Veneziano-Type Amplitudes with Form Factors for πN Scattering

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The crossing-symmetric amplitude possessing Regge behavior for πN scattering is constructed using suitable combinations of beta-function terms and t-dependent form factors. The input parameters obtained from nucleon-pole and low-energy pion-nucleon resonances are being used to predict the charge-exchange polarizations, forward differential cross sections, and π^+p and π^-p backward differential cross sections. The theoretical predictions agree reasonably well with experimental data.

I. INTRODUCTION

The most attractive feature of a Veneziano-type amplitude[1] is that it can describe both the high-energy and the low-energy regions of any scattering process. The conventional high-energy Regge-pole amplitude describes the average behavior of the low-energy region.[2] In general, the Regge amplitude can successfully predict the large-s behavior for small t or small u only. The amplitude when extrapolated should give the observed properties of the t- and u-channel resonances. But the amplitude parametrized in terms of Regge poles does not show the observed behavior for any process of the low-energy region with the accuracy demanded from the principle of duality. The Veneziano-type expression for the amplitude can be expressed either in terms of low-energy resonance parameters of the s channel or in terms of residue functions of crossed channels. In principle, the resonance parameters alone should be able to predict the high-energy forward, backward differential cross sections, polarizations, etc. Several authors[3] have met with partial success with the Veneziano-type amplitude in the pion-nucleon system where extensive experimental information is available. In spite of good results at high s and small t predicted from the low-energy resonance parameters, the π^-p backward differential cross sections are predicted to be 2000 times larger than the observed values.[3] We have attempted here with the Veneziano-type amplitude to predict the observed π^-p backward differential cross sections, charge-exchange forward differential cross sections, polarizations, and π^+p backward differential cross sections. We have introduced t-dependent form factors to the Δ_δ trajectory consistent with the requirement of Regge asymptotic behavior, crossing symmetry, and poles at points corresponding to resonances.

In this investigation, the elastic scattering polarization is not considered at all. The polarization for any process is caused by the interference between the spin-flip and the spin-nonflip amplitudes. In the nonflip amplitudes, the dominating part comes from the "Pomeranchukon" amplitude. There is no "Pomeranchukon" amplitude in the flip amplitudes. In the dual approach to this investigation, the "Pomeranchukon" amplitude is absent. Therefore, the elastic polarization arises largely due to interference with a dominating "Pomeranchukon" amplitude in the nonflip amplitudes with the flip amplitudes. Since our present investigation uses the dual approach, the elastic polarization is not considered.

Usually, a Veneziano-type amplitude allows only constant residues for its representations. Since the constant residues for the Δ_δ resonance fail completely to predict backward differential cross sections for πN scattering, simple combinations of beta-function terms for such an amplitude are not adequate to predict simultaneously all data which are observed these days. Therefore, Veneziano-type amplitudes with constant residues require modifications for a full description of the real world. The minimum change we want to make for the amplitude without violating crossing symmetry and asymptotic behavior is to introduce exponentially t-dependent form factors for the residues of the Δ_δ resonance in order to predict simultaneously π^-p, π^+p backward differential cross sections, charge-exchange polarizations, and forward differential cross sections.

The aim of the present investigation is to correlate successfully the low-energy resonance parameters with the high-energy differential cross sections and polarizations. For this purpose, we have introduced two fermion trajectories N_α and Δ_δ. Since the contribution due to the N_γ trajectory[4] is insignificant according to the work of Barger and Olsson, we have not considered it here at all. The usual ρ and P' along with their daughters ρ' and P'' occur in our formulations of Veneziano-type beta-function terms. Both ρ and P' trajectories choose nonsense with no extra power of α. Another ρ_1 trajectory half a unit below the ρ trajectory is introduced here. Our work resembles to a great extent that of Fenster and Wali[3] except that we have introduced t-dependent form factors to Δ_δ trajectories.

In the u channel, the MacDowell symmetry relates $F^{(+)}(J, \sqrt{u})$ to $F^{(-)}(J, -\sqrt{u})$ such that $F^{(+)}(J, \sqrt{u}) = -F^{(-)}(J, -\sqrt{u})$. This symmetry can be continued for complex J and, therefore, places a significant restriction on u-channel Regge trajectories. The trajectories and residues of (+) and (−) poles are related by

$$\alpha^+(\sqrt{u}) = \alpha^-(-\sqrt{u})$$

and

$$\beta^+(\sqrt{u}) = -\beta^-(-\sqrt{u}) \ .$$

Nature does not seem to obey the MacDowell reflection symmetry. For the nucleon $N_\alpha(938, J^P = \frac{1}{2}^+)$, $N_\beta(938, J^P = \frac{1}{2}^-)$ should be observed due to MacDowell symmetry. Since there is no $\frac{1}{2}^-$ particle with mass $\simeq$ nucleon, we are left with no alternative but not to consider the N_β trajectory at all. In other words, one should assume N_β trajectory chooses nonsense at $\alpha^-(\sqrt{u}) = \frac{1}{2}$ in order that the residue $\gamma^-(\sqrt{u})$ vanish there. The five dominant $I = \frac{3}{2}$ resonances fall nicely on a Δ_δ trajectory with its lowest state $\Delta_\delta(1236, \frac{3}{2}^+)$ resonance. But there is no D_{33} state with mass $\simeq$1236 for the MacDowell reflection of the $P_{33}(1236)$. Therefore, we do not strictly consider the N_γ trajectory. Also the contributions due to N_γ trajectory[4] is insignificant according to the work of Barger and Olsson.

Since in the Veneziano representations the collisions between different Regge trajectories to produce cuts are not allowed, we are free to choose as many trajectories as we require to predict observed facts. The ρ_1 trajectory with the same quantum numbers of the usual ρ is essential to predict the charge-exchange (CEX) polarization data. Indeed, it is intended to reproduce some features of cuts in absorption fits translated into Regge-pole-plus-cut parametrizations. In the πN system, the ϕ-$(f - f')$ cut is equivalent to the ρ_1 trajectory in the Veneziano representations in some sense, although they play significant parts in different models.

In Sec. II, we construct the invariant amplitudes with suitable choice of beta-function terms. Section III deals with determination of parameters and numerical evaluation of the model.

II. CONSTRUCTION OF AMPLITUDE

To construct a Veneziano-type pion-nucleon scattering amplitude, one has to take care of $s \leftrightarrow u$ crossing symmetry. It is well known that

$$A^{(\pm)}(s, t, u) = \pm A^{(\pm)}(u, t, s) \ .$$

Regge asymptotic behavior for large s and fixed t gives

$$A(s, t) \to s^{\alpha(t)}, \quad B(s, t) \to s^{\alpha(t)-1} \ , \tag{1}$$

where $\alpha(t)$ is the appropriate exchanged meson trajectory, and for large s and fixed u gives

$$A(s, u) \to s^{\alpha(u)-1/2}, \quad B(s, u) \to s^{\alpha(u)-1/2} \ , \tag{2}$$

where $\alpha(u)$ refers to the exchanged fermion trajectory.

We are using the following notations for the Euler functions[3]:

$$B^{\pm}(n, \tfrac{1}{2}m) = \frac{\Gamma(n - \alpha(t))\,\Gamma(\frac{1}{2}m - \alpha_x(s))}{\Gamma(n + \frac{1}{2}m - \alpha(t) - \alpha_x(s))} \pm \frac{\Gamma(n - \alpha(t))\,\Gamma(\frac{1}{2}m - \alpha_x(u))}{\Gamma(n + \frac{1}{2}m - \alpha(t) - \alpha_x(u))} \ , \tag{3}$$

$$B^{\pm}(\tfrac{1}{2}m, \tfrac{1}{2}n) = \frac{\Gamma(\frac{1}{2}m - \alpha_x(s))\,\Gamma(\frac{1}{2}n - \alpha_x(u))}{\Gamma(\frac{1}{2}m + \frac{1}{2}n - \alpha_x(s) - \alpha_x(u))} \pm \frac{\Gamma(\frac{1}{2}m - \alpha_x(u))\,\Gamma(\frac{1}{2}n - \alpha_x(s))}{\Gamma(\frac{1}{2}m + \frac{1}{2}n - \alpha_x(u) - \alpha_x(s))} \ , \tag{4}$$

$$B_x(\tfrac{1}{2}m, \tfrac{1}{2}m) = \frac{\Gamma(\frac{1}{2}m - \alpha_x(s))\,\Gamma(\frac{1}{2}m - \alpha_x(u))}{\Gamma(m - \alpha_x(s) - \alpha_x(u))} \ . \tag{5}$$

Here x denotes the fermion trajectories N_α and Δ_δ. The C functions, where $C(x, y) = \Gamma(x)\Gamma(y)/\Gamma(x + y - 1)$, are defined in a similar fashion. Below, we write $A^{(-)}$, $A^{(+)}$, $B^{(-)}$, and $B^{(+)}$ which satisfy s-u crossing and display Regge behavior and have resonance poles on Regge trajectories.

$$\frac{A^{(-)}}{4\pi} = (\alpha_1^- - \theta_1^-)[C^-_{\rho N_\alpha}(1, \tfrac{3}{2}) + C^-_{N_\alpha}(\tfrac{3}{2}, \tfrac{5}{2})] + \theta_1^-[C^-_{\rho_1 N_\alpha}(1, \tfrac{3}{2}) + C^-_{N_\alpha}(\tfrac{3}{2}, \tfrac{5}{2})]$$
$$+ \left\{\left[d^-\left(1 + \frac{t}{2q^2}\right) + c^-\right] C^-_{\rho\Delta_\delta}(1, \tfrac{3}{2}) + c^- C^-_{\Delta_\delta}(\tfrac{3}{2}, \tfrac{5}{2}) - d^-\left(1 + \frac{t}{2q^2}\right) C^-_{\rho_1\Delta_\delta}(2, \tfrac{3}{2})\right\} e^{c(t - t^*)} \ , \tag{6}$$

$$\frac{A^{(+)}}{4\pi}=\alpha_1^+[C^+_{P'N_\alpha}(1,\tfrac{3}{2})+C_{N_\alpha}(\tfrac{3}{2},\tfrac{3}{2})]$$

$$+\left\{\left[d^+\left(1+\frac{t}{2q^2}\right)+c^+\right]C^+_{P'\Delta_\delta}(1,\tfrac{3}{2})-c^+C_{\Delta_\delta}(\tfrac{3}{2},\tfrac{3}{2})-d^+\left(1+\frac{t}{2q^2}\right)C^+_{P''\Delta_\delta}(2,\tfrac{3}{2})\right\}e^{D(t-t^*)}\ , \qquad (7)$$

$$\frac{B^{(-)}}{4\pi}=(B_1^- - \phi_1^-)[B^+_{\rho N}(1,\tfrac{1}{2})+B_N(\tfrac{1}{2},\tfrac{1}{2})]+\phi_1^-[B^+_{\rho_1 N_\alpha}(1,\tfrac{1}{2})+B_{N_\alpha}(\tfrac{1}{2},\tfrac{1}{2})]$$

$$+\left\{\left[a^-+b^-\left(1+\frac{t}{2q^2}\right)\right]B^+_{\rho'\Delta_\delta}(1,\tfrac{1}{2})+\left[b^-\left(1+\frac{t}{2q^2}\right)-a^-\right]B_{\Delta_\delta}(\tfrac{1}{2},\tfrac{1}{2})\right.$$

$$\left.-b^-\left(1+\frac{t}{2q^2}\right)[B^+_{\rho'\Delta_\delta}(2,\tfrac{1}{2})+B^+_{\Delta_\delta}(\tfrac{1}{2},\tfrac{3}{2})]\right\}e^{A(t-t^*)}\ , \qquad (8)$$

$$\frac{B^{(+)}}{4\pi}=B_1^+[B^-_{\rho N_\alpha}(1,\tfrac{1}{2})+B^-_{N_\alpha}(\tfrac{1}{2},\tfrac{3}{2})]$$

$$+\left\{\left[a^++b^+\left(1+\frac{t}{2q^2}\right)\right]B^-_{P'\Delta_\delta}(1,\tfrac{1}{2})-a^+B^-_{\Delta_\delta}(\tfrac{1}{2},\tfrac{3}{2})-b^+\left(1+\frac{t}{2q^2}\right)B^-_{P''\Delta_\delta}(2,\tfrac{1}{2})\right\}e^{B(t-t^*)}\ , \qquad (9)$$

where t^* is the value of t at the Δ_δ resonance and q is the s-channel center-of-mass momenta of the πN system. For large s and fixed t, we have

$$\frac{A^{(-)}}{4\pi}=(\theta_1^- - \alpha_1^-)\Gamma(1-\alpha_\rho(t))(bs)^{\alpha_\rho(t)}(1-e^{-i\pi\alpha_\rho(t)})-\theta_1^-\Gamma(1-\alpha_{\rho_1}(t))(bs)^{\alpha_{\rho_1}(t)}(1-e^{-i\pi\alpha_{\rho_1}(t)})$$

$$-\left[d^-\left(1+\frac{t}{2q^2}\right)+c^-\right]\Gamma(1-\alpha_\rho(t))(bs)^{\alpha_\rho(t)}(1-e^{-i\pi\alpha_\rho(t)})e^{c(t-t^*)}$$

$$+d^-\left(1+\frac{t}{2q^2}\right)\Gamma(2-\alpha_\rho(t))(bs)^{\alpha_\rho(t)-1}(1+e^{-i\pi\alpha_\rho(t)})e^{c(t-t^*)}\ , \qquad (10)$$

$$\frac{A^{(+)}}{4\pi}=\alpha_1^+\Gamma(1-\alpha_{P'}(t))(bs)^{\alpha_{P'}(t)}(1+e^{-i\pi\alpha_{P'}(t)})$$

$$+\left[d^+\left(1+\frac{t}{2q^2}\right)+c^+\right]\Gamma(1-\alpha_{P'}(t))(bs)^{\alpha_{P'}(t)}(1+e^{-i\pi\alpha_{P'}(t)})e^{D(t-t^*)}$$

$$-d^+\left(1+\frac{t}{2q^2}\right)\Gamma(2-\alpha_{P'}(t))(bs)^{\alpha_{P'}(t)-1}(1-e^{-i\pi\alpha_{P'}(t)})e^{D(t-t^*)}\ , \qquad (11)$$

$$\frac{B^{(-)}}{4\pi}=(\beta_1^- - \phi_1^-)\Gamma(1-\alpha_\rho(t))(bs)^{\alpha_\rho(t)-1}(1-e^{-i\pi\alpha_\rho(t)})+\phi_1^-\Gamma(1-\alpha_{\rho_1}(t))(bs)^{\alpha_{\rho 1}(t)-1}(1-e^{-i\pi\alpha_{\rho 1}(t)})$$

$$+\left[a^-+b^-\left(1+\frac{t}{2q^2}\right)\right]\Gamma(1-\alpha_\rho(t))(bs)^{\alpha_\rho(t)-1}(1-e^{-i\pi\alpha_\rho(t)})e^{A(t-t^*)}$$

$$-b^-\left(1+\frac{t}{2q^2}\right)\Gamma(2-\alpha_\rho(t))(bs)^{\alpha_\rho(t)-2}(1+e^{-i\pi\alpha_\rho(t)})e^{A(t-t^*)}\ , \qquad (12)$$

$$\frac{B^{(+)}}{4\pi}=-\beta_1^+\Gamma(1-\alpha_{P'}(t))(bs)^{\alpha_{P'}(t)-1}(1+e^{-i\pi\alpha_{P'}(t)})$$

$$-\left[a^++b^+\left(1+\frac{t}{2q^2}\right)\right]\Gamma(1-\alpha_{P'}(t))(bs)^{\alpha_{P'}(t)-1}(1+e^{-i\pi\alpha_{P'}(t)})e^{B(t-t^*)}$$

$$+b^+\left(1+\frac{t}{2q^2}\right)\Gamma(2-\alpha_{P'}(t))(bs)^{\alpha_{P'}(t)-2}(1-e^{-i\pi\alpha_{P'}(t)})e^{B(t-t^*)}\ . \qquad (13)$$

For large s and fixed u, we have

$$\frac{A^{(-)}}{4\pi} = -\alpha_1^- \Gamma(\tfrac{3}{2} - \alpha_{N_\alpha}(u))\,(bs)^{\alpha_{N\alpha}(u)-1/2}(1 + e^{-i\pi[\alpha_{N\alpha}(u)-1/2]})$$

$$- c^- \Gamma(\tfrac{3}{2} - \alpha_{\Delta_\delta}(u))\,(bs)^{\alpha_{\Delta\delta}(u)-1/2}(1 - e^{-i\pi[\alpha_{\Delta\delta}(u)-1/2]})e^{C(m_{\Delta\delta}{}^2 - s)}\ , \tag{14}$$

$$\frac{A^{(+)}}{4\pi} = \alpha_1^+ \Gamma(\tfrac{3}{2} - \alpha_{N_\alpha}(u))\,(bs)^{\alpha_{N\alpha}(u)-1/2}(1 + e^{-i\pi[\alpha_{N\alpha}(u)-1/2]})$$

$$+ c^+ \Gamma(\tfrac{3}{2} - \alpha_{\Delta_\alpha}(u))\,(bs)^{\alpha_{\Delta\delta}(u)-1/2}(1 - e^{-i\pi[\alpha_{\Delta\delta}(u)-1/2]})e^{D(m_{\Delta\delta}{}^2 - s)}\ , \tag{15}$$

$$\frac{B^{(-)}}{4\pi} = \beta_1^- \Gamma(\tfrac{1}{2} - \alpha_{N_\alpha}(u))\,(bs)^{\alpha_{N\alpha}(u)-1/2}(1 + e^{-i\pi[\alpha_{N\alpha}(u)-1/2]})$$

$$+ a^- \Gamma(\tfrac{1}{2} - \alpha_{\Delta_\delta}(u))\,(bs)^{\alpha_{\Delta\delta}(u)-1/2}(1 - e^{-i\pi[\alpha_{\Delta\delta}(u)-1/2]})e^{A(m_{\Delta\delta}{}^2 - s)}\ , \tag{16}$$

$$\frac{B^{(+)}}{4\pi} = -\beta_1^+ \Gamma(\tfrac{1}{2} - \alpha_{N_\alpha}(u))\,(bs)^{\alpha_{N\alpha}(u)-1/2}(1 + e^{-i\pi[\alpha_{N\alpha}(u)-1/2]})$$

$$- a^+ \Gamma(\tfrac{1}{2} - \alpha_{\Delta_\delta}(u))\,(bs)^{\alpha_{\Delta\delta}(u)-1/2}(1 - e^{-i\pi[\alpha_{\Delta\delta}(u)-1/2]})e^{B(m_{\Delta\delta}{}^2 - s)}\ . \tag{17}$$

III. EVALUATION OF CONSTANTS AND RESULTS

In this section, we discuss the evaluation of the parameters and compare our predictions with experimental data.

In $\pi^- p$ backward scattering, only Δ_δ resonance exchange takes place. Therefore it yields $\alpha_1^+ = \alpha_1^-$. The residues of β_1^- and β_1^+ can be expressed easily in terms of the pion-nucleon coupling constant g at the nucleon pole,

$$2\beta_1^- = 2\beta_1^+ = g^2\ (\text{GeV}/c)^{-2}\ .$$

The constants a^-, a^+, b^-, b^+, c^-, c^+, d^-, and d^+ can be evaluated from the residues of Δ_δ resonance from the expressions[5]

$$A^{3/2} = \frac{g^{*2}}{\mu^2}\, q^{*2}\left((m_{\Delta_\delta} + m)\cos\theta + \frac{\frac{1}{3}(E^* + m)(m_{\Delta\delta} - m)}{E^* - m}\right)\frac{1}{m_{\Delta\delta}{}^2 - s}\ ,$$

$$B^{3/2} = \frac{g^{*2}}{\mu^2}\, q^{*2}\left(\cos\theta - \frac{1}{3}\,\frac{E^* + m}{E^* - m}\right)\frac{1}{m_{\Delta\delta}{}^2 - s}\ , \tag{18}$$

where g^*/μ is the coupling constant of Δ_δ decaying in $N + \pi$. E^* is the energy of nucleon at Δ_δ resonance and q^* is the center-of-mass momentum of πN system at Δ_δ resonance. The α_{Δ_δ} trajectory is given by

$$\alpha_{\Delta_\delta}(u) = -0.03 + u\ . \tag{19}$$

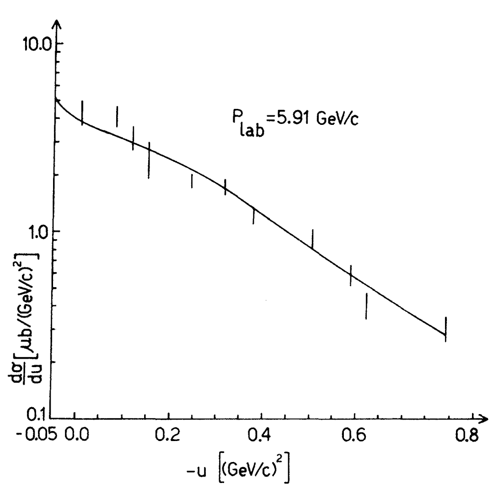

FIG. 1. Backward differential cross section for $\pi^- p \to \pi^- p$ at laboratory momentum 5.91 GeV/c. Data from Ref. 6.

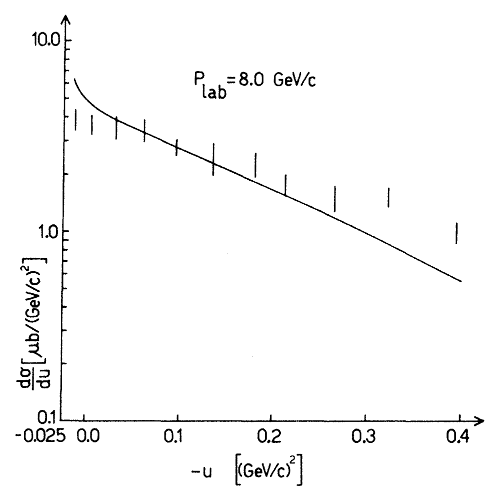

FIG. 2. Backward differential cross section for $\pi^- p \to \pi^- p$ at laboratory momentum 8.0 GeV/c. Data from Ref. 7.

The constant a's, b's, c's, and d's obtained from the residues of Δ_δ resonance are given below:

$$\begin{aligned} a^- &= -16.7978\ (\mathrm{GeV}/c)^{-2}, \\ a^+ &= -28.0065\ (\mathrm{GeV}/c)^{-2}, \\ b^- &= 1.62\ (\mathrm{GeV}/c)^{-2}, \\ b^+ &= 6.65\ (\mathrm{GeV}/c)^{-2}, \\ c^- &= 1.47\ (\mathrm{GeV}/c)^{-1}, \\ c^+ &= -8.35\ (\mathrm{GeV}/c)^{-1}, \\ d^- &= 0.4188\ (\mathrm{GeV}/c)^{-1}, \\ d^+ &= 0.3152\ (\mathrm{GeV}/c)^{-1}. \end{aligned} \tag{20}$$

In π^-p backward elastic scattering all the parameters are known except the parameters occurring in the form factors of Δ_δ resonance. At the Δ_δ resonance, these are equal to unity and, therefore, they cannot be determined. We want to evaluate the parameters A, B, C, and D from the best fit of χ^2 of backward π^-p differential cross sections. In backward elastic scattering, the invariant amplitudes B and A are linear combinations of $B^{(+)}(s,u)$, $B^{(-)}(s,u)$, $A^{(+)}(s,u)$, and $A^{(-)}(s,u)$ defined earlier:

$$B(\pi^-p\to p\pi^-) = B^{(+)}(s,u) + B^-(s,u)$$

and

$$A(\pi^-p\to p\pi^-) = A^{(+)}(s,u) + A^{(-)}(s,u)\,. \tag{21}$$

$$\left(\frac{d\sigma}{du}\right)_{\text{backward }\pi^-p} = \frac{\pi}{p^2}\left[\,|f_1(s,u)+f_2(s,u)\cos\theta|^2 + |f_2(s,u)\sin\theta|^2\right],$$

where

$$f_1(s,u) = \frac{E+m}{8\pi W}\{A^{(+)}(s,u)+A^{(-)}(s,u) + (W-m)[B^{(+)}(s,u)+B^{(-)}(s,u)]\}$$

and

$$f_2(s,u) = \frac{E+m}{8\pi W}\{-[A^{(+)}(s,u)+A^{(-)}(s,u)] + (W+m)[B^{(+)}(s,u)+B^{(-)}(s,u)]\}\,. \tag{22}$$

The π^-p backward differential cross sections are evaluated from best fit of χ^2 at laboratory momentum 5.91 GeV/c (Ref. 6) and 8.0 GeV/c (Ref. 7). The exponentially t-dependent parameters obtained, thereby, are $A\simeq 0.1\ (\mathrm{GeV}/c)^{-2}$, $B\simeq 0.15\ (\mathrm{GeV}/c)^{-2}$, $C\simeq 20.0\ (\mathrm{GeV}/c)^{-2}$, and $D\simeq 0.20\ (\mathrm{GeV}/c)^{-2}$. Figures 1 and 2 give quantitative features of predictions. For charge-exchange (CEX) πN forward scattering, we have

$$\begin{aligned} B_{\mathrm{CEX}}(s,t) &= \sqrt{2}\,B^{(-)}(s,t)\,, \\ A_{\mathrm{CEX}}(s,t) &= \sqrt{2}\,A^{(-)}(s,t)\,. \end{aligned} \tag{23}$$

Here

$$\left(\frac{d\sigma}{dt}\right)_{\mathrm{CEX}} = \frac{\pi}{p^2}\left[\,|f(s,t)_{\mathrm{CEX}}|^2 + |g(s,t)_{\mathrm{CEX}}|^2\right]$$

and

$$\vec{\mathrm{p}} = \frac{2\,\mathrm{Im}[f^*(s,t)g(s,t)]_{\mathrm{CEX}}}{|f(s,t)_{\mathrm{CEX}}|^2 + |g(s,t)_{\mathrm{CEX}}|^2}\,\hat{n}\,, \tag{24}$$

where $\hat{n}$ is the unit vector perpendicular to the plane of scattering. We have defined $f(s,t)$ and $g(s,t)$ in the following way:

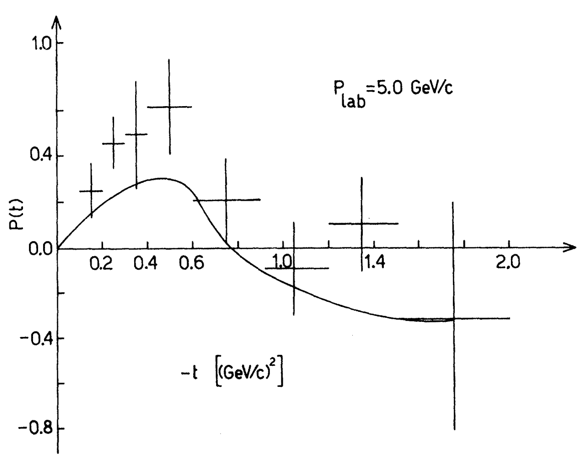

FIG. 3. Polarization of the recoil neutron in $\pi^-p\to\pi^0n$ at laboratory momentum 5.0 GeV/c. Data from Ref. 8.

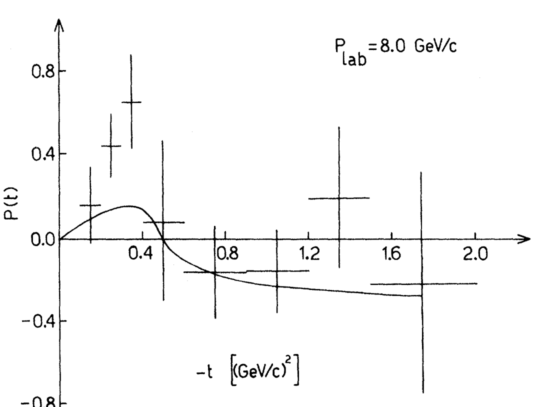

FIG. 4. Polarization of the recoil neutron in $\pi^-p\to\pi^0n$ at laboratory momentum 8.0 GeV/c. Data from Ref. 8.

$$f(s,t)=f_1(s,t)+f_2(s,t)\cos\theta$$

and (25)

$$g(s,t)=f_2(s,t)\sin\theta \ .$$

The only unknown parameters occuring in πN forward charge-exchange differential cross sections and polarizations are α_1^-, θ_1^-, and ϕ_1^-. We want to evaluate them in a similar fashion from the best fit of χ^2. They turned out to be

$$\alpha_1^- \simeq -2.13\ (\text{GeV}/c)^{-1}\ ,$$

$$\theta_1^- \simeq -0.64\ (\text{GeV}/c)^{-1}\ , \qquad (26)$$

$$\phi_1^- \simeq -8.05\ (\text{GeV}/c)^{-2}\ .$$

Figures 3–7 compare our predictions with the experimental data.

Finally in π^+p backward differential cross sections, we have no adjustable parameters to fit in. We predict π^+p backward differential cross sections in Figs. 8 and 9. The α_{N_α} trajectory is

$$\alpha_{N_\alpha}(u) = -0.38 + u \ .$$

Except in the neighborhood of $u=0$, our model can predict the backward differential cross sections of both π^-p and π^+p processes accurately and consistently with the experimental data. The qualitative features of the polarizations are exhibited over a wide range of t. Our work cannot reproduce the large polarizations observed recently.[8] Our model, however, has been able to describe all the processes of πN scattering with reasonable success.

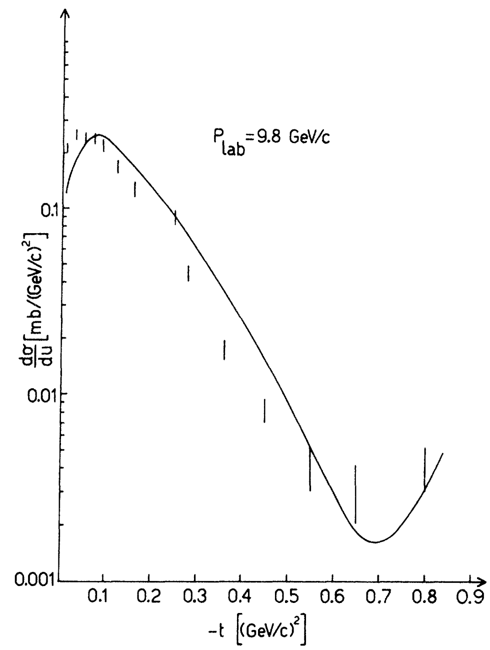

FIG. 6. Differential cross section for $\pi^-p\to\pi^0n$ at laboratory momentum 9.8 GeV/c. Data from Ref. 9.

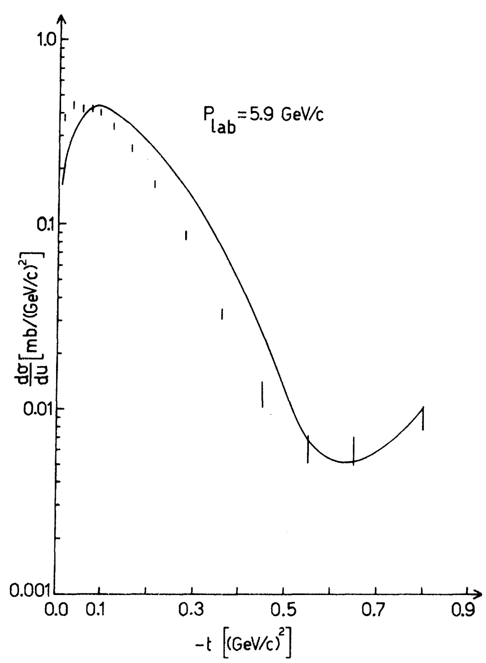

FIG. 5. Differential cross section for $\pi^-p\to\pi^0n$ at laboratory momentum 5.9 GeV/c. Data from Ref. 9.

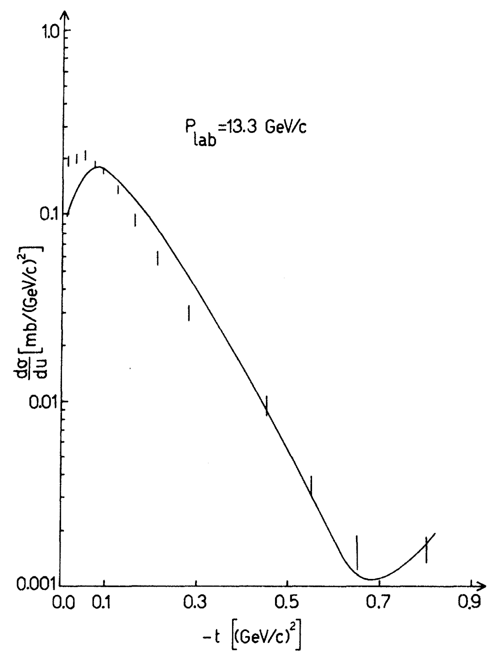

FIG. 7. Differential cross section for $\pi^-p\to\pi^0n$ at laboratory momentum 13.3 GeV/c. Data from Ref. 9.

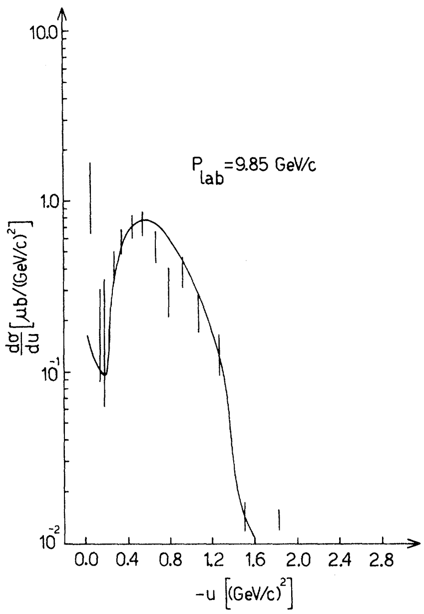

FIG. 8. Backward differential cross section for $\pi^+p \to \pi^+p$ at laboratory momentum 9.85 GeV/c. Data from Ref. 10.

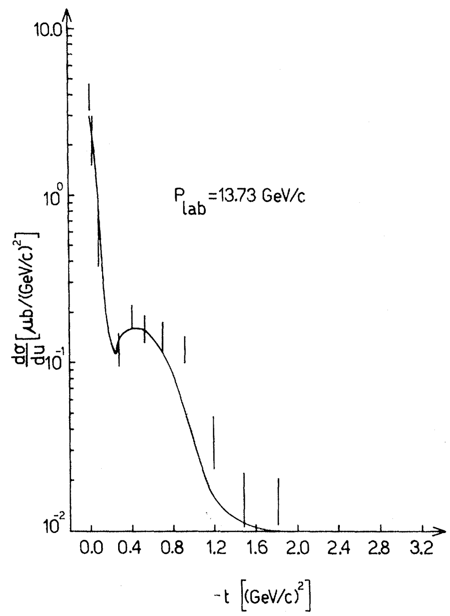

FIG. 9. Backward differential cross section for $\pi^+p \to \pi^+p$ at laboratory momentum 13.73 GeV/c. Data from Ref. 10.

IV. CONCLUSIONS

We have studied in this paper the modified Veneziano-type amplitude for πN scattering. In our model, we have introduced daughter trajectories of ρ and P'. Since the daughter trajectories are parallel to their parent trajectories and lie one unit below their parents, we have simplified our notations in the asymptotic limit to denote them in terms of their parents. The signatures of the daughter trajectories are of opposite sign. A second trajectory of ρ quantum numbers half a unit below ρ called ρ_1 trajectory together with ρ prevents the vanishing of charge-exchange differential cross sections when $\alpha_\rho(t)$ goes through zero. Furthermore, the ρ and ρ_1 together provide an explanation for the nonzero polarizations for $\pi^-p \to \pi^0 n$.

The simple combinations of beta functions with constant-residue parameters satisfy the principle of duality. Earlier attempts to describe π^-p backward differential cross sections failed completely for such a combination of beta functions with constant residues (without any form factors). Any phenomenological model should be partially successful to describe all processes simultaneously. In order to satisfy this requirement, we believe that the Veneziano-type amplitude should be modified for fuller description of nature. Without violating the crossing symmetry and Regge asymptotic behavior, we have made minimum modifications for our amplitude by introducing t-dependent form factors for the residues of Δ_δ resonance only. Conventional Veneziano-type amplitude does not allow such a modification, but this seems to work for our model to describe all processes of πN scattering.

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