ρ-Meson Properties in a Nonpolynomial Lagrangian Theory

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Two possible explanations of the existence and the properties of the ρ meson are analyzed within the framework of the nonpolynomial Lagrangian. It is found that the ρ meson bootstrapping reciprocally with the B meson in the static $\pi\omega$ scattering provides a qualitatively satisfactory description of the ρ-meson properties. On the other hand, the forces in the $\pi\pi$ scattering are far too weak to generate the ρ meson. The calculations are performed within the framework of the N/D method, and are free of a cutoff parameter.

I. INTRODUCTION

In a recent paper[1] it was argued that the nonpolynomial Lagrangian given by Weinberg[2] can provide a satisfactory description of at least the low-energy πN interaction. A static model calculation of πN scattering in the $I=\frac{1}{2}, \frac{3}{2}$ and $J=\frac{1}{2}, \frac{3}{2}$ channels was performed within the framework of the N/D method and the Bethe-Salpeter equation. As a consequence of including the multipion intermediate states implied by the nonpolynomial Lagrangian,[2,3] the theory did not require a cutoff and allowed a calculation of all the low-energy parameters of πN scattering, i.e., $\bar{N}N\pi$ and $\bar{N}N^*\pi$ coupling constants and N^*-N mass difference. The agreement of the results of these calculations with the experimental results is good enough to justify some amount of cautious optimism regarding the validity of the nonpolynomial Lagrangian description for the πN system.

It is certainly legitimate to inquire as to whether the same Lagrangian can yield a correct description of the ρ-meson parameters. Here, we are faced with two alternatives:

(1) The ρ and B (assumed to be a 2^- state) mesons evolve together,[4] from a reciprocal bootstrap mechanism, in the $\pi\omega$ channel, with ρ coming out as a bound state and B as a resonance state of the $\pi\omega$ system. This might be demonstrated within the framework of the static model.

(2) The ρ meson bootstraps itself[5] as a resonance state of the $\pi\pi$ system, justification for which would demand a relativistic calculation of $\pi\pi$ scattering, elastic or including $\pi\omega$ production.

The earlier attempts to solve this problem were hampered by the need for a cutoff, which prevented a detailed description of scattering and hence a definite commitment to either of the two above possibilities. However, with the development of the nonpolynomial Lagrangian techniques, the situation is more encouraging. It is now feasible to carry out calculations of $\pi\omega$ scattering, and of $\pi\pi$ scattering, without introducing a cutoff, and the resulting predictions, being more complete, would provide a less ambiguous description of the ρ-meson properties.

We first present a static model calculation of $\pi\omega$ scattering, in $J=1$ and 2 channels, including the multipion intermediate states within the framework of the nonpolynomial Lagrangian. The method of calculation is essentially the same as for πN system, with the ρ-meson exchange playing the role of the nucleon exchange and B-meson exchange that of the N^* exchange. However, the reciprocal bootstrap of the ρ, B system differs from that of N and N^* in that ρ is not one of the external particles. Therefore, because the two equations for the coupling constants are linearly dependent, we end up with three equations for four unknowns – two coupling constants and two masses. Nevertheless, these equations imply that, assuming the B meson to be a resonance, its mass must be less than 1350 MeV, the ρ meson is lighter than the B meson, and the renormalized $\pi\rho\omega$ coupling constant f as defined by Peierls[6] has the value

$$f^2/4\pi \approx 0.7,$$

which should be compared with the value of 0.45 obtained by a semiphenomenological analysis.[7] We also have the standard relation between the coupling constants of ρ and B mesons to the $\pi\omega$, i.e., $\gamma_{\rho\pi\omega} \approx \frac{5}{3}\gamma_{B\pi\omega}$.

These results are quite reasonable, but suggest that while the ρ and B mesons are essential for the existence of each other, the forces arising from their exchanges are not quite strong enough to produce a completely satisfactory closed reciprocal bootstrap. Perhaps the additional forces from the $\pi\pi$ channel may be sufficient. Unfortunately a coupled-channel relativistic-bootstrap calculation with nonpolynomial Lagrangian is very involved and we leave it for a more ambitious future effort.

However, as a first step towards a relativistic bootstrap, we do carry out an approximate calculation of $\pi\pi$ scattering in the p-wave state with the Weinberg interaction. The forces turn out to be far too weak to produce a reasonable ρ-meson resonance, lending support to our assertion that ρ is primarily composed of the $\pi\omega$ state, though the forces in the $\pi\pi\to\pi\omega$ process may contribute. This perhaps is the reason for the failure[5] of some of the early efforts to bootstrap the ρ meson without considering the B meson.

II. THE STATIC N/D EQUATIONS FOR $(n\pi)\omega\to(m\pi)\omega$

We carry out the reciprocal bootstrap calculation of the ρ and B mesons within the static N/D method. The techniques are similar to the ones used earlier[1] for the πN system. The $\pi\rho\omega$ interaction part of the chiral Lagrangian is described by the interaction Hamiltonian

$$H_{\rm int}=f\epsilon_{\mu\nu\alpha\beta}\frac{(\partial_\mu\omega_\nu)\vec{\rho}_\alpha\cdot(\partial_\beta\vec{\phi})}{1+a^2\phi^2} \tag{1}$$

where f is the renormalized $\pi\rho\omega$ coupling constant whose value is determined from a semiphenomenological analysis[7] of ω decay to be $f^2/4\pi=0.45$, $a=1/F_\pi=0.8/m_\pi$; and ω_ν, ρ_α, and ϕ describe the ω, ρ, and π fields, respectively. The term $\partial_\beta\phi$ will create or annihilate a p-wave pion, while the $a^2\phi^2$ terms create or annihilate s-wave pions.

We start with the unitarity relation for the scattering amplitude T for one static ω, one p-wave pion, and $2(r'+p'+n')$ s-wave pions going to one static ω, one p-wave pion, and $2(r+p+n)$ s-wave pions:

$$\begin{aligned}\mathrm{Im}T(\omega_0',\omega_1',\ldots,\omega'_{2(r'+p'+n')};\omega_0,\omega_1,\ldots,\omega_{2(r+p+n)})\\ =\sum_{R,P,N}\int T^*(\omega_0',\omega_1',\ldots,\omega'_{2(r'+p'+n')};\omega_0'',\omega_1'',\ldots,\omega''_{2(R+P+N)})\\ \times T(\omega_0'',\omega_1'',\ldots,\omega''_{2(R+P+N)};\omega_0,\omega_1,\ldots,\omega_{2(r+p+n)})\\ \times\frac{d^3q_0''d^3q_1''\cdots d^3q''_{2(R+P+N)}\omega_0{}^2\delta(\omega_0''+\omega_1''+\cdots+\omega''_{2(R+P+N)}-\omega)}{(2\omega_0'')(2\omega_1'')\cdots(2\omega''_{2(R+P+N)})(2\pi)^{6(R+P+N)}},\end{aligned} \tag{2}$$

where we have taken the pion mass $\mu=0$. The Born term from the exchange of the ρ or B meson in the u channel is of the form

$$T^B(\omega_0',\omega_1',\ldots,\omega'_{2(r'+p'+n')};\omega_0,\omega_1,\ldots,\omega_{2(r+p+n)})=\frac{\gamma^x F(r',p',n')F(r,p,n)}{m_x+\sum\omega_i'+\sum\omega_i-E}, \tag{3}$$

where

$$\begin{aligned}F(r,p,n)&=\frac{(r+p+n)!}{r!\,p!\,n!}[(2r)!(2p)!(2n)!]^{1/2}a^{2(r+p+n)},\\ \gamma^x&=\beta_{JJ'}\gamma_{J'},\end{aligned} \tag{4}$$

with m_x being the mass of the exchanged particle, $\gamma_{J'}$ being the coupling constant of the particle exchanged to the $\pi\omega$ system, and $\beta_{JJ'}$ being the crossing matrix for angular momentum:

$$\beta_{JJ'}=\frac{1}{6}\begin{pmatrix}2&-6&10\\-2&3&5\\2&3&1\end{pmatrix}. \tag{5}$$

The total energy is $E=m_\omega+\sum\omega_i$. We note that the Born term is a function only of $\sum\omega_i'$ and $\sum\omega_i$, and also that the r',p',n' and r,p,n dependence factorizes. These features are exploited by defining

$$T(\omega_0',\omega_1',\ldots,\omega'_{2(r'+p'+n')};\omega_0,\omega_1,\ldots,\omega_{2(r+p+n)})\equiv S(\omega',\omega)F(r',p',n')F(r,p,n), \tag{6}$$

where

$$\omega=\sum\omega_i,\qquad \omega'=\sum\omega_i'.$$

The factorizability of T is discussed in Ref. 1. Then the index-free amplitude $S(\omega,\omega)$ satisfies the unitarity relation

$$\mathrm{Im}S(\omega,\omega)=|S(\omega,\omega)|^2\rho(\omega)\,, \tag{7}$$

where

$$\rho(\omega)=\sum_{R,P,N}\rho_{2(R+P+N)}(\omega)\,\frac{(R+P+N)!(2R)!(2P)!(2N)!}{(R!P!N!)^2}\,a^{4(R+P+N)}\,, \tag{8}$$

$$\rho_l(\omega)=\int\frac{d^3q_0d^3q_1\cdots d^3q_l\,\omega_0{}^2\delta(\omega_0+\omega_1+\cdots+\omega_l-\omega)}{(2\omega_0)(2\omega_1)\cdots(2\omega_l)(2\pi)^{3l}} \tag{9}$$

The l-particle phase space can be written in a factorizable form by using the representation

$$\delta(\omega_0+\omega_1+\cdots+\omega_l-\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\exp[-it(\omega-\omega_0-\omega_1-\cdots-\omega_l)]dt\,,$$

so that

$$\rho_l(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-i\omega t}g_0(t)[g_1(t)]^l dt\,, \tag{10}$$

where

$$\begin{aligned}g_0(t)&=\int_0^\infty\omega_0{}^3e^{it\omega_0}d\omega_0\,,\\ g_1(t)&=\frac{1}{4\pi^2}\int_0^\infty\omega_1e^{it\omega_1}d\omega_1\,.\end{aligned} \tag{11}$$

With this form for $\rho_l(\omega)$, summation in (8) can be carried out by interchanging the order of summation and integration, so that

$$\begin{aligned}\rho(\omega)&=\frac{1}{2\pi}\int_{-\infty}^{\infty}dt\,e^{-it\omega}g_0(t)\sum_{R,P,N}\frac{[(R+P+N)!]^2(2R)!(2P)!(2N)!}{[R!P!N!]^2}[a^2g_1(t)]^{2(R+P+N)}\\ &=\frac{1}{2\pi}\int_{-\infty}^{\infty}dt\,e^{-it\omega}g_0(t)\sum_{n=0}^{\infty}(2n+1)![a^2g_1(t)]^{2n}\\ &=\frac{1}{2\pi}\int_{-\infty}^{\infty}dt\,e^{-it\omega}g_0(t)\int_0^\infty\frac{ue^{-u}du}{1-u^2a^4[g_1(t)]^2}\,.\end{aligned} \tag{12}$$

Of course the summation in (12) is only formal since $g_1(t)$ in (11) is not well defined. We give meaning to (12) by interchanging the order of integrations in (12) and redefining

$$\begin{aligned}g_0(t)&=\int_0^\infty\omega_0{}^3e^{it\omega_0}e^{-u^{1/2}\omega_0t_0}d\omega_0\\ &=\frac{6}{(t+iu^{1/2}t_0)^4}\,,\\ g_1(t)&=\frac{1}{4\pi^2}\int_0^\infty\omega_1e^{it\omega_1}e^{-u^{1/2}\omega_1t_0}d\omega_1\\ &=-\frac{1}{4\pi^2(t+iu^{1/2}t_0)^2}\,,\end{aligned} \tag{13}$$

and

$$\rho(\omega)=\lim_{t_0\to0}\frac{3}{\pi}\int_0^\infty ue^{-u}du\int_{-\infty}^{\infty}\frac{e^{-it\omega}dt}{(t+iu^{1/2}t_0)^4-u^2(a/2\pi)^4}\,. \tag{14}$$

It should be noted that the summation in (12) is legitimate, provided $t_0>a/2\pi$. This is later reflected by the scattering amplitude having a branch point at $t_0=a/2\pi$, requiring a careful choice of the physical branch.

We define

$$S(\omega)=N(\omega)/D(\omega)\,, \tag{15}$$

where $N(\omega)$ has only the left-hand singularities and $D(\omega)$ has the right-hand singularities. If $S_l(\omega)$ describes the left-hand singularities of $S(\omega)$, we can write

$$N(\omega)=\frac{1}{\pi}\int\frac{\mathrm{Im}S_l(\omega')D(\omega')}{\omega'-\omega}d\omega', \tag{16}$$

$$D(\omega)=1-\frac{\omega-\bar{\omega}}{\pi}\int_0^\infty\frac{\rho(\omega')N(\omega')d\omega'}{(\omega'-\bar{\omega})(\omega'-\omega-i\epsilon)}, \tag{17}$$

where $\bar{\omega}$ is the subtraction point. We analytically continue (17) from $t_0>a/2\pi$ to $t_0=0$, which with the proper choice of the branch leads to

$$D(\omega)=1-i\rho(\omega)N(\omega)\theta(\omega)-\frac{\omega-\bar{\omega}}{\pi}\,\mathrm{p.v.}\int_0^\infty d\omega'\left[\frac{N(\omega')\rho_1(\omega')}{(\omega'-\bar{\omega})(\omega'-\omega)}-\frac{N(-\omega')\rho_2(\omega')}{(\omega'+\bar{\omega})(\omega'+\omega)}\right], \tag{18}$$

where p.v. denotes "principal value" and

$$\rho(\omega)=\tfrac{3}{2}\int_0^\infty\frac{ue^{-u}}{\delta^3}(e^{\omega\delta}-e^{-\omega\delta}-2\sin\omega\delta)du, \tag{19}$$

with $\delta=(a/2\pi)u^{1/2}$, and

$$\rho_1(\omega)=-\tfrac{3}{2}\int_0^\infty\frac{ue^{-u}}{\delta^3}(2\sin\omega\delta+e^{-\omega\delta})du,$$
$$\rho_2(\omega)=\tfrac{3}{2}\int_0^\infty\frac{ue^{-u}}{\delta^3}e^{-\omega\delta}du. \tag{20}$$

Asymptotically the functions $\rho_i(\omega)$ go as $1/\omega$ so that Eq. (18) does not need a cutoff.

For $\pi\omega$ scattering in the $J=2$ channel, the contribution to S_l comes from ρ and B meson exchanges, which can be approximated as

$$\mathrm{Im}S_l(\omega')=-\tfrac{1}{2}(\pi\gamma)\delta(\omega'+\omega_\rho)-\tfrac{1}{6}(\pi\gamma^*)\delta(\omega'+\omega_B), \tag{21}$$

where $\gamma=\frac{1}{6}f^2$, γ^* is the $B\pi\omega$ coupling constant, $\omega_\rho=m_\rho-m_\omega$, and $\omega_B=m_B-m_\omega$. If we take $\bar{\omega}=-\omega_\rho$, we get

$$N(\omega)=\frac{\gamma}{2(\omega+\omega_\rho)}+\frac{\gamma^*D(-\omega_B)}{6(\omega+\omega_B)}. \tag{22}$$

With this expression for $N(\omega)$, the zeros of $D(\omega)$ can be studied and the B-meson mass and coupling constant can be obtained as a function of the input parameters. The relations are particularly simple for a linearized $D(\omega)=1+(\omega+\omega_\rho)c$ obtained by expanding $D(\omega)$ at $\omega=-\omega_\rho$,

$$\gamma^*=\tfrac{1}{2}\gamma+\tfrac{1}{6}\gamma^*, \tag{23}$$

$$1-(\omega_B+\omega_\rho)(6.3-1.5\omega_\rho)\left(\tfrac{1}{2}\gamma+\frac{\gamma^*\omega_B}{3(\omega_B+\omega_\rho)}\right)=0. \tag{24}$$

These calculations are repeated for the $J=1$ channel for which the contribution to S_l is given as

$$\mathrm{Im}S_l(\omega')=-\pi\tfrac{1}{6}5\gamma^*\delta(\omega'+\omega_B)-\tfrac{1}{2}\pi\gamma(\omega'+\omega_\rho), \tag{25}$$

which for $\bar{\omega}=-\omega_B$ leads to

$$N(\omega)=\frac{5\gamma^*}{6(\omega+\omega_B)}+\frac{\gamma D(-\omega_\rho)}{2(\omega+\omega_\rho)}. \tag{26}$$

The zero of the corresponding $D(\omega)$ is assumed to yield the ρ meson. Therefore, for a linearized $D(\omega)=1+(\omega+\omega_B)b$ obtained by expanding $D(\omega)$ at $\omega=-\omega_B$,

$$\gamma=\tfrac{1}{6}5\gamma^*+\tfrac{1}{2}\gamma, \tag{27}$$

$$1-(\omega_B+\omega_\rho)(6.3-1.5\omega_B)\left(\tfrac{1}{6}5\gamma^*+\frac{\gamma\omega_\rho}{\omega_B+\omega_\rho}\right)=0. \tag{28}$$

The results (23) and (27) are standard,[4] but (24) and (28) are new and are obtained since our theory does not require a cutoff.

Unfortunately, using $\gamma=\frac{1}{3}5\gamma^*$ from (23) and (27), we are left with two equations in 3 unknowns, γ, ω_ρ, and ω_B. We nevertheless can obtain some broad results. We find, for example, that for $m_B>m_\omega$, the value of γ is rather stable, and $m_\rho<m_B$. In particular, for $\omega_B\approx 2m_\pi$, we have

$$\omega_\rho\approx 1.2m_\pi, \tag{29}$$

and

$$\gamma=\tfrac{1}{6}(f^2/4\pi)\approx 0.11, \tag{30}$$

which should be compared with the semiphenomenological value[7] of 0.07. These results are quite reasonable but indicate that the forces for the ρ channel are not quite strong enough to produce the physical ρ meson. They suggest that ρ is primarily made up of $\pi\omega$. The additional forces required for a more satisfactory description of the ρ meson come, perhaps, from the $\pi\pi$ channel. However, it is beyond our present ability to incorporate this manifestly relativistic feature.

The effect of taking nonzero pion mass may be estimated from the conventional static theory with a cutoff. With a fixed cutoff of about $8m_\pi$, we find that increasing the pion mass from zero to m_π increases m_B by about 10%.

III. THE N/D EQUATION FOR $n\pi \to m\pi$

The $\pi\pi$ interaction is given by the Weinberg Hamiltonian[2]

$$H = -\frac{(\partial_\mu\vec{\phi})\cdot(\partial_\mu\vec{\phi})}{2(1+a^2\phi^2)^2} - \frac{m_\pi{}^2\phi^2}{2(1+a^2\phi^2)} - \tfrac{1}{2}m_\rho{}^2\left[\vec{\rho}_\mu + \frac{2a^2\vec{\phi}\times\partial_\mu\vec{\phi}}{g_0(1+a^2\phi^2)}\right]^2 . \qquad (31)$$

The contribution of the last term to $\pi\pi$ scattering is suppressed by the mutual cancellation[2] (the cancellation being exact in the zero-momentum-transfer limit) of the two terms in the bracket, while that of the second term is small because of the small pion mass. The leading part of the interaction, therefore, is the first term, and that is the only term we will consider for $n\pi \to m\pi$ scattering.

Similar to our earlier discussions, we consider the process of two p-wave pions and $2(r'+p'+n')$ pions in the s-wave state going to two p-wave pions and $2(r+p+n)$ pions in the s-wave state. We start with the unitarity relation

$$\begin{aligned}\mathrm{Im}T(q_1', q_2', p_1', \ldots, p'_{2(r'+p'+n')}; q_1, q_2, p_1, \ldots, p_{2(r+p+n)}) &= \tfrac{1}{2}\sum_{R,P,N}\int T^*(q_1', q_2', p_1', \ldots, p'_{2(r'+p'+n')}; q_1'', q_2'', p_1'', \ldots, p''_{2(R+P+N)}) \\ &\quad \times T(q_1'', q_2'', p_1'', \ldots, p''_{2(R+P+N)}; q_1, q_2, p_1, \ldots, p_{2(r+p+n)}) \\ &\quad \times \frac{d^3q_1''\,d^3q_2''\,d^3p_1''\cdots d^3p''_{2(R+P+N)}(2\pi)^4\delta^4(q_1''+q_2''+p_1''+\cdots+p''_{2(R+P+N)}-K)}{(2q_{10}'')(2q_{20}'')(2p_{10}'')\cdots(2\pi)^{3(2R+2P+2N+2)}} , \qquad (32)\end{aligned}$$

where

$$K = q_1+q_2+\sum p_i = q_1'+q_2'+\sum p_i' .$$

The $I=1$ Born term for the T matrix is

$$T_B^{I=1}(q_1', q_2', p_1', \ldots, p'_{2(r'+p'+n')}; q_1, q_2, p_1, \ldots, p_{2(r+p+n)}) = \tfrac{2}{3}a^2(q_1'-q_2')\cdot(q_1-q_2)J(r', p', n'; r, p, n), \qquad (33)$$

where

$$J(r', p', n'; r, p, n) = (a^2)^{r+r'+p+p'+n+n'}(r+p+n+r'+p'+n'+2)!\,\frac{[(2r)!(2r')!(2p)!(2p')!(2n)!(2n')!]^{1/2}}{r!r'!p!p'!n!n'!} . \qquad (34)$$

In order to simplify the problem we make the following factorization approximation:

$$\begin{aligned}&J(r', p', n'; r, p, n) \approx j(r', p', n')j(r, p, n), \\ &j(r, p, n) = [J(r, p, n; r, p, n)]^{1/2} .\end{aligned} \qquad (35)$$

This mutilates only the factor $(r+p+n+r'+p'+n'+2)!$ in (34) while leaving other terms unaffected. It is reasonable for r', p', n' near r, p, n, and plays the same simplifying role as the factorization of the potential, for example, in the Lippmann-Schwinger equation. We define

$$T(q_1', q_2', p_1', \ldots, p'_{2(r'+p'+n')}; q_1, q_2, p_1, \ldots, p_{2(r+p+n)}) = j(r', p', n')T(q_1', q_2'; q_1, q_2; K)j(r, p, n), \qquad (36)$$

which, with the use of the identity

$$(2\pi)^4\delta^4(q_1''+q_2''+p_1''+\cdots+p''_{2(R+P+N)}-K) = \int d^4x\, e^{ix(q_1''+q_2''+p_1''+\cdots+p_{2(R+P+N)}-K)} , \qquad (37)$$

leads to

$$\begin{aligned}\mathrm{Im}\tilde{T}(q_1', q_2'; q_1, q_2; K) &= \tfrac{1}{2}\int \frac{d^3q_1''\,d^3q_2''}{(2q_{10}'')(2q_{20}'')(2\pi)^6}\,\tilde{T}^*(q_1', q_2'; q_1'', q_2''; K) \\ &\quad \times \tilde{T}(q_1'', q_2''; q_1, q_2; K)\int d^4x\, e^{i(q_1''+q_2''-K)\cdot x}\int_0^\infty \frac{u^2e^{-u}\,du}{1-[2f(x)u]^2} , \qquad (38)\end{aligned}$$

where

$$f(x)=a^2\int\frac{d^3q''\,e^{iq''\cdot x}}{2q_0''(2\pi)^3}\,, \tag{39}$$

and we have used[1]

$$\begin{aligned}\sum_{R,P,N=0}^{\infty} f^{2(R+P+N)}\,\frac{(2R)!\,(2P)!\,(2N)!\,(2R+2P+2N+2)!}{[R!P!N!]^2} &= \sum_{n=0}^{\infty} f^{2n}(2n+1)!\,\frac{(2n+2)!}{n!\,n!}\\ &\approx \sum_{n=0}^{\infty}(2f)^{2n}(2n+2)!\\ &= \int_0^\infty \frac{u^2e^{-u}\,du}{1-(2fu)^2}\,. \end{aligned} \tag{40}$$

In order to give meaning to the summation in (40), we redefine

$$f(x)=\lim_{t_0\to 0} a^2\int\frac{d^3q''\,e^{iq''\cdot x}\,e^{-u^{1/2}q_0''t_0}}{2q_0''(2\pi)^3}\,, \tag{41}$$

which, for $m_\pi=0$, simplifies to

$$f(x)=\lim_{t_0\to 0}\frac{a^2}{4\pi^2[\vec{x}^2-(t+iu^{1/2}t_0)^2]}\,. \tag{42}$$

Our final approximation is to take

$$f(x)=\lim_{t_0\to 0}\left[-\frac{a^2}{4\pi^2(t+iu^{1/2}t_0)^2}\right], \tag{43}$$

which also guarantees that the s-wave mesons do not take away any angular momentum. This approximation is not expected to distort the analysis seriously, since the s-wave multipions bring about the most serious changes for large momenta, and therefore, by the uncertainty principle, small $\vec{x}$ values constitute the most important region. Furthermore the approximation does not change the asymptotic behavior in the Euclidean space.

The unitarity condition (38) in the center-of-mass frame reduces to

$$\begin{aligned}\mathrm{Im}\tilde{T}(q_1',q_2';q_1,q_2;K_0)=\tfrac{1}{2}\int&\frac{d^3q_1''\,d^3q_2''}{(2q_{10}'')(2q_{20}'')(2\pi)^3}\,\tilde{T}^*(q_1',q_2';q_1'',q_2'';K_0)\\ &\times\tilde{T}(q_1'',q_2'';q_1,q_2;K_0)\,\delta(\vec{q}_1''+\vec{q}_2'')\\ &\times\int dt\,e^{-it(K_0-q_{10}''-q_{20}'')}\int_0^\infty\frac{u^2e^{-u}(t+iu^{1/2}t_0)^4\,du}{(t+iu^{1/2}t_0)^4-(a^2u/2\pi^2)^2}\,.\end{aligned} \tag{44}$$

We project out the $I=1$ p-wave amplitude, which with the definition

$$\tilde{T}^{I=1}(q_1',q_2';q_1,q_2;K_0)=\vec{q}_1'\cdot\vec{q}_1R(s) \tag{45}$$

gives

$$\mathrm{Im}\,R(s)=\rho(K_0)\,|R(s)|^2\,, \tag{46}$$

where $s=K_0^{\,2}$ and

$$\begin{aligned}\rho(K_0)&=\lim_{t_0\to 0}\frac{1}{48(2\pi)^2}\int_{-\infty}^{\infty}dt\,e^{-itK_0}\int_0^\infty du\,\frac{u^2e^{-u}(t+iu^{1/2}t_0)}{(t+iu^{1/2}t_0)^4-(a^2u/2\pi^2)^2}\cdots\\ &=\lim_{t_0\to 0}\frac{1}{48(2\pi)^2}\int\frac{\pi}{2\delta^2}(e^{K_0\delta}+e^{-K_0\delta}-2\cos K_0\delta)u^2e^{-u}e^{-u^{1/2}t_0K_0}\,du\,,\end{aligned} \tag{47}$$

where

$$\delta=\frac{a}{\pi}(\tfrac{1}{2}u)^{1/2}\,.$$

The Born approximation for $R(s)$ is $\frac{1}{3}8a^2$, which leads us to write

$$R(s)=\frac{\frac{1}{3}8a^2}{D(s)}\,, \tag{48}$$

where

$$D(s)=1-\frac{8a^2}{3\pi}\int_0^\infty \frac{\rho(K_0')}{s'-s}ds'. \tag{49}$$

The $D(s)$ function is analytically continued from $t_0>(a/2^{1/2}\pi)$ to $t_0=0$, which with the proper choice of the branch leads to

$$D(s)=1-i\rho(K_0)\tfrac{1}{3}8a^2\theta(s)-\frac{8a^2}{3\pi}\int_0^\infty \frac{ds'}{s'-s}[\rho_1(K_0')+\rho_2(K_0')], \tag{50}$$

with

$$\rho_1(K_0)=\frac{1}{48(2\pi)^2}\int_0^\infty (du)u^2e^{-u}\,\frac{\pi}{2\delta^2}(e^{-K_0\delta}-2\cos K_0\delta),$$
$$\rho_2(K_0)=\frac{1}{48(2\pi)^2}\int_0^\infty (du)u^2e^{-u}\,\frac{\pi}{2\delta^2}\,e^{-K_0\delta}. \tag{51}$$

Asymptotically, the phase-space functions $\rho_i(K_0)$ go as $1/K_0$, so that our integral in (50) is finite. The $D(s)$ can be evaluated in a straightforward manner. However, it has no zeros in the region of interest. In particular, even though $D(s)$ decreases as s increases, it does so very slowly. For example, at $s=0$, it has a value of 0.90 and changes to 0.84 at $s\approx(5.5m_\pi)^2$. Within our framework, therefore, the ρ meson has little chance of coming out as an exhaustive resonance of the $\pi\pi$ interaction.

It is true that we have made serious approximations of factorizability (35) and ignoring $\bar{\tilde{x}}$ dependence in (42). Nevertheless we feel that our result, that the $\pi\pi$ interaction by itself contributes little to the existence of the ρ meson, is qualitatively correct within the framework of Weinberg's chiral interaction. It should be noted that though we have worked with the N/D method, the Bethe-Salpeter equation also leads to the same final equations and conclusions.

IV. CONCLUSIONS

We have examined the problem of generation of the ρ meson from $\pi\omega$ and $\pi\pi$ systems within the framework of the nonpolynomial Lagrangian. The reciprocal bootstrap of ρ and B mesons in static $\pi\omega$ scattering provides a very promising explanation of the ρ- and B-meson parameters. Since the theory does not need a cutoff, we are able to predict, in addition to the usual relation between the $\pi\rho\omega$ coupling and the $\pi B\omega$ coupling, the value of the $\pi\rho\omega$ coupling constant and the approximate masses of ρ and B mesons, which are in qualitative agreement with the empirical values. Some additional forces are needed for a more satisfactory description, perhaps from the coupling of the $\pi\omega$ to the $\pi\pi$ channel.

It should, however, be emphasized that this analysis is carried out under the assumption that the B meson is a 2^- particle, and would be invalid otherwise. Of course, if some other particle has these quantum numbers, and couples strongly to the $\pi\omega$ state, the analysis would go through with this particle replacing the B meson.

The relativistic calculation of $\pi\pi$ scattering requires some serious approximations, but the forces are too weak, even qualitatively, to yield the ρ meson. A coupled-channel calculation of $\pi\pi$ and $\pi\omega$ systems is beyond us for the present and we leave it for the future.

[1] S. H. Patil and S. K. Sharma, Phys. Rev. D 6, 2574 (1972).
[2] S. Weinberg, Phys. Rev. 166, 1568 (1968).
[3] S. Okubo, Progr. Theoret. Phys. (Kyoto) 11, 80 (1954); G. V. Efimov, Zh. Eksp. Teor. Fiz. 44, 2107 (1963) [Sov. Phys. JETP 17, 1417 (1963)]; E. S. Fradkin, Nucl. Phys. 49, 624 (1968); A. Salam and J. Strathdee, Phys. Rev. D 1, 3296 (1970); B. W. Lee and B. Zumino, Nucl. Phys. B13, 671 (1969); S. H. Patil, *ibid.* B38, 614 (1972).
[4] J. Franklin, Phys. Rev. 137, B995 (1965).
[5] F. Zachariasen and C. Zemach, Phys. Rev. 128, 849 (1963).
[6] R. F. Peierls, Phys. Rev. Letters 12, 50 (1964); 12, 119(E) (1964).
[7] M. Gell-Mann, D. Sharp, and W. D. Wagner, Phys. Rev. Letters 8, 261 (1962); W. R. Frazer, S. Patil, and H. L. Watson, *ibid.* 11, 231 (1963).