete canonical analysis and setting up the Hamiltonian. However, we shall not attempt this here.

The gauge field system involves, *a priori*, a large number of independent components. Many of these can be eliminated through the imposition of $SL(6, C)$ covariant constraints. Since the gauge symmetry must in any case be spontaneously broken, the imposition of such constraints will not lead to any further loss of symmetry. Their use is optional. We shall make heavy use of such constraints in order to simplify the structure of our Lagrangian.

The plan of the paper is as follows. In Sec. II the various fields are introduced, and the action of the group on them defined. A gauge-invariant Lagrangian is exhibited. [As a simple illustrative example the $SL(2, C)$ gauge-invariant Lagrangian of Weyl is given in Appendix A.] The notion of covariant constraint is introduced and gauge

conditions are discussed. In Sec. III the vacuum solution and bare-particle spectrum are obtained. (The same is done for an alternative Lagrangian in Appendix B.) Section IV contains some general remarks about the structure of the gauge system and what is likely to happen when interactions are taken into account. The extension to $SL(6, C)$ $\otimes SL(6, C)$ gauge symmetry is sketched briefly. Section V discusses the existence of conserved currents which close on the algebra of $SL(6, C)$. Section VI considers the interaction of the gauge fields with matter. It is suggested that, though the new theory is unitarity-preserving, the predictions of the old phenomenological $SL(6, C)$ theory may perhaps be expected to persist in the present theory.

II. AN SL(6,C)-INVARIANT LAGRANGIAN

We propose to set up a Lagrangian which is invariant under $SL(6, C)$ transformations of the second kind. Although we shall be concerned only with the gauge field part of the system, it is useful in establishing the notation to introduce a 12-component quark field ψ and its adjoint $\bar{\psi}$. These transform under the action of the full symmetry group

$$(P \otimes SU(3)) \cdot SL(6, C)$$

according to

$$\psi(x) \to \psi'(x') = a(\Lambda)\omega\,\Omega(x)\psi(x),$$
$$\bar{\psi}(x) \to \bar{\psi}'(x') = \bar{\psi}(x)\Omega^{-1}(x)\omega^{-1}a(\Lambda)^{-1}, \tag{2.1}$$

where $x'_\mu = \Lambda_{\mu\nu} x_\nu + b_\mu$ is a Poincaré transformation and the matrices $a(\Lambda)$, ω, and Ω are expressible by

$$a(\Lambda) = \exp[\tfrac{1}{4} i\,\theta_{\alpha\beta}(\Lambda)\sigma_{\alpha\beta}],$$
$$\omega = \exp(\tfrac{1}{2} i\,\omega^k \lambda^k), \tag{2.2}$$
$$\Omega(x) = \exp\{\tfrac{1}{2} i\,[\,\Omega^k(x) + \tfrac{1}{2}\Omega^k_{\alpha\beta}(x)\sigma_{\alpha\beta} + \Omega^k_5(x)\gamma_5\,]\lambda^k\},$$

with real parameters. The transformations (2.1) form a group in the sense that two successive actions combine according to the rule

$$a_1\omega_1\Omega_1 a_2\omega_2\Omega_2 = a_1 a_2\omega_1\omega_2\Omega'_1\Omega_2,$$

where Ω'_1 is given by

$$\Omega'_1 = a_2^{-1}\omega_2^{-1}\Omega_1 a_2\omega_2.$$

This rule is consistent only if the transformations $a^{-1}\omega^{-1}\Omega a\omega$ are themselves in $SL(6, C)$ (which of course they are).

The system of gauge fields to be used in constructing the Lagrangian comprises three distinct types, B_μ, S, and L_μ, which are expressed in the Dirac basis by

$$B_\mu = (B_\mu^k + \tfrac{1}{2} B_{\mu[\alpha\beta]}^k \, \sigma_{\alpha\beta} + B_{\mu5}^k \gamma_5)\tfrac{1}{2}\lambda^k \,,$$

$$S = \exp[\, i \, (P^k + \tfrac{1}{2} P_{[\alpha\beta]}^k \, \sigma_{\alpha\beta} + P_5^k \gamma_5)\tfrac{1}{2}\lambda^k] \,, \qquad (2.3)$$

$$L_\mu = (L_{\mu\,\alpha}^k \gamma_\alpha + L_{\mu\alpha5}^k i \, \gamma_\alpha\gamma_5)\lambda^k \,.$$

The components B_μ^k, $B_{\mu5}^k$, P^k, P_5^k $(k=1,2,\ldots,8)$, and $B_{\mu[\alpha\beta]}^k$, $P_{[\alpha\beta]}^k$, $L_{\mu\,\alpha}^k$, $L_{\mu\alpha5}^k$ $(k=0,1,\ldots,8)$ are all real.

The gauge fields transform according to

$$B_\mu \to \Omega B_\mu \Omega^{-1} - (1/i)\Omega \, \partial_\mu \Omega^{-1} \,,$$

$$S \to \Omega S \,, \qquad\qquad\qquad (2.4)$$

$$L_\mu \to \Omega L_\mu \Omega^{-1}$$

under the gauge group SL(6, C), and according to

$$B_\mu \to \Lambda_{\mu\nu} a\omega B_\nu a^{-1}\omega^{-1} \,,$$

$$S \to a\omega S a^{-1}\omega^{-1} \,, \qquad\qquad (2.5)$$

$$L_\mu \to \Lambda_{\mu\nu} a\omega L_\nu a^{-1}\omega^{-1}$$

under the asymptotic symmetry group P⊗SU(3).

The vector B_μ is the basic gauge field. It is used in the forming of covariant derivatives, viz.,

$$\nabla_\mu \psi = \partial_\mu \psi + i B_\mu \psi \,,$$

$$\nabla_\mu S = \partial_\mu S + i B_\mu S \,, \qquad\qquad (2.6)$$

$$\nabla_\mu L_\nu = \partial_\mu L_\nu + i \,[\, B_\mu, L_\nu] \,.$$

The analog of B_μ in Yang-Mills theory is the field which carries the gauge quanta and couples to the conserved isospin current. In gravitation theory B_μ plays the role of a connection but there it is essentially the derivative of the field which carries the gauge quanta. (It will turn out to have a similar status in the present model.)

The scalar, $S(x)$, is more subtle. If there were no gauge symmetry of the second kind but only the spontaneously broken *rigid* SL(6, C), then S would carry the Goldstone modes. These would be 70 massless excitations represented by the components $P(x)$ in (2.3). In systems where gauge symmetry of the second kind is present these Goldstone modes are not excited. More precisely, they are exactly compensated by the longitudinal modes in $B_\mu(x)$ insofar as gauge-independent quantities are concerned. They can appear in gauge-dependent quantities. [However, it is possible to choose a gauge in which $P(x) \equiv 0$ (that is, $S \equiv 1$) and the corresponding modes in $B_\mu(x)$ are suppressed. This is the so-called unitary gauge.] Notice that the components of S transform as bosons under the asymptotic symmetry P⊗SU(3), but as quarks under the gauge symmetry SL(6, C).

The third set of fields, $L_\mu(x)$, can be usefully introduced into the gauge system. They are not strictly necessary from the group-theoretical point of view, but they can be made to play the

role of canonically conjugate variables to the $B_\mu(x)$. It is just the existence of this possibility which distinguishes gauge theories based on a spin-containing symmetry, such as Weyl's SL(2, C) gauge-invariant *vierbein* version of gravity theory, from those, such as Maxwell or Yang-Mills theory, which are based on a purely internal symmetry. In being conjugate to the connection B_μ, the field L_μ has many resemblances to the *vierbein* field in general relativity theory. In fact, it will become the carrier of gauge quanta. Moreover, it will be made to have a nonvanishing value in the ground state,

$$\langle L_\mu \rangle = \gamma_\mu \,. \qquad\qquad (2.7)$$

This equation reflects the absence of SL(6, C) symmetry in the ground state. It is, of course, invariant under P⊗SU(3). In gravity theory the analogous equation would be

$$\langle L_{\mu\alpha} \rangle = \eta_{\mu\alpha} \,, \qquad\qquad (2.8)$$

where η denotes the Minkowski tensor. This equation (first introduced by Einstein, though not in this quantum formulation), while being Poincaré-invariant, exhibits the lack of SL(2, C) *gauge* invariance in the vacuum. (It also exhibits the breaking of the coordinate transformation group in Einstein's theory, but from our present point of view this is not the significance of this spontaneous symmetry breaking.)

Note that the covariant derivative of the *vierbein* field L_μ given by (2.6) cannot be made to vanish identically and here the resemblance to general relativity is lost. There is no analog of Riemannian geometry in the present scheme.

The variety of gauge-invariant Lagrangians which can be invented for the system, B_μ, S, and L_μ, is large. A fairly simple example, closest in form to the Einstein-Weyl Lagrangian for SL(2,C), is obtained by requiring that no more than four fields and two derivatives occur in each term. This is given by

$$\mathcal{L} = \tfrac{1}{8} \mathrm{Tr}\left[-\frac{1}{\kappa^2}(\nabla_\mu L_\nu \nabla_\nu L_\mu - \nabla_\mu L_\mu \nabla_\nu L_\nu) \right.$$

$$+ \alpha \, L_\mu (S\nabla_\nu S^{-1})L_\mu(S\nabla_\nu S^{-1})$$

$$\left. + \beta_1 L_\mu L_\mu + \beta_2 L_\mu L_\mu L_\nu L_\nu + \beta_3 L_\mu L_\nu L_\mu L_\nu \right],$$

$$(2.9)$$

in which Greek indices are to be saturated with the Minkowski tensor.[3] The real parameters α, β_1, β_2, β_3 are at present arbitrary. Our problem is to find suitable values for them. That is, we should like the Lagrangian (2.9) to yield a stable P⊗SU(3)-invariant vacuum in which a 2^+ massive nonet propagates.

Since the *vierbein* field is not playing a fundamental group-theoretic role in this system, we are free to reduce the number of independent components contained in it by imposing constraints. Consider the quantities, $l^k_{\mu\alpha}$ and $l^k_{\mu\alpha 5}$, defined by

$$S^{-1}L_\mu S = (l^k_{\mu\alpha}\gamma_\alpha + l^k_{\mu\alpha 5}\, i\,\gamma_\alpha\gamma_5)\lambda^k \ . \qquad (2.10)$$

This structure is *gauge-invariant*. Indeed, according to (2.3) and (2.6) we have

$$S^{-1}L_\mu S \to a\omega S^{-1}L_\mu S a^{-1}\omega^{-1} \ .$$

That is, $l^k_{\mu\alpha}$ is a nonet of second-rank tensors while $l^k_{\mu\alpha 5}$ is a nonet of pseudotensors with respect to $P\otimes SU(3)$.

To simplify the Lagrangian (2.9) and, what is more important, to ensure that the ground state value $\langle L_\mu \rangle = \gamma_\mu$ is a stable solution, we shall impose the constraints[4]

$$l^k_{\mu\alpha} - l^k_{\alpha\mu} = 0 , \qquad (2.11)$$

$$l^k_{\mu\alpha 5} = 0 . \qquad (2.11')$$

For the constraints (2.11′) to be realized, one can show that $\langle L_\mu \rangle = \gamma_\mu$ is a sufficient condition. In the unitary gauge these SL(6, C)-covariant constraints take the simple form

$$L^k_{\mu\alpha} - L^k_{\alpha\mu} = 0 ,$$

$$L^k_{\mu\alpha 5} = 0 .$$

It will be shown in Sec. III that the free-field approximation to the Lagrangian (2.9) reduces to the Pauli-Fierz[5] form for the following special values of the parameters:

$$\beta_1 = -3M^2/2\kappa^2 ,$$

$$\beta_2 = -\beta_3 = M^2/8\kappa^2 . \qquad (2.12)$$

III. VACUUM SOLUTION AND PARTICLE SPECTRUM

Having adopted the Lagrangian (2.10) and the constraints (2.11), our problem now is to determine the free parameters in such a way that a consistent perturbative scheme of stable solutions can be set up. This means, first, that the Euler-Lagrange equations should have a $P\otimes SU(3)$-invariant solution, and, second, that small perturbations about this solution should carry positive energy. We shall require, further, that these small excitations have the character of 2^+ nonet.

Because of the SL(6,C) gauge invariance, the classical equations of motion are underdetermined. They must be supplemented by a set of 70 gauge conditions. A convenient choice of gauge for the considerations which follow is given by $P(x)=0$ or, equivalently,

$$S(x)=1 . \qquad (3.1)$$

These conditions specify the so-called *unitary* gauge.[6] This gauge has the advantage that no zero-mass excitations appear in it. In other gauges such as, for example, the Landau gauge, $\partial_\mu B_\mu = 0$, zero-mass excitations do arise but only in gauge-dependent quantities.[7]

Rather than deal directly with the equations of motion, we shall require that the values

$$\langle L_\mu \rangle = \gamma_\mu , \quad \langle B_\mu \rangle = 0 , \quad \langle S \rangle = 1 \qquad (3.2)$$

represent an extremum of the action with respect to small variations. The parameters $\alpha, \beta_1, \beta_2, \beta_3$ must be adjusted so as to ensure this. The small variations must, of course, be compatible with the constraints (2.11) and with the gauge condition (3.1). Into the Lagrangian substitute the expressions

$$L_\mu = \gamma_\mu + \varphi^k_{\mu\alpha}\gamma_\alpha\lambda^k ,$$
$$B_\mu = (B^k_\mu + \tfrac{1}{2}B^k_{\mu[\alpha\beta]}\sigma_{\alpha\beta} + B^k_{\mu 5}\gamma_5)\tfrac{1}{2}\lambda^k , \qquad (3.3)$$

where $\varphi^k_{\mu\alpha} = \varphi^k_{\alpha\mu}$. Treat the components φ and B as small quantities and retain only the first- and second-order contributions. The single first-order term is proportional to $\varphi^0_{\mu\mu}$ and will vanish if we take

$$\beta_1 + 8\beta_2 - 4\beta_3 = 0 . \qquad (3.4)$$

The second-order terms then assume the form

$$\mathcal{L}_{(2)} = -\frac{2}{\kappa^2}B^k_{\mu[\nu\alpha]}(\varphi^k_{\mu\alpha,\nu} - \eta_{\mu\nu}\varphi^k_{\lambda\alpha,\lambda})$$
$$-\frac{1}{\kappa^2}(B^k_{\mu[\nu\alpha]}B^k_{\nu[\mu\alpha]} - B^k_{\mu[\mu\alpha]}B^k_{\nu[\nu\alpha]})$$
$$+\alpha(B^k_\mu B^k_\mu + B^k_{\mu 5}B^k_{\mu 5}) + 8\beta_3\varphi^k_{\mu\nu}\varphi^k_{\mu\nu}$$
$$+4(\beta_2 - \beta_3)\varphi^k_{\mu\mu}\varphi^k_{\nu\nu} . \qquad (3.5)$$

The excitation spectrum to which the Lagrangian (3.5) gives rise can best be analyzed by eliminating the algebraic variables B. That is, one solves the algebraic equation $\partial\mathcal{L}_{(2)}/\partial B = 0$ to obtain (provided $\alpha \neq 0$)

$$B^k_\mu = 0 ,$$
$$B^k_{\mu 5} = 0 , \qquad (3.6)$$
$$B^k_{\mu[\nu\alpha]} = \varphi^k_{\mu\nu,\alpha} - \varphi^k_{\mu\alpha,\nu} - \tfrac{1}{2}\eta_{\mu\nu}\varphi^k_{\lambda\lambda,\alpha} + \tfrac{1}{2}\eta_{\mu\alpha}\varphi^k_{\lambda\lambda,\nu} ,$$

and substitutes these expressions back into (3.5). The result is

$$\mathcal{L}_{(2)} = \frac{1}{\kappa^2}(\varphi^k_{\mu\nu,\alpha}\varphi^k_{\mu\nu,\alpha} - 2\varphi^k_{\alpha\mu,\mu}\varphi^k_{\alpha\nu,\nu} - \tfrac{1}{2}\varphi^k_{\mu\mu,\alpha}\varphi^k_{\nu\nu,\alpha})$$
$$+8\beta_3\varphi^k_{\mu\nu}\varphi^k_{\mu\nu} + 4(\beta_2 - \beta_3)\varphi^k_{\mu\mu}\varphi^k_{\nu\nu} . \qquad (3.7)$$

The values (2.12) have been chosen such that (3.7) takes the well-known Pauli-Fierz form

$$\mathcal{L}_{(2)} = \frac{1}{\kappa^2} \Big[\varphi^k_{\mu\nu,\alpha} \varphi^k_{\mu\nu,\alpha} - 2\varphi^k_{\alpha\mu,\mu}\varphi^k_{\alpha\nu,\nu} - \tfrac{1}{2}\varphi^k_{\mu\mu,\alpha}\varphi^k_{\nu\nu,\alpha}$$

$$- M^2(\varphi^k_{\mu\nu}\varphi^k_{\mu\nu} - \varphi^k_{\mu\mu}\varphi^k_{\nu\nu}) \Big] . \tag{3.8}$$

The parameter κ, which can be removed from the bilinear expression $\mathcal{L}_{(2)}$ by the rescaling $\varphi \to \kappa\varphi$, $B \to \kappa B$, is to be interpreted as the universal coupling constant of our gauge theory.

The arbitrary parameter α must not vanish. In the simpler gauge model in which SL(6,C) is replaced by SL(2,C) this parameter can vanish, in which case the Weyl-Einstein Lagrangian results.[2]

A different SL(6,C)-invariant Lagrangian, which also contains one free parameter (in addition to κ and M), is treated in Appendix B.

IV. INDEPENDENT DEGREES OF FREEDOM

Let us consider what happens to the first two equations of motion (3.6) when we take into account the complete Lagrangian $\mathcal{L}$, rather than just its bilinear part $\mathcal{L}_{(2)}$.

The fields B^k_μ and $B^k_{\mu5}$ no longer vanish as a consequence of the equations of motion. However, since the complete Lagrangian contains no derivatives of B_μ, the modified equations will simply express the fields B^k_μ and $B^k_{\mu5}$ as implicit but purely *algebraic* functions of the other fields in the theory ($\varphi^k_{\mu\nu}$ and $B^k_{\mu[\alpha\beta]}$). The two fields B^k_μ and $B^k_{\mu5}$ are thus algebraic composites of the other independent fields, with no propagation character of their own. Indeed, it was to guarantee this algebraic-composite character for B^k_μ and $B^k_{\mu5}$ that the S-field-containing term was introduced into the Lagrangian in the first place.

It is perfectly possible to introduce one simple additional term in the Lagrangian which *would* ensure a propagation of 1^- and 1^+ particles corresponding to these Yang-Mills-like fields B^k_μ and $B^k_{\mu5}$. The term in question has the same form as the third term in (2.9) except that $(S\nabla_\nu S^{-1})$ is replaced by $B_{\mu\nu}$; i.e., take

$$L_{\text{YM}} = \tfrac{1}{8}\alpha' \operatorname{Tr} L_\alpha B_{\mu\nu} L_\alpha B_{\mu\nu} , \tag{4.1}$$

where $B_{\mu\nu}$ denotes the covariant curl

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + i[B_\mu, B_\nu] . \tag{4.2}$$

The bilinear generated by (4.1) is proportional to

$$\alpha' \big[(\partial_\mu B^k_\nu - \partial_\nu B^k_\mu)^2 + (\partial_\mu B^k_{\nu5} - \partial_\nu B^k_{\mu5})^2 \big] . \tag{4.3}$$

The important point to observe is that (4.3) involves B^k_μ and $B^k_{\mu5}$ fields alone. There is *no* contribution to the spin-2^+ bilinears. This is not, of course, true of the trilinear and quadrilinear terms which arise in the interaction Lagrangian given by (4.1). These will in general involve the spin-2^+-describing fields $B^k_{\mu[ab]}$ as well as their

first derivatives in combinations like $(\varphi_{\alpha\beta})^2 \times (\partial_\nu B_{\mu[\alpha\beta]})^2$.

This appearance of first derivatives of $B^k_{\mu[ab]}$ is a new feature, peculiar to the Lagrangian (4.1). It is to be stressed that these derivatives do *not* appear in the bilinear terms, where their presence would signal ghosts, but only in the trilinear and quadrilinear interaction terms. Their existence must mean that $B^k_{\mu[ab]}$ are no longer algebraic variables canonically conjugate to the $\varphi^k_{\mu\nu}$'s, but are independent dynamical variables. The particle spectrum given by the bilinear terms would thus probably be altered. Also, the stability of the vacuum can no longer be taken for granted.

In view of these uncertainties, the safest procedure seems to be to discard (4.1). The theory as it stands will therefore *not* allow for the propagation of Yang-Mills-like[8] degrees of freedom B^k_μ and $B^k_{\mu5}$.

To summarize, of the eight sets of fields introduced into the theory (viz., $\varphi^k_{\mu\nu}$, $\varphi^k_{\mu\nu5}$, $B^k_{\mu[\alpha\beta]}$, B^k_μ, $B^k_{\mu5}$, $P^k_{\alpha\beta}$, P^k, and P^k_5) two sets ($\varphi^k_{\mu\nu5}$ and the antisymmetric parts of $\varphi^k_{\mu\nu}$) are removed by imposing SL(6,C) covariant constraints. Three sets, $P \equiv P^k_{\alpha\beta}$, P^k, and P^k_5, are eliminated by choice of a special gauge. Two further sets, B^k_μ and $B^k_{\mu5}$, are completely determined as algebraic functions of the rest, through the Lagrangian field equations. There then remain in the theory just the symmetric parts of $\varphi^k_{\mu\nu}$ ($\varphi^k_{\mu\nu} = \varphi^k_{\nu\mu}$) and their canonically conjugate variables $B^k_{\mu[\alpha\beta]}$. Our choice of the constants $\beta_1, \beta_2, \beta_3$ was dictated by the requirement that the bilinear part of the Lagrangian should coincide with the Pauli-Fierz Lagrangian describing the propagation of a nonet of 2^+ particles *only*.[9]

Does this requirement guarantee that in addition to these 2^+ degrees of freedom, represented in the rest frame by the traceless symmetric fields, $\varphi^k_{ij} - \tfrac{1}{3}\delta_{ij}\varphi^k_{mm}$, no further degrees of freedom [corresponding to rest-frame spin-one fields φ_{0i} ($= \varphi_{i0}$) and spin-zero fields φ^k_{00} and φ^k_{ii}] will ever get excited *as a consequence of the interaction terms* and propagate, for example, as *composite bound-state fields*?

It is clear that, in the absence of a higher-gauge symmetry affecting just these degrees of freedom, we cannot answer this question in any general manner. For Einstein's generally covariant theory for an SU(3) singlet helicity-two particle, general covariance does provide just the requisite higher symmetry GL(4,R) which guarantees that the spin-one φ_{0i} fields do not propagate. But for the zero-spin degrees of freedom φ_{00} and φ_{ii} (even in the Pauli-Fierz theory) there is no such higher symmetry, nor any guarantee of absence

of propagation, as has recently been stressed by Boulware and Deser.[10]

We conjecture that if the SL(6,C) symmetry of the present paper is extended to a still higher symmetry—possibly U(6,6)—we shall be able to construct a theory where the nonpropagation of all the unwanted degrees of freedom is guaranteed by the extended gauge invariance of the theory, rather than by the choosing of special values for the parameters. In this sense the theory presented in the present paper will need further elaboration.

One last remark: One can construct a theory of 2$^+$ and 2$^-$ nonets which utilize both $\varphi_{\mu\nu}^{(k)}$ and $\varphi_{\mu\nu 5}^k$ degrees of freedom by extending the gauge group to SL(6,C) $\otimes$ SL(6,C). The formalism is a simple extension of that presented in Sec. II with

$$\Omega(x) = \exp \tfrac{1}{2}i\left[\Omega^k(x) + \tfrac{1}{2}\Omega^k_{\alpha\beta}(x)\sigma_{\alpha\beta} + \Omega^k_5(x)\gamma_5\right]\lambda^k$$
$$\times \exp\left[\Omega'^k(x) + \tfrac{1}{2}\Omega'^k_{\alpha\beta}(x)\sigma_{\alpha\beta} + \Omega'^k_5(x)\gamma_5\right]\lambda^k .$$

$$(4.4)$$

Note that Ω is not pseudounitary; $\bar{\Omega}(x) \neq \Omega^{-1}(x)$. We introduce the two distinct gauge fields, B_μ and C_μ, and two sets of *vierbein* fields, L^1_μ and L^2_μ, which transform as

$$(B + iC)_\mu \rightarrow \Omega(B + iC)_\mu \Omega^{-1} - \frac{1}{i}\Omega\,\partial_\mu\Omega^{-1} , \qquad (4.5)$$

$$(B - iC)_\mu \rightarrow (\bar{\Omega})^{-1}(B - iC)_\mu \bar{\Omega} - \frac{1}{i}(\bar{\Omega})^{-1}\partial_\mu\bar{\Omega}, \qquad (4.6)$$

and

$$L^1_\mu \rightarrow \Omega L^1_\mu \bar{\Omega} , \qquad (4.7)$$

$$L^2_\mu \rightarrow (\bar{\Omega})^{-1}L^2_\mu \Omega^{-1} . \qquad (4.8)$$

We also introduce the field S, which is expressed in the Dirac basis by

$$S = \exp\left[i\,(P^k + \tfrac{1}{2}P^k_{\alpha\beta}\sigma_{\alpha\beta} + P^k_5\gamma_5)\lambda^k/2\right]$$
$$\times \exp\left[(Q^k + \tfrac{1}{2}Q^k_{\alpha\beta}\sigma_{\alpha\beta} + Q^k_5\gamma_5)\lambda^k/2\right] ,$$

which transforms according to $S \rightarrow \Omega S$. The covariant derivatives

$$\nabla_\mu L^1_\nu = \partial_\mu L^1_\nu + i[B_\mu, L^1_\nu] - \{C_\mu, L^1_\nu\} , \qquad (4.9)$$

$$\nabla_\mu L^2_\nu = \partial_\mu L^2_\nu + i[B_\mu, L^2_\nu] + \{C_\mu, L^2_\nu\} , \qquad (4.10)$$

transform like L^1_μ and L^2_μ, with quantities like $(\nabla_\mu L^1_\mu \nabla_\nu L^2_\mu)$ transforming as scalars under SL(6,C) $\otimes$ SL(6,C).

To replace the covariant constraint (2.11), we shall require that L^2_μ be expressible as a function L^1_μ and S. The new constraints analogous to (2.11$'$) are given by

$$S^{-1}L^1_\mu \bar{S}^{-1} = (l^k_{\mu\alpha}\gamma_\alpha + l^k_{\mu\alpha 5}i\,\gamma_\alpha\gamma_5)\lambda^k ,$$
$$\bar{S}\,L^2_\mu S = (l^k_{\mu\alpha}\gamma_\alpha - l^k_{\mu\alpha 5}i\,\gamma_\alpha\gamma_5)\lambda^k . \qquad (4.11)$$

Note that these constraints guarantee that in the expression

$$\tfrac{1}{8}\,\mathrm{Tr}L^1_\mu L^2_\mu = l^k_{\mu\alpha}l^k_{\mu\alpha} + l^k_{\mu\alpha 5}\,l^k_{\mu\alpha 5}$$

the two fields $l^k_{\mu\alpha}$ and $l^k_{\mu\alpha 5}$ occur symmetrically with the same metric. In addition, the larger gauge group SL(6,C) $\otimes$ SL(6, C) permits the imposition of constraints similar to (2.11):

$$l^k_{\mu\alpha} = l^k_{\alpha\mu} ,$$
$$l^k_{\mu\alpha 5} = l^k_{\alpha\mu 5} ,$$

which makes the symmetry between $l^k_{\mu\alpha}$ and $l^k_{\mu\alpha 5}$ even more piquant. It is an easy matter now to construct the required Lagrangian (replace $L_\mu L_\nu$ by $L^1_\mu L^2_\nu$ everywhere) which ensures the propagation of 2$^+$ and 2$^-$ nonets.

V. SL(6,C) CURRENT ALGEBRA

So far in this paper we have been dealing with purely field-theoretic matters. To conclude, we shall make a few brief remarks on the algebraic potential of this kind of model and, in particular, indicate how a set of conserved SL(6,C) currents could be constructed.

The first step is to define canonical momenta and impose the usual Poisson-bracket relations. Of course, when a Lagrangian carries a gauge symmetry of the second kind, the canonical momenta are not all independent. However, this problem is easily circumvented by the standard method of imposing a gauge-choosing mechanism; one merely adds to the gauge-invariant Lagrangian a Lagrange-multiplier term which breaks symmetry transformations of the second kind while respecting those of the first kind. In this way one can arrange that, for example, the canonical momenta

$$\Pi_\mu = \partial\mathcal{L}/\partial L_{\mu,0} \qquad (5.1)$$

are all independent. The infinitesimal (rigid) transformations,

$$\delta L_\mu = i\,[\delta\Omega, L_\mu] ,$$

are then generated (in the sense of Poisson brackets) by the functional

$$\delta G = \int d^3x\,\tfrac{1}{8}\,\mathrm{Tr}(i\delta\Omega[\,L_\mu,\Pi_\mu\,]) .$$

This suggests that a suitable definition for the current 4-vector would be

$$J_\nu = i\left[L_\mu, \frac{\partial\mathcal{L}}{\partial L_{\mu,\nu}}\right] + \cdots . \qquad (5.2)$$

(We have not written in all the independent contributions to this current. Each field whose time derivative appears in the Lagrangian would make such a contribution.) It is well known[11] that, for

gauge theories, currents constructed in this way either vanish when the equations of motion are used or take the form

$$J_\nu = \partial_\mu F_{\mu\nu}, \qquad (5.3)$$

where $F_{\mu\nu}$ is antisymmetric.[12] The form of $F_{\mu\nu}$ can be modified by adding to the original Lagrangian a 4-divergence which is variationally insignificant. Whatever the chosen form, however, one can be sure that, in view of their canonical derivation, the time components of these currents must satisfy the SL(6,C) algebra.

For illustration, we construct currents for the Weyl-Einstein Lagrangian which is SL(2,C)-gauge-invariant. In the Lagrangian (2.9), set $\alpha = \beta_1 = \beta_2 = \beta_3 = 0$ to obtain

$$\mathcal{L} = \tfrac14 \text{Tr}(\nabla_\mu L_\nu \nabla_\nu L_\mu - \nabla_\mu L_\mu \nabla_\nu L_\nu). \qquad (5.4)$$

A little algebra[2] then shows that it can be cast into the more familiar form

$$\mathcal{L} = \tfrac14 \text{Tr}\left(\frac{1}{i}[L_\mu, L_\nu]B_{\mu\nu}\right) + \text{surface term}$$

$$= \frac{1}{4i}\text{Tr}(2B_\nu \partial_\mu[L_\mu, L_\nu] - i[B_\mu, B_\nu][L_\mu, L_\nu])$$

$$+ \text{surface term}. \qquad (5.5)$$

Discarding the surface term, we compute the derivatives

$$\frac{\partial \mathcal{L}}{\partial B_\nu} = \frac{1}{i}\partial_\mu[L_\mu, L_\nu] + [B_\mu, [L_\mu, L_\nu]],$$

$$\frac{\partial \mathcal{L}}{\partial L_{\mu,\nu}} = \frac{1}{i}\eta_{\mu\nu}[L_\lambda, B_\lambda] + \frac{1}{i}[B_\mu, L_\nu].$$

According to (5.2),

$$J_\nu = [L_\nu, [L_\mu, B_\mu]] + [L_\mu, [B_\mu, L_\nu]]$$

$$= [B_\mu, [L_\mu, L_\nu]]. \qquad (5.6)$$

We adopt this as the definition of the current. One can easily verify that, barring Schwinger terms, it generates the algebra of SL(2, C). Note that it equals

$$\frac{\partial \mathcal{L}}{\partial B_\nu} - \frac{1}{i}\partial_\mu[L_\mu, L_\nu],$$

so that *when the field equations are satisfied* the first term vanishes and

$$J_\nu = -\frac{1}{i}\partial_\mu[L_\mu, L_\nu]. \qquad (5.7)$$

In this form J_ν is identically conserved. (Our derivation is incomplete, of course, because we have not explicitly shown how to overcome the gauge difficulties through inclusion of the contribution of the Lagrange multiplier.)

In connection with the matter field contributions to the current, it should be remarked that the variable which is canonically conjugate to the quark field ψ (for example) is not $\bar\psi\gamma_0$ but, rather, $\bar\psi L_0$. (The Poisson bracket of ψ and $\bar\psi$ is therefore quite complicated.) Nevertheless, the current retains the simple form $-L_\nu \psi\bar\psi$. That is, the SU(3) currents, in particular, take the form

$$\bar\psi L_\nu \tfrac12 \lambda^i \psi, \qquad (5.8)$$

which is obtained from the naive current by the simple replacement $\gamma_\nu = \langle L_\nu \rangle \to L_\nu$.

VI. SL(6,C) GAUGE-INVARIANT QUARK LAGRANGIAN AND SU(6) SYMMETRY

The SL(6, C) gauge-invariant quark Lagrangian is simple to write down:

$$\mathcal{L} = \tfrac12 i \bar\psi L_\mu \nabla_\mu \psi + \text{H.c.} - m\bar\psi\psi. \qquad (6.1)$$

Likewise, the Lagrangian describing a $\underline{35}$ of SU(6) interacting through the exchange of the gauge nonet of 2^+ gluons is given by

$$\mathcal{L} = \tfrac12 i \text{Tr}\Phi[L_\mu, \nabla_\mu \Phi] - m\Phi\Phi, \qquad (6.2)$$

where Φ is the second-rank multispinor. Both these are familiar Lagrangians from the Bargmann-Wigner[13] theory of SU(6) supermultiplets, except that the Dirac matrix γ_μ has been replaced by the field L_μ, and the ordinary derivative ∂_μ by the covariant derivative ∇_μ. This change, however, is decisive in that whereas previously the kinetic-energy terms in the multispinor Lagrangians were notoriously noninvariant for the SL(6, C) symmetry, the replacement of $\gamma_\mu = \langle L_\mu \rangle$ by L_μ and ∂_μ by ∇_μ makes the kinetic-energy terms also part of a gauge-invariant construct. A parallel change which occurs in the SL(6, C) currents has been noted in the last section. The variable canonically conjugate to ψ is not $\bar\psi\gamma_0$ but $\bar\psi(x)L_0(x)$, so that the correct (conserved) set of SL(6, C) quark currents involves replacing $\bar\psi\gamma_\mu$, in the earlier versions of SL(6, C) theory, by $\bar\psi(x)L_\mu(x)$.

The important question which arises here is this. We possess now an SL(6, C) gauge-invariant Lagrangian, describing a positive-frequency nonet of 2^+ particles; we also possess SL(6,C)-gauge-invariant Lagrangians describing quarks (6.1) and the quark-antiquark system (6.2). Presumably, with some effort one can construct gauge-invariant Lagrangians for higher SU(6) quark supermultiplets, where everywhere the Dirac matrix $\gamma_\mu = \langle L_\mu \rangle$ will be replaced by $L_\mu(x)$, and ∂_μ by ∇_μ. In contrast with the old SL(6, C) theory, in the new Lagrangian the kinetic-energy terms are part of an SL(6, C) gauge-invariant structure. The question is: How in practice does the availability of this unitarity-preserving SL(6, C)-gauge-invariant theory [or, equivalently, of a spin–unitary-spin-containing[14] algebra of SL(6, C) conserved cur-

rents] modify the earlier results of the phenom-
enological SL(6, C)? The answer seems to be:
hardly at all—except for the new couplings of the
2⁺ nonet with matter. The reason is that crucial
to our theory is the spontaneous symmetry-break-
ing mechanism of Einstein, which replaces $L_\mu(x)$
by $\langle L_\mu(x) \rangle = \gamma_\mu$ in the leading approximation. This
mechanism ensures that the physical particle
states are indeed Poincaré⊗SU(3) states. The
spontaneous symmetry-breaking mechanism im-
plies that the symmetry of the Lagrangian is not
reflected in the symmetry of the S matrix, which
for all quark multiplets still proceeds through the
familiar progression of the SU(6) residual sym-
metry for the one-particle states and the collinear
U(3)⊗U(3) for the residual symmetry of the ver-
tices. The only surprise in the situation is that the
purely gauge part of the Lagrangian described by
the L, B, and S fields gives rise to a multiplet of
pure spin-two particles, which constitutes only an
incomplete multiplet of the quark-based phenom-
enological SU(6). The situation here is completely
analogous to nonlinearly realized chiral theories,
which also display incomplete multiplets of the
larger symmetry group. Clearly the nonlinear
constraints like (2.11') or (4.11) are playing a role
in producing our incomplete multiplet.

VII. SUMMARY

The principal ideas set out in this paper may be
summarized as follows:

1. To describe a nonet of 2⁺ particles we pro-
pose the SL(6, C) gauge-invariant Lagrangian

$$\mathcal{L}_1 = \tfrac{1}{8} \operatorname{Tr}\left[-\frac{1}{\kappa^2}(\nabla_\mu L_\nu \nabla_\nu L_\mu - \nabla_\mu L_\mu \nabla_\nu L_\nu) \right.$$
$$ - \frac{3M^2}{2\kappa^2}(L_\mu L_\mu + \tfrac{1}{24}[L_\mu, L_\nu][L_\mu L_\nu]) $$
$$ \left. + \alpha L_\mu S \nabla_\nu S^{-1} L_\mu S \nabla_\nu S^{-1} \right] . $$

The fields L_μ and S satisfy the SL(6, C) covariant
constraints (2.11) and (2.11') of the text. Crucial
to the emergence of the particle spectrum is the
concept of spontaneous symmetry breaking which
is implemented through the relation $L_\mu = \gamma_\mu + \kappa \phi_\mu$
where $\langle L_\mu \rangle = \gamma_\mu$ and ϕ_μ is the physical field which
describes the propagation of 2⁺ particles.

2. The interaction of the gauge field with quarks
is governed by the Lagrangian

$$\mathcal{L}_2 = \tfrac{1}{2} i(\bar\psi L_\mu \nabla_\mu \psi - \nabla_\mu \bar\psi L_\mu \psi) - m\bar\psi \psi .$$

3. The Lagrangian which describes a 35 of SU(6)
and its interactions with the gauge fields is given
by

$$\mathcal{L}_3 = \operatorname{Tr}(\tfrac{1}{2} i \Phi [L_\mu, \nabla_\mu \Phi] - m\Phi\Phi) ,$$

where Φ is a second-rank multispinor. Similar
Lagrangians could possibly be written for higher-
rank multispinors.

4. A conserved set of currents for L, B, and
the quark field, which close on the algebra of
SL(6, C), is given by

$$J_\nu = \frac{1}{\kappa^2}[B_\mu, [L_\mu, L_\nu]] + \tfrac{1}{4}\{\psi\bar\psi, L_\nu\} - i\alpha L_\mu(S\nabla_\nu S^{-1})L_\mu .$$

5. The SL(6, C) gauge-invariant matter Lagran-
gians such as $\mathcal{L}_2$ and $\mathcal{L}_3$, which are constructed
from SL(6, C) multispinors, will describe the prop-
agation of multiplets of the rest symmetry SU(6).
Also, multispinor vertices [like $\operatorname{Tr}(\Phi\Phi\Phi)$] will ex-
hibit collinear SU(3)⊗SU(3) symmetry. However,
the propagation of the (nonlinearly realized) in-
complete SU(6) multiplet of 2⁺ gluons described by
L_μ and B_μ will break this symmetry, though in a
well-defined manner.

6. We do not wish to insist on the superiority of
any one SL(6, C) gauge-invariant Lagrangian over
another.

7. As is well known, the theory of spin-2 inter-
actions is not renormalizable in the conventional
sense. The spontaneous symmetry-breaking mech-
anism used here does not appear to change this
situation—unlike the case of spin-1 gauge theories.
Different (possible nonpolynomial Lagrangian)
methods will need to be developed to deal with the
ultraviolet infinites. We feel that the important
point to stress is that SL(6, C)—or indeed SL(6, C)
⊗SL(6, C) or U(6, 6)⊗U(6, 6)—gauge-invariant La-
grangians can be constructed and that (through the
operation of the spontaneous symmetry-breaking
mechanism) they describe propagation of positive
frequency particles. Corresponding to each one of
these Lagrangians there exists an algebra of con-
served currents closing on SL(6, C) or SL(6, C)
⊗SL(6, C), etc. It is the existence of such Lagran-
gians and such algebras—rather than their partic-
ular form—which may possibly be significant for
symmetry physics.

APPENDIX A

For completeness we summarize here the treat-
ment of SL(2, C) gauge invariance given in Ref. 2.
The SL(2, C) gauge transformations $\psi \to \Omega\psi$, are
respresentable in the form

$$\Omega(x) = \exp[\tfrac{1}{4} i\Omega_{\alpha\beta}(x)\sigma_{\alpha\beta}] .$$

The matrix Ω is pseudounitary, $\Omega^{-1} = \gamma_0 \Omega^+ \gamma_0$. The
gauge fields B_μ and L^μ are respresented in the
Dirac basis by

$$B_\mu = \tfrac{1}{2} B_{\mu[\alpha\beta]}\sigma_{\alpha\beta} ,$$
$$L^\mu = L^\mu_\alpha \gamma_\alpha .$$

These fields transform according to

$$B_\mu \to \Omega B_\mu \Omega^{-1} - \frac{1}{i}\Omega \partial_\mu \Omega^{-1},$$

$$L^\mu \to \Omega L^\mu \Omega^{-1}.$$

Covariant derivatives are formed as in the text. For example,

$$\nabla_\mu L^\nu = \partial_\mu L^\nu + i[B_\mu, L^\nu],$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + i[B_\mu, B_\nu].$$

The simplest SL(2, C) gauge-invariant Lagrangian for L and B is given by

$$i\,\mathrm{Tr}[L^\mu, L^\nu]B_{\mu\nu}. \tag{A1}$$

There is no need to introduce the Goldstone fields $S(x)$ in this case. Thus, the field values

$$\langle L^\mu(x)\rangle = \gamma^\mu, \qquad \langle B_\mu(x)\rangle = 0$$

are indeed a solution of the Euler-Lagrange equa-tions, and small perturbations on this solution can be shown to carry positive energy. The expres-sion (A1) is equivalent to the Palatini form of Einstein's Lagrangian. The remarkable fact about it is that it is also a scalar density under general coordinate transformations if we adopt the trans-formation rules

$$B_\mu(x) \to B'_\mu(x) = \frac{\partial x^\nu}{\partial x'^\mu}B_\nu(x),$$

$$L^\mu(x) \to L'^\mu(x) = \left|\det\frac{\partial x}{\partial x'}\right|^{1/2}\frac{\partial x'^\mu}{\partial x^\nu}L^\nu(x).$$

APPENDIX B

The Lagrangian (2.9) is not the only gauge-invariant one which gives rise to a pure massive 2^+ nonet. A different one-parameter family is arrived at by considering the expression

$$\mathcal{L} = \tfrac{1}{8}\,\mathrm{Tr}(a_1\nabla_\mu L_\nu \nabla_\nu L_\mu + a_2\nabla_\mu L_\mu \nabla_\nu L_\nu + a_3\nabla_\mu L_\nu \nabla_\mu L_\nu + 2a_4\nabla_\mu S^{-1}\{L_\mu, L_\nu\}\nabla_\nu S$$

$$+ 2a_5\nabla_\mu S^{-1}[L_\mu, L_\nu]\nabla_\nu S + a_6 L_\mu L_\mu + a_7 L_\mu L_\mu L_\nu L_\nu + a_8 L_\mu L_\nu L_\mu L_\nu). \tag{B1}$$

This Lagrangian also reduces to the Pauli-Fierz form in the free-field approximation if the follow-ing values for the parameters $a_1, \ldots, a_8$ are adopted:

$$a_1 = \frac{1}{\kappa^2}\frac{1-3\alpha}{2\alpha}, \qquad a_5 = -\frac{1}{\kappa^2}\frac{1-3\alpha}{1+3\alpha},$$

$$a_2 = \frac{1}{\kappa^2}\frac{1-3\alpha}{2}, \qquad a_6 = -\frac{3}{2}\frac{M^2}{\kappa^2},$$

$$a_3 = -\frac{1}{\kappa^2}\frac{1+\alpha}{2\alpha}, \qquad a_7 = \frac{1}{8}\frac{M^2}{\kappa^2}, \tag{B2}$$

$$a_4 = \frac{6}{\kappa^2}\frac{1+\alpha}{1+3\alpha}, \qquad a_8 = -\frac{1}{8}\frac{M^2}{\kappa^2}.$$

The components of B are given in this approxi-mation by

$$B^k_\mu = 0,$$

$$B^k_{\mu 5} = 0,$$

$$B^k_{\mu[\nu\alpha]} = -\frac{1+3\alpha}{1-3\alpha}(\varphi^k_{\mu\nu,\alpha} - \varphi^k_{\mu\alpha,\nu}) \tag{B3}$$

$$+\frac{\alpha}{1-3\alpha}(\eta_{\mu\nu}\varphi^k_{\lambda\lambda,\alpha} - \eta_{\mu\alpha}\varphi^k_{\lambda\lambda,\nu}),$$

provided $\alpha \neq -1, \tfrac{1}{3}, +1$.

[1] C. N. Yang and R. L. Mills, Phys. Rev. **96**, 191 (1954); R. Shaw, thesis, Cambridge University, 1954 (unpub-lished).

[2] C. J. Isham, Abdus Salam, and J. Strathdee, Nuovo Cimento Lett. **5**, 969 (1972).

[3] Although (2.9) is not, as it stands, invariant with re-spect to the general coordinate transformations of Einstein, this could be easily arranged. One could, for example, define a metric tensor

$$g_{\mu\nu} = \tfrac{1}{12}\,\mathrm{Tr}\,(L_\mu L_\nu)$$

with which to saturate Greek indices, and a Christoffel symbol $\{^\lambda_{\mu\nu}\}$ with which to form covariant derivatives. Typically,

$$\nabla_\mu L_\nu = \partial_\mu L_\nu + i[B_\mu, L_\nu] - \{^\lambda_{\mu\nu}\}L_\lambda.$$

However, we shall not pursue this refinement here.

[4] The importance of constraints in SL(6, C)-invariant theories (not gauged) was first recognized by F. Gürsey, in *International Symposium on Contemporary Physics, Trieste, Italy, 1968* (International Atomic Energy Agency, Vienna, Austria, 1968), Vol. 2.

[5] W. Pauli and M. Fierz, Proc. Roy. Soc. Lond. **A73**, 211 (1939).

[6] The unitary gauge has been discussed extensively in recent treatments of spontaneously broken gauge sym-metry. See, for example, T. W. B. Kibble, Phys. Rev. **155**, 1554 (1967); B. W. Lee and J. Zinn-Justin, Phys. Rev. D **5**, 3137 (1972); Abdus Salam and J. Strathdee, Nuovo Cimento **11A**, 397 (1972).

[7]The Landau gauge, however, does have the advantage that it respects the rigid SL(6, C) transformations whereas the unitary gauge does not. It is therefore much easier, for example, to define the currents which generate such transformations in the Landau gauge.

[8]We call these Yang-Mills-like degrees of freedom because the corresponding fields occur in the covariant and universal combination $\nabla_\mu = (\partial_\mu + iB_\mu)$.

[9]Of course, by choosing a different covariant Lagrangian, a different set of covariant constraints, and a different set of relations between the parameters, it may be possible to arrange that, besides the 2$^+$ nonet, other 1$^+$ and 0$^+$ particles may also propagate with positive frequencies, at least so far as the bilinear part of the Lagrangian is concerned. We have not attempted such extensions of the theory.

[10]D. G. Boulware and S. Deser, Seattle Report No. RLO-1388-825, 1972 (unpublished).

[11]A Trautman, in *Lectures on Particles and Field Theory*, 1964 Brandeis Summer Institute in Theoretical Physics, edited by S. Deser and K. W. Ford (Prentice-Hall, Englewood Cliffs, N. J., 1965).

[12]This means that the total SL(6, C) charges are represented by 2-dimensional surface integrals. Their matrix elements between physical states must therefore vanish since the gauge-dependent massless excitations cannot contribute. We have an algebra of currents but not of charges. This phenomenon is familiar in Yang-Mills-like theories where spontaneous symmetry breaking gives masses to the gauge particles. In such theories the "broken" charges are zero. [See B. Zumino, CERN Report No. Th 1550, 1972 (unpublished).] This is presumably in accord with Coleman's theorem. [However, by splitting off from the current those terms which depend on spontaneous symmetry-breaking parameters, it seems possible to define new (partially conserved) "currents" which do close on the algebra but do not yield zero charges. We do not elaborate on this here.]

[13]R. Delbourgo, M. A. Rashid, Abdus Salam, and J. Strathdee, in *Seminar on High-Energy Physics and Elementary Particles* (International Atomic Energy Agency, Vienna, Austria, 1965), p. 455. In the present paper the notation used is that of this reference. See also F. Gürsey, Ref. 4.

[14]It is important to stress that our group, which includes O(3, 1)⊗SL(6,C), contains spin in two places: "orbital" spin within the O(3, 1) subgroup of the Poincaré group P, typified by the 4-vector indices $\mu, \nu, \ldots$ attached to L_μ and B_μ, and also "intrinsic" spin within the SL(2, C) contained inside SL(6, C). [See also A. O. Barut, P. Budini, and C. Fronsdal, Proc. R. Soc. Lond. A291, 106 (1965).] In a crude sense, ours is the gauge theory of the O(3)⊗SU(6) particle-generating symmetry.

Unitarity in Gauge Theories with Nonlinear Gauge Conditions*

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We consider the Abelian gauge theories in which the transverse and the longitudinal photons are coupled. The unitarity relation holds because the unwanted absorptive part of the amplitude due to this coupling is canceled by the fictitious particle. It is pointed out that, by introducing the Lagrange multiplier, one can see the cancellations for all orders in perturbation expansions by looking at the equation of motion for the Lagrange multiplier. The possible implication of this on the massless Yang-Mills fields is indicated.

I. INTRODUCTION

The gauge field theories with nonlinear gauge conditions appear very much different from the usual formulation with linear gauge conditions. Let us take, for example, Abelian gauge theory. In the theory of electromagnetic interactions of charged particles with nonlinear gauge conditions (we shall call it the NGC theory), the longitudinal photon couples to the transverse photon. The new Feynman rules involve two gauge parameters, three new vertices, and a fictitious particle. The unitarity and the gauge invariance of the NGC theory can no longer be proved to all orders in the usual manner within the framework of canonical quan-

tization. Nevertheless, using the Feynman path integral one can still formally prove that the NGC theory is unitary and gauge-invariant in the same way as in the usual theory with linear gauge conditions.

It has been pointed out that the general properties of the S-matrix element formally proved by using the path integral may not be correct if the boundary terms due to integration by parts are not properly taken into account.[1] Therefore, we carry out explicit calculations of the fourth-order diagrams to show that the extra absorptive amplitude due to the new vertices is indeed canceled by the fictitious particle in any gauge.[2] Furthermore, by introducing the Lagrange multiplier $\chi(x)$, one can derive