PHYSICAL REVIEW D VOLUME 8, NUMBER 8 15 OCTOBER 1973

Dynamical Model of Spontaneously Broken Gauge Symmetries*

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(Received 3 May 1973)

We demonstrate how a theory consisting of massless Fermi and vector-meson fields can lead to an excitation spectrum of solely massive particles. We eschew spinless Bose fields in the fundamental Lagrangian, contrary to current practice. A detailed model is presented and solved in lowest order. Fermion and axial-vector-meson masses are spontaneously generated, and the vector particle's mass is computed in terms of the fermion mass.

I. INTRODUCTION

An attractive idea for field-theoretic models involving massive vector mesons is that the mass arises from spontaneous breakdown of a gauge symmetry. Indeed the currect activity in the theory of weak interactions is centered around such a possibility.[1] In these examples, both fermions and vector mesons become massive because a canonical scalar field, already present in the Lagrangian, acquires a symmetry-violating vacuum expectation value. In a now familiar fashion, this vacuum expectation value immediately leads to a fermion mass and a massless excitation. The massless excitation, however, does not correspond to a particle, but rather combines with a massless vector gauge field to produce a massive vector-meson particle. For brevity we shall refer to this phenomenon as the Higgs mechanism.[2]

In this paper we examine the possibility that masses of fermions and vector mesons can arise spontaneously, *without* the presence of canonical scalar fields in the Lagrangian.[3] Thus we are extending the work of Nambu and Jona-Lasinio,[4] who showed that the Goldstone mechanism can take place even when the Lagrangian does not include spin-zero fields.

The fundamental reason why an apparently massless vector meson acquires a mass was given a decade ago by Schwinger.[5] The reason is that the vacuum polarization tensor, that is, the proper two-point correlation function of the conserved current J^μ to which the meson couples with strength g, acquires a pole at zero momentum transfer. To see this explicitly, consider the complete vector-meson propagator,

$$D^{\mu\nu}(q) = -i\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) D(q^2),$$

which is given by[6]

$$D^{\mu\nu}(q) = -i\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\frac{1}{q^2 - q^2\Pi(q^2)},$$

where

$$\Pi^{\mu\nu}(q) = (g^{\mu\nu}q^2 - q^\mu q^\nu)i\Pi(q^2) = -g^2\int d^4x\, e^{iqx}\langle 0|T^*J^\mu(x)J^\nu(0)|0\rangle|_{\text{pp}}. \quad (1.1)$$

Here the subscript pp indicates the proper part; i.e., only one-vector-meson-irreducible graphs contribute to $\Pi^{\mu\nu}(q)$. [We insist that $\Pi^{\mu\nu}(q)$ be transverse, since the current is conserved; this entails a judicious choice of seagull terms.] Clearly, if $\Pi(q^2)$ has a pole at $q^2=0$, the vector meson is massive, even though it is massless in the absence of interactions ($g=0$, $\Pi=0$). We shall call this the Schwinger mechanism.

There is no physical principle which would prevent $\Pi(q^2)$ from acquiring a pole. Indeed, for massless spinor electrodynamics in two dimensions,[7] Schwinger found that $\Pi(q^2)$ *does* acquire a pole, as can be seen on purely dimensional grounds: In that theory g has units of mass. (The occurrence of the vector-meson mass in this model is also related to the anomaly of the axial-vector fermion current.[8] The axial-vector current will also be important in our four-dimensional analysis.) The Higgs mechanism, now seen as a special realization of the Schwinger mechanism, provides an explicit reason for a pole in $\Pi(q^2)$: The vacuum expectation value of a canonical scalar field coupled to the vector meson gives rise to tadpole contributions to $\Pi(q^2)$ which produce a pole. We show that such a pole can occur for purely dynamical reasons, even in the absence of canonical scalar fields.

In Sec. II, it is demonstrated that a pole in $\Pi(q^2)$ can arise whenever a massless *fermion* acquires a mass through spontaneous symmetry breaking. This is an elaboration on ideas of Englert and Brout.[2,3] As in Ref. 4, this symmetry breaking is not due to a vacuum expectation value of canonical scalar field; rather it is *assumed* to be a consequence of a symmetry-breaking solution to the integral equations of the theory. This gives rise to a zero-mass bound excitation in the two-fermion sector, which produces a pole in $\Pi(q^2)$. Consequently, as explained above, the vector meson also acquires a mass. Furthermore, we show that the zero-mass excitation decouples from the theory. We give in Sec. III a "phenomenological" description of the previous "fundamental" theory. The phenomenological theory, valid at low energy, is realized by a nonlinear Lagrangian, which is not renormalizable, even though the fundamental theory possesses this property. In an appendix we show how the various unrenormalized equations encountered in the text should be renormalized. Also we solve them in the lowest-order Bethe-Salpeter approximation, and exhibit explicitly the workings of the Schwinger mechanism. We find a formula for the spontaneously generated vector-meson mass.

II. SPONTANEOUS MASS GENERATION

A. Preliminaries

We consider a theory with a massless fermion field ψ and a neutral vector-meson field A^μ, interacting through a conserved current J^μ. (No internal symmetry degrees of freedom are included in this simplified discussion.) Furthermore we assume that the Schwinger-Dyson equation for the fermion mass operator $\Sigma(p)$ has a symmetry-breaking solution, $\{\gamma^5, \Sigma(p)\} \neq 0$. It is well known that this can happen if there is a massless, bound excitation in the fermion-antifermion channel. Since a fermion mass is being generated, this zero-mass excitation couples to $\bar\psi\psi$ or $\bar\psi\gamma^5\psi$. Ultimately we shall want the zero-mass state to combine with the massless vector-meson field and generate a meson mass. This can only happen if a transition between J^μ and $\bar\psi\psi$ or $\bar\psi\gamma^5\psi$ is allowed. Therefore, as long as charge-conjugation invariance is not spontaneously broken, J^μ *must contain the axial-vector current.*

We are thus led to consider a theory described by the Lagrangian

$$\begin{aligned} \mathcal{L} &= i\bar\psi\partial\!\!\!/\psi - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + gJ_5^\mu A_\mu\,, \\ J_5^\mu &= i\bar\psi\gamma^\mu\gamma^5\psi\,, \\ F^{\mu\nu} &= \partial^\mu A^\nu - \partial^\nu A^\mu\,. \end{aligned} \tag{2.1}$$

(A vector interaction may also be included. This complication will be discussed further in Sec. II D.) The axial-vector anomaly occurs in the above theory by virtue of the axial-vector coupling.[9] For the time being, we shall ignore this; the anomaly can be eliminated by introducing additional fermions.[10] Indeed the anomaly *must* be removed; otherwise the equation of motion

$$\partial_\nu F^{\mu\nu} = gJ_5^\mu \tag{2.2}$$

is not consistent with the antisymmetry of $F^{\mu\nu}$. (It is interesting that the axial-vector current, together with the anomaly — which, to be sure, must be removed — is central to the present discussion, just as it is central in Schwinger's two-dimensional model.) In Sec. II E, the effect of the anomaly will be considered further.

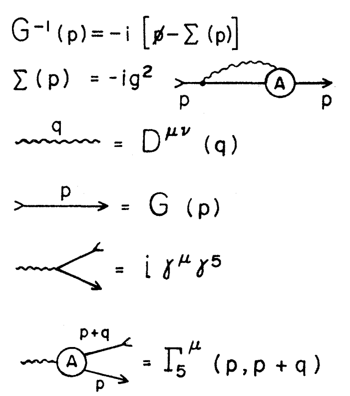
The theory (2.1) is chirally invariant and renormalizable, in the sense that off-mass-shell Green's functions can be computed in perturbation theory. The normalization point will not be taken at $k^2=0$ because of possible infrared divergences; rather an arbitrary value $k^2=k_0{}^2$ will be chosen.

B. Spontaneous Mass Generation

The proper vertex function $\Gamma_5^\mu(p,p')$ associated with J_5^μ satisfies a Ward-Takahashi identity:

$$q_\mu\Gamma_5^\mu(p,p+q) = \gamma^5G^{-1}(p+q) + G^{-1}(p)\gamma^5\,. \tag{2.3}$$

There must not be any anomalous exceptions to this equation, since the current, a source of the gauge field, must be always conserved; see (2.2).[11] In (2.3), $G(p)$ is the complete fermion Green's function, which is given by the following Schwinger-Dyson equation, shown diagrammatically as Eq. (2.4) in Fig. 1. We assume that a



(2.4)

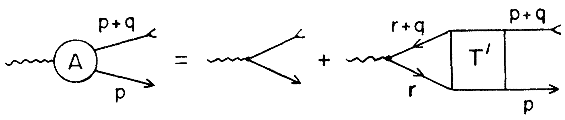
FIG. 1. Schwinger-Dyson equation for fermion propagator.

chiral symmetry-breaking solution for Eq. (2.4) exists such that $\{\gamma^5, \Sigma(p)\}$ is not zero. Then (2.3) implies that $\Gamma_5^\mu(p,p+q)$ has a pole at $q=0$ with residue $i\Gamma_5(p)$, where

$$\Gamma_5(p) = \{\gamma^5, \Sigma(p)\} \neq 0\,. \tag{2.5}$$

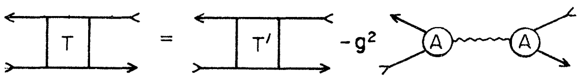
This is our principal assumption. In the Appendix we show that to lowest order in the coupling a symmetry-breaking solution exists.

The proper vertex function is also related to a portion of the fermion-fermion scattering amplitude. The relation is exhibited as Eq. (2.6) in Fig. 2, where T' is the "one-vector-meson-irreducible" fermion-fermion scattering amplitude; i.e., the



(2.6)

FIG. 2. Proper vertex function expressed in terms of the scattering amplitude.



(2.7)

FIG. 3. Relation between complete and one-vector-meson-irreducible scattering amplitudes.

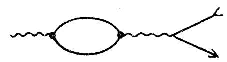
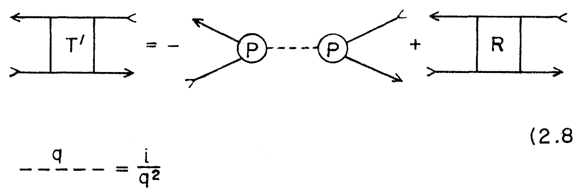


FIG. 4. Example of graph excluded from proper vertex function.



(2.8)

FIG. 5. Singular contribution to T'.

full scattering amplitude T is given by T' supplemented by one-vector-meson exchange graphs; see Eq. (2.7) in Fig. 3. The reason why T' rather than T appears in (2.6) is that Γ_5^μ is the *proper* vertex function and does not contain graphs like that of Fig. 4. [The minus sign in (2.7) arises from the fact that the graph is second-order in the coupling, i.e., $(ig)^2$ is the correct factor.]

The pole in Γ_5^μ is to be attributed to a pole in T'.[12] Thus we represent T' by a pole term plus a regular term R, as in Eq. (2.8) of Fig. 5. In other words, we attribute the chiral-symmetry breaking of the fermion mass to the existence of a massless Goldstone excitation. This excitation is a bound state in the fermion-antifermion channel; the proper vertex which describes the coupling of the bound state to $\psi\bar{\psi}$ is represented by Fig. 6 and is given by $P(p,p')$. This set of circumstances is different from that envisioned for massless spinor electrodynamics.[11] In the latter theory the electron acquires a mass spontaneously, but the Goldstone theorem is evaded due to anomalous nonconservation of the axial-vector current. In the present theory, as we have repeatedly stated, the axial-vector current must be conserved, and the Goldstone theorem holds.

Upon inserting (2.8) into (2.6), we determine the pole term in Γ_5^μ. The relation is

$$\Gamma^\mu_{5\,\text{pole}}(p,p+q) = -\left[-\operatorname{Tr}\int \frac{d^4r}{(2\pi)^4}\, G(r) i\gamma^\mu\gamma^5 G(r+q) P(r+q,r)\right] \frac{i}{q^2}\, P(p,p+q)\,, \tag{2.9a}$$

which is given graphically in Fig. 7. By Lorentz invariance, the integral in (2.9a) is proportional to q^μ:

$$q^\mu I(q^2) = \operatorname{Tr}\int \frac{d^4r}{(2\pi)^4} G(r) i\gamma^\mu\gamma^5 G(r+q) P(r+q,r)\,,$$

$$I(0) = \lambda\,. \tag{2.9b}$$

Thus

$$\Gamma^\mu_{5\,\text{pole}}(p,p+q) = \frac{iq^\mu}{q^2}\,\lambda P(p,p+q) \tag{2.9c}$$

and comparison with (2.3) yields

$$\Gamma_5(p) = \lambda P(p,p)\,. \tag{2.10}$$

Note that Eq. (2.9c) establishes the result that the singularity in Γ_5^μ is a pole in q^2, as well as a pole in q. As a consequence $P(p,p+q)$ is ambiguous up to terms of $O(q^2)$; however, terms of $O(q)$ are well defined.

It is clear that $P(p,p+q)$ must be an odd-parity vertex, and that $P(p,p)$ must be nonvanishing. Therefore the massless excitation must be a pseudoscalar, with some nonderivative coupling to the fermion antifermion channel.

Let us now examine the vacuum polarization tensor, which is given by the Schwinger-Dyson equation shown in Fig. 8, apart from seagull terms. Since the proper vertex function occurring in the equation for $\Pi^{\mu\nu}$ has a pole, $\Pi^{\mu\nu}$ also develops this singularity. The pole contribution to $\Pi^{\mu\nu}$ can be obtained by inserting the singular part of the vertex function [Fig. 7 and Eqs. (2.9)] into the Schwinger-Dyson equation of Fig. 8. The result is given diagrammatically in Fig. 9. The integrals occurring in Fig. 9 are already introduced in (2.9b). We find therefore

$$\Pi^{\mu\nu}_{\text{pole}}(q) = g^2[q^\mu I(q^2)]\frac{i}{q^2}[-q^\nu I(q^2)]$$

$$\to -ig^2\lambda^2 q^\mu q^\nu/q^2\,. \tag{2.11}$$

[The minus sign in the last factor in the first equation above arises as follows. The second pseudo-

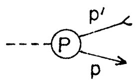


FIG. 6. Dynamical pseudoscalar vertex.

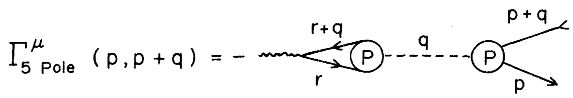
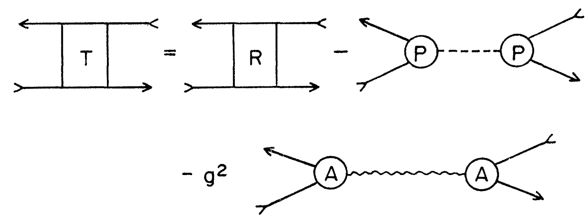


FIG. 7. Pole term in proper vertex function.



(2.13)

FIG. 10. The complete scattering amplitude.

scalar vertex in Fig. 9 occurs with arguments $P(r, r+q)$, rather than $P(r+q, r)$ as in the definition $q^\mu I(q^2)$ in (2.9b). To make contact with (2.9b) the integration variable r is shifted to $r-q$, and q is replaced by $-q$.] Eq. (2.11) shows that $\Pi(q^2)$, defined in (1.1), has the pole $\Pi_{\rm pole}(q^2)=g^2\lambda^2/q^2$. This indicates that the vector meson acquires a mass μ. In the approximation where only the pole term is kept, $\mu=g\lambda$. An exact formula

$$D(0)=-1/g^2\lambda^2 \qquad (2.12)$$

may also be given.

C. Decoupling of the Massless Excitation

The amplitude T' contains a massless pole; nevertheless, it is true that the full on-mass shell scattering amplitude does not possess this pole. Hence the massless excitation decouples from the theory. To establish this important result, we combine (2.7) and (2.8) to obtain (2.13) as shown in Fig. 10.

Only the last two terms in (2.13) contain poles at $q^2=0$. The external fermions are on mass shell; thus the $q^\mu q^\nu$ term of the vector-meson propagator in the last term in (2.13) gives zero contribution since the on-mass-shell vertex is transverse. Consequently $D^{\mu\nu}(q)$ may be set equal to $-ig^{\mu\nu}D(q^2)$. Furthermore, if the leftmost vertex function in the last term in (2.13) is decomposed into a regular piece and the pole term (2.9c), one sees that the pole term does not contribute: The pole is proportional to q^μ, which contracts with the rightmost vertex function and annihilates it. Thus the last two terms in (2.13), which potentially contain a zero-mass pole, are represented by (2.14) of Fig. 11. The right-hand part of the top diagram in Fig. 11 is the regular part of the axial-vertex function which we call $\tilde{\Gamma}^\mu_5$. The pole term (2.9a) in the rightmost vertex function of the last diagram in Fig. 11 has been explicitly exhibited.

We concentrate on the last term in (2.14). We may set $\tilde{\Gamma}^\mu_5=\Gamma^\mu_5-\Gamma^\mu_{5\,\rm pole}$. Since the remaining part of the graph is proportional to q^μ, Γ^μ_5 does not contribute, and we are left with (2.15) given in Fig. 12. Evaluating this at $q^2=0$, and recalling that $q^2\Pi(q^2)|_{q^2=0}=g^2\lambda^2$, we find that (2.15) is

$$P(p'+q, p')\frac{i}{q^2}P(p, p+q)\,.$$

This is precisely the negative of the second term in (2.14); hence all the pole terms cancel.

Clearly a similar analysis can be performed in the crossed channel, and we conclude that T on the mass shell is free from massless poles.

D. The Problem of Other Symmetries

The theory which we have studied is formally scale-invariant. Consequently the emergence of mass terms appears to violate that symmetry and to lead to further massless excitations. It is clear that these potential Goldstone bosons are spin-zero, positive-parity objects. They could couple to the energy-momentum tensor, but not to a vector or axial-vector meson.

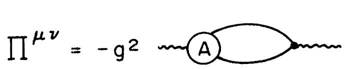


FIG. 8. Schwinger-Dyson equation for vacuum polarization tensor.

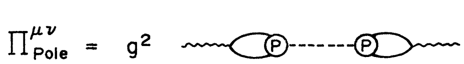
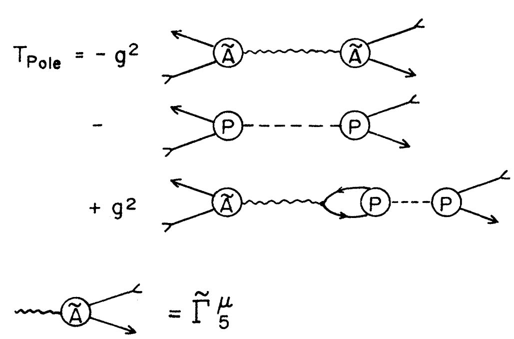


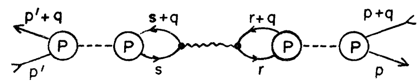
FIG. 9. Pole term in vacuum polarization tensor.



(2.14)

FIG. 11. Potentially singular portion of scattering amplitude.

$$-g^2\left[-\Gamma_{5\,\text{pole}}^{\mu}\right]\left[-ig_{\mu\nu}D(q^2)\right]\left[\Gamma_{5\,\text{pole}}^{\nu}\right]$$

$= g^2$ 

$$=-g^2\left[\frac{iq^{\mu}\lambda}{q^2}P(p'+q,p')\right]\left[\frac{-ig_{\mu\nu}}{q^2-q^2\Pi(q^2)}\right]\left[\frac{iq^{\nu}\lambda}{q^2}P(p,p+q)\right] \quad (2.15)$$

FIG. 12. Diagrammatic representation of Eq. (2.15).

However, the above circumstances very likely do not occur, since scale invariance does not appear to be realized in the solutions of a field theory, in spite of the formal symmetry of the Lagrangian. The reason is not spontaneous violation, but rather the presence of anomalies, analogous to those of the axial-vector current.[13] Thus we shall ignore any considerations of scale invariance.

Indeed it would appear that scalar massless mesons coupled to the energy-momentum tensor must be avoided. Such mesons would lead to a pole in the vacuum polarization tensor of two energy-momentum tensors. If one then considers coupling the energy-momentum tensor to a graviational gauge field, the pole would produce a mass for the graviton, in a fashion entirely analogous to our previous discussions. Since massive gravity apparently is not realized in nature, this state of affairs must be avoided.

The model in (2.1) possesses two other currents which are formally conserved. These are $\bar{\psi}\gamma^{\mu}\gamma^5\psi^c$ and its Hermitian conjugate. Here ψ^c is the charge-conjugate field $\psi_i^c = C_{ij}\bar{\psi}_j$; C is the charge-conjugation matrix $i\gamma^2\gamma^0$ which satisfies $C=-\tilde{C}=-C^{-1}=-C^{\dagger}$, and which transposes the γ matrices: $C^{-1}\gamma^{\mu}C=-\tilde{\gamma}^{\mu}$, $C^{-1}\gamma^5 C=\tilde{\gamma}^5$.

If these two currents are conserved in the solutions of the theory, one obtains Ward identities which at zero momentum transfer require that $G(p)\gamma^5 C+\gamma^5 C\tilde{G}(-p)=0$. By charge-conjugation invariance of the theory, it is also true that $C\tilde{G}(-p)=G(p)C$. Hence the conservation of these odd currents requires massless fermions: $\{\gamma^5,\Sigma(p)\}=0$.

Clearly this result is not acceptable, nor is its evasion with Goldstone particles satisfactory. We do not wish to deal with a theory which possesses massless, doubly charged scalars. The additional, unwanted symmetries may be disposed of by one of two devices.

Firstly, one may argue that in the solutions of the theory the currents are not conserved, again because of anomalies. The envisioned situation is analogous to the discussion of massless electrodynamics, where the electron acquires a mass spontaneously.[11,14] The axial-vector current is formally conserved in that model, yet one can show that it is consistent that the equations determining the matrix elements of the axial-vector current and of its divergence admit a nonconserved solution. This comes to pass because of the divergences of perturbation theory. (Unlike the anomalies of the triangle graph and of scale invariance which occur in a low order of perturbation theory, examples of this mechanism require infinite summation of graphs.)

The second way of destroying the additional symmetry is by introducing vector couplings into the Lagrangian (2.1). This vector coupling can be a gauge coupling to the vector field A^{μ}: The interaction term in (2.1) is replaced by $\bar{\psi}\gamma^{\mu}(g'+ig\gamma^5)\psi A_{\mu}$. This leads to a parity-violating theory. If one wishes to maintain parity, one can introduce an additional (massive) vector field B_{μ} coupled to $\bar{\psi}\gamma^{\mu}\psi$. In the presence of a vector interaction, $\bar{\psi}\gamma^{\mu}\gamma^5\psi^c$ is no longer conserved. A massive vector field also destroys formal scale invariance.

E. The Problem of the Triangle Anomaly

Of course our theory (2.1) is inconsistent because of the triangle anomaly.[9] In order to remove this contradiction, the number of fermion fields must be increased.[10,15] We introduce a multiplet of n fields

$$\Psi=\begin{pmatrix}\psi_1\\ \vdots\\ \psi_n\end{pmatrix},$$

coupled to the axial-vector field by the interaction $i\bar{\Psi}\gamma^{\mu}\gamma^5\underline{g}\Psi A_{\mu}$. Provided the $n\times n$ Hermitian matrix $\underline{g}$ satisfies $\mathrm{Tr}\underline{g}^3=0$, the anomaly is absent. We shall limit the discussion to the simplest, two-fermion case:

$$\Psi=\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix},\quad \underline{g}=\begin{pmatrix}1&0\\0&-1\end{pmatrix}. \quad (2.16)$$

The theory now possesses many conserved currents. Two, given by $\bar{\Psi}\gamma^\mu \underline{g}\Psi$ and $\bar{\Psi}\gamma^\mu\Psi$, or $\bar{\psi}_i\gamma^\mu\psi_i$, $i=1,2$, ensure the conservation of the individual species of fermions. In order to avoid further Goldstone bosons, this continuous symmetry must not be spontaneously broken. Fortunately even if we allow each of the fermions to acquire a *different* mass, and a *different* coupling to the massless bound state, the new symmetry remains operative. The model possesses additional, formally conserved, charged currents as discussed in the previous subsection. These must be disposed of by one of the two models mentioned above. Finally, the axial-vector current $\bar{\Psi}\gamma^\mu\gamma^5\Psi$, though beset by the triangle anomaly, would also lead to Goldstone bosons. The reason is that the anomaly does not contribute to the Ward identity at zero momentum, while it is only the zero-momentum Ward identity that is required to establish the existence of massless excitations. Thus conservation of this current must be broken, by the first method discussed above, just as in massless quantum electrodynamics.

There are also discrete symmetries present in the model; for example,

$$\psi_1 \to \psi_2, \quad \psi_2 \to \psi_1, \quad A^\mu \to -A^\mu$$

and (2.17)

$$\psi_1 \to i\psi_2, \quad \psi_2 \to -i\psi_1, \quad A^\mu \to -A^\mu .$$

(This eliminates 3-meson couplings, and the anomaly.) The discrete symmetry implies that $G_1(p)=G_2(p)$ and $P_1(p,p')=-P_2(p,p')$ [subscripts refer to fermion species]. Clearly, as a consequence of the symmetry the masses of the two fermions are the same, and the coupling of the pseudoscalar massless excitation to the fermions is equal in magnitude and opposite in sign. If we wish that the masses of the two fermions be different, this discrete symmetry must be broken. This breaking does no violence to the continuous symmetries, and since the symmetry which is being boken is discrete, no new massless excitations are required.

The discrete symmetry which we have discussed is not unlike ordinary charge-conjugation invariance. Indeed, the analogy can be made explicit by redefining the fields Ψ as follows:

$$\Psi = \tfrac{1}{2}(1+i\gamma^5)\Psi' + \tfrac{1}{2}(1-i\gamma^5)M\Psi' ,$$

where $M = \left(\begin{smallmatrix}0 & 1\\ 1 & 0\end{smallmatrix}\right)$. The Lagrangian (2.1) becomes now

$$\mathcal{L} = i\bar{\psi}'\not{\partial}\Psi' - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \Psi'\gamma^\mu \underline{g}\,\psi' A_\mu . \qquad (2.18)$$

In the general case, when there are n fermions, care must be exercised that the spontaneously generated masses do not violate any new continuous symmetries. This would lead to massless excitations which may not decouple from the system.

F. Discussion

An interesting sum rule for the vector-meson mass may be derived in our model. Let us begin with (2.9b):

$$q^\mu I(q^2) = \mathrm{Tr}\int \frac{d^4r}{(2\pi)^4}[G(r)i\gamma^\mu\gamma^5 G(r+q)P(r+q,r)]$$

$$= -\mathrm{Tr}\int \frac{d^4r}{(2\pi)^4}[G(r+q)i\gamma^\mu\gamma^5 G(r)P(r,r+q)] , \qquad (2.19a)$$

$$g^{\mu\nu}\lambda = -\mathrm{Tr}\int \frac{d^4r}{(2\pi)^4}\left[\partial^\nu G(r)i\gamma^\mu\gamma^5 G(r)P(r,r) + G(r)i\gamma^\mu\gamma^5 G(r)\frac{\partial}{\partial q_\nu}P(r,r+q)\bigg|_{q=0}\right] . \qquad (2.19b)$$

We also know from (2.3) and (2.9a) that

$$q_\mu \tilde{\Gamma}_5^\mu(p,p+q) = \gamma^5 G^{-1}(p+q) + G^{-1}(p)\gamma^5 - iP(p,p+q)\lambda , \qquad (2.20)$$

where $\tilde{\Gamma}_5^\mu(p,p+q)$ is the vertex function, *without* its pole at $q^2=0$. Hence it is true that

$$i\lambda P(p,p) = \{\gamma^5, G^{-1}(p)\} ,$$

$$i\lambda\frac{\partial}{\partial q_\nu}P(p,p+q)\bigg|_{q=0} = \gamma^5\partial^\nu G^{-1}(p) - \tilde{\Gamma}_5^\nu(p,p) . \qquad (2.21)$$

Inserting (2.21) in (2.19b) gives

$$g^{\mu\nu}\lambda^2 = -\mathrm{Tr}\int \frac{d^4r}{(2\pi)^4}[G(r)\partial^\nu G^{-1}(r)G(r)\gamma^\mu - G(r)\partial^\nu G^{-1}(r)G(r)\gamma^\mu\gamma^5 G(r)\gamma^5 G^{-1}(r)$$
$$+ G(r)\gamma^\mu\gamma^5 G(r)\gamma^5\partial^\nu G^{-1}(r) - G(r)\gamma^\mu\gamma^5 G(r)\tilde{\Gamma}_5^\nu(r,r)] . \qquad (2.22a)$$

The middle two terms cancel against each other, while $\partial^\nu G^{-1}(r)$ may be related to the vertex function Γ^ν

of the vector current:

$$\partial^\nu G^{-1}(r) = -i\Gamma^\nu(r,r)\,.$$

Hence

$$g^{\mu\nu}\lambda^2 = i\,\mathrm{Tr}\int \frac{d^4r}{(2\pi)^4}[G(r)\Gamma^\nu(r,r)G(r)\gamma^\mu - G(r)\Gamma_5^\nu(r,r)G(r)i\gamma^\mu\gamma^5]\,. \quad (2.22\mathrm{b})$$

Recalling the definition of the vacuum polarization tensor (1.1) and its representation in terms of the Schwinger-Dyson equation of Fig. 8, we recognize that (2.22b) is equivalent to

$$g^{\mu\nu}g^2\lambda^2 = -i[\tilde\Pi_A^{\mu\nu}(0) - \Pi_V^{\mu\nu}(0)]\,. \quad (2.23)$$

$\tilde\Pi_A^{\mu\nu}$ is the vacuum polarization tensor associated with the nonsingular, nonconserved axial-vector current, while $\Pi_V^{\mu\nu}$ is the vacuum polarization of the conserved vector current, apart from seagull terms.

Formally, if one ignores seagull terms, $\Pi_V^{\mu\nu}(0) = 0$, since the vector current is conserved. This is also seen from (2.22a), where the integrand in the first term on the right-hand side is a total divergence $-\partial^\nu G(r)\gamma^\mu$. However, if $G(r) \to 1/r$ for large r, $\Pi_V^{\mu\nu}(0)$ is in fact a quadratically divergent constant which cancels a similar quadratic divergence in $\tilde\Pi_A^{\mu\nu}(0)$. In other words, $\Pi_V^{\mu\nu}(0)$ is the seagull term, which is necessary to convert the formal Schwinger-Dyson equation for $\tilde\Pi_A^{\mu\nu}$ into a correct expression. Thus we may replace (2.23) by

$$g^{\mu\nu}g^2\lambda^2 = -i\tilde\Pi_A^{\mu\nu}(0)\,, \quad (2.24)$$

where $\tilde\Pi_A^{\mu\nu}$ is now understood to include the requisite seagull term.

The formula (2.24) may also be understood from (1.1) and (2.11). We have

$$\Pi^{\mu\nu}(q) = (g^{\mu\nu}q^2 - q^\mu q^\nu)\left[i\tilde\Pi(q^2) + i\frac{g^2\lambda^2}{q^2}\right], \quad (2.25)$$

where $\tilde\Pi(q^2)$ is by definition regular. It is natural to identify $\tilde\Pi_A^{\mu\nu}$ with the regular part of (2.25). Thus the following nontransverse expression is found:

$$\tilde\Pi_A^{\mu\nu}(q) = ig^2\lambda^2 g^{\mu\nu} + (g^{\mu\nu}q^2 - q^\mu q^\nu)i\tilde\Pi(q^2)\,, \quad (2.26)$$

and $-i\tilde\Pi_A^{\mu\nu}(0) = g^{\mu\nu}g^2\lambda^2$, which agrees with (2.24). Note also that $\tilde\Pi_A^{\mu\nu}(q)$, though nonconserved, has a conserved absorptive part. Of course in the approximation $\mu^2 = g^2\lambda^2$, (2.23) and (2.24) become mass formulas.

Equation (2.23) is also related to Weinberg's first sum rule.[16] If we identify $i\Pi_V^{\mu\nu}(0)$ with $g^2 g^{\mu\nu}$ times the Schwinger term S_V of the vector current commutator, and similarly for the Schwinger term S_A of the *regular* part of the axial-vector current, then (2.23) reads $\lambda^2 + S_A = S_V$. This is recognized as Weinberg's first sum rule, when it is recalled that the Schwinger term of the *total* axial-vector current differs from that of the regular part by the square of the current-pseudoscalar coupling, which in our case is λ.

It is useful to estimate the various quantities that have been encountered. We set $\Sigma(p) = \not p A(p^2) + mB(p^2)$. Since $iG^{-1}(p) = \not p - \Sigma(p)$, it follows that $\Sigma(p)|_{\not p = m} = m$, or $A(m^2) + B(m^2) = 1$. From (2.5) and (2.10) we have

$$2m\gamma^5 B(m^2) = \lambda P(p,p)|_{\not p = m}\,. \quad (2.27)$$

Define $\lambda = m\bar\lambda$, where $\bar\lambda$ is a dimensionless number. Also, $P(p,p)|_{\not p = m}$ is defined to be $2\gamma^5 f$, again a dimensionless quantity. Thus

$$B(m^2) = \bar\lambda f\,. \quad (2.28)$$

The vector-meson mass satisfies $\mu^2 = g^2\lambda^2$, or

$$\mu = g\,\frac{B(m^2)}{f}\,m\,. \quad (2.29)$$

Thus if we imagine μ to be an order of magnitude larger than m, and g to be of electromagnetic strength, then $B(m^2)/f$ is of order 100. Moreover, if $B(m^2)$ is of order unity, then f is very small. Such a small coupling constant is perhaps unnatural, if it is a fundamental parameter. However, in the present context, f is of dynamical origin, and no preconceptions about its magnitude exist.

One may attempt a sort of tree approximation by setting $\Sigma(p)$ equal to m and $P(p, p+q)$ to $2\gamma^5 f$. It is then consistent, according to (2.21) that $\bar\lambda = f^{-1}$, $\mu = gmf^{-1}$, $\Gamma^\mu(r,r) = \gamma^\mu$, and $\tilde\Gamma_5^\mu(r,r) = i\gamma^\mu\gamma^5$. From (2.19b) or (2.22b) one may compute $\bar\lambda$. The former gives

$$g^{\mu\nu}\bar\lambda m = -2if\,\mathrm{Tr}\int\frac{d^4r}{(2\pi)^4}\frac{(\not r + m)\gamma^\nu(\not r + m)\gamma^\mu\gamma^5(\not r + m)\gamma^5}{(r^2 - m^2)^3}$$

$$= \frac{g^{\mu\nu}fm}{2\pi^2}\left(\ln\frac{\Lambda^2}{m^2} - 1\right). \quad (2.30\mathrm{a})$$

From (2.22b) we find

$$g^{\mu\nu}\bar\lambda^2 m^2 = -i\,\mathrm{Tr}\int\frac{d^4r}{(2\pi)^4}\frac{1}{(r^2 - m^2)^2}[(\not r + m)\gamma^\nu(\not r + m)\gamma^\mu + (\not r + m)\gamma^\nu\gamma^5(\not r + m)\gamma^\mu\gamma^5]$$

$$= g^{\mu\nu}\frac{m^2}{2\pi^2}\left(\ln\frac{\Lambda^2}{m^2} - 1\right). \quad (2.30\mathrm{b})$$

Equations (2.30) imply that $\bar{\lambda}^2 = f^2L^2 = L$, where L is the logarithmically divergent quantity

$$\frac{1}{2\pi^2}\left(\ln\frac{\Lambda^2}{m^2}-1\right).$$

Consistency with $\bar{\lambda}^2 = f^{-2}$ is achieved if $L = f^{-2}$.

These results are not satisfactory, since they indicate that the dynamical quantity $\bar{\lambda}$ is divergent, for which there does not appear a possibility of renormalization. In a truly consistent theory $\bar{\lambda}$ should be finite. It is not difficult to see the source of the problem. Our approximation $\Sigma(p) = m$ commits us to the result that $\lim_{p\to\infty}\Sigma(p) = m$, which would indicate that the *bare* mass is nonzero. Clearly the solution of the Schwinger-Dyson equation for $\Sigma(p)$ should have the property that, apart from terms proportional to $\not{p}$, $\Sigma(p)$ vanishes asymptotically; i.e.,

$$\lim_{p\to\infty}\lambda P(p,p) = \lim_{p\to\infty}\{\gamma^5, \Sigma(p)\} = 0\,.$$

Thus we *must* not set $P(p, p+q) = 2\gamma^5 f$, and $\Sigma(p) = m$.

A more plausible approximation for $\Sigma(p)$ is $m\,(|p^2|/m^2)^{-\epsilon(g^2)}$,[17] where $\epsilon(g^2)$ is a positive, coupling-constant-dependent quantity. We expect that for small g^2, $\epsilon(g^2) = c^2g^2$. With this approximation, it is still consistent to leading order in g^2, to set $\Gamma^\mu(r,r) = \gamma^\mu$ and $\tilde{\Gamma}_5^\mu(r,r) = i\gamma^\mu\gamma^5$. From (2.20b) we now get, instead of (2.30b), to leading order in g,

$$g^{\mu\nu}\bar{\lambda}^2 m^2 = g^{\mu\nu}\frac{m^2}{2\pi^2}\frac{1}{2\epsilon(g^2)}\,. \tag{2.31}$$

We thus find that $\bar{\lambda}$ is of order $1/g$, and $\mu = m/2\pi c$ is independent of g. Consequently, f becomes of order g. In the Appendix we discuss in greater detail these results, with special emphasis on the problem of renormalization. We justify the above approximation for $\Sigma(p)$, and evaluate $\epsilon(g^2)$ in lowest order.

III. PHENOMENOLOGICAL LAGRANGIANS

The mechanism which we have discussed is attractive in that it dispenses with fundamental scalar fields as the agents responsible for a pole in the vacuum polarization tensor. Furthermore, since the vector meson propagator has the form

$$-i\left(g^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\right)[q^2-q^2\Pi(q^2)]^{-1},$$

its high-energy behavior is no more divergent than that of the free propagator, and the theory should be renormalizable.

An obvious shortcoming of the theory, as developed so far, is that no effective method of computation has been found. Ordinary perturbation theory will never expose a bound state: To any finite order in g we shall always have massless fermions and vector mesons.

One possible, though incomplete, approach is to describe the physical system by an effective Lagrangian. The effective Lagrangian, in tree approximation, should reproduce some of the dynamics of the complete theory. The description, necessarily limited to a low-energy domain, should take into account the following features of the complete theory: (1) the excitation spectrum, (2) couplings of the states to each other, and (3) the symmetries of the problem.

In the simple model considered in Sec. II, the excitation spectrum consists of a massive fermion, a massive axial-vector meson, and a massless pseudoscalar meson, which, however, decouples from the theory. There may be other massive bound states in the theory. Presumably these are important only at higher energies, and for a low-energy phenomenology may be ignored.

The interactions of the theory involve fermion–vector-meson couplings, fermion-pseudoscalar couplings, and vector-meson–pseudoscalar couplings. The latter two, however, are removable by an appropriate choice of fields.

Finally, the symmetries which are to be maintained are vector and axial-vector-current conservation and parity conservation. The axial-vector current should be related to a gauge symmetry.

Guided by the above considerations, we are led to the Lagrange function

$$\begin{aligned}\mathcal{L} = {} & i\bar{\psi}\not{\partial}\psi + \tfrac{1}{2}\partial^\mu\phi\partial_\mu\phi - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}\mu^2A^2 \\ & + ig\bar{\psi}\gamma^\mu\gamma^5\psi A_\mu - \bar{\psi}\psi S(\phi) + \bar{\psi}\gamma^5\psi P(\phi) \\ & - 2gA^\mu\partial_\mu\phi a(\phi) + b(\phi)\,.\end{aligned} \tag{3.1}$$

The real functions $a(\phi)$, $b(\phi)$, $S(\phi)$, and $P(\phi)$ are to be determined. By parity conservation, the first three are even in ϕ; the last is odd. Also $S(0) = m$. Since we wish to realize chiral symmetry, with a *single* boson field ϕ, it is to be expected that the Lagrangian will not be a polynomial in ϕ. The equations of motion are

$$i\not{\partial}\psi = -ig\gamma^\mu\gamma^5\psi A_\mu + \psi S(\phi) - \gamma^5\psi P(\phi)\,, \tag{3.2}$$

$$\Box\phi = 2g\partial_\mu A^\mu a(\phi) - \bar{\psi}\psi S'(\phi) + \bar{\psi}\gamma^5\psi P'(\phi) + b'(\phi)\,, \tag{3.3}$$

$$\partial_\mu F^{\mu\nu} = -ig\bar{\psi}\gamma^\nu\gamma^5\psi - \mu^2A^\nu + 2g\partial^\nu\phi a(\phi)\,. \tag{3.4}$$

Clearly the vector current $\bar{\psi}\gamma^\mu\psi$ is conserved. The axial-vector current which must be conserved is

$$J_5^\mu = i\bar{\psi}\gamma^\mu\gamma^5\psi - 2a(\phi)\partial^\mu\phi\,. \tag{3.5}$$

By virtue of the equations of motion the divergence of J_5^μ is

$$\partial_\mu J_5^\mu = -2\bar{\psi}\gamma^5\psi[S(\phi)+a(\phi)P'(\phi)] - 2\bar{\psi}\psi[P(\phi)-a(\phi)S'(\phi)] - 4g\partial_\mu A^\mu a^2(\phi) - 2a(\phi)b'(\phi) - 2\partial^\mu\phi\partial_\mu\phi a'(\phi) . \qquad (3.6)$$

Since ultimately we shall wish to arrive at a gauge theory, $\partial_\mu A^\mu$ may be set to zero. This is also consistent with (3.4), when it is assumed that $\partial_\mu J_5^\mu = 0$. In order that (3.6) be zero, we choose $b(\phi)=0$, $a(\phi)=$constant, and

$$S(\phi) = -aP'(\phi) , \quad P(\phi) = aS'(\phi) . \qquad (3.7)$$

The solution of these equations which conserves parity and leads to a properly normalized $S(0)$ is

$$S(\phi) = m\cos\phi/a , \quad P(\phi) = -m\sin\phi/a . \qquad (3.8)$$

Consequently the Lagrangian (3.1) is given by

$$\mathcal{L} = i\bar{\psi}\not\partial\psi + \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}\mu^2 A^2 + ig\bar{\psi}\gamma^\mu\gamma^5\psi A_\mu - m\bar{\psi}\psi\cos\phi/a - m\bar{\psi}\gamma^5\psi\sin\phi/a - 2agA^\mu\partial_\mu\phi . \qquad (3.9)$$

The theory (3.9) possesses a global symmetry with the transformations

$$\delta\psi = \gamma^5\psi , \quad \delta\bar{\psi} = \bar{\psi}\gamma^5 , \quad \delta\phi = -2a , \quad \delta A^\mu = 0 . \qquad (3.10a)$$

However, we require that (3.10a) should be extendible to local symmetry transformation:

$$\delta\psi = \gamma^5\psi\theta , \quad \delta\bar{\psi} = \bar{\psi}\gamma^5\theta , \quad \delta\phi = -2a\theta , \quad \delta A^\mu = -\frac{1}{g}\partial^\mu\theta . \qquad (3.10b)$$

This can only happen if $2ga=\mu$. Thus the final phenomenological Lagrangian is

$$\mathcal{L} = i\bar{\psi}\not\partial\psi + \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}\mu^2A^2 + ig\bar{\psi}\gamma^\mu\gamma^5\psi A_\mu - m\bar{\psi}\left(\exp\frac{2g}{\mu}\gamma^5\phi\right)\psi - \mu A^\mu\partial_\mu\phi . \qquad (3.11)$$

The expression (3.11) may also be arrived at by a different argument. Since we seek a Lagrange function which gives a realization of a theory with chiral U(1)×U(1) symmetry, we are naturally led to a σ-type model, without isospin[18]:

$$\mathcal{L} = i\bar{\psi}\not\partial\psi + \tfrac{1}{2}\partial_\mu\pi\partial^\mu\pi + \tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + ig\bar{\psi}\gamma^\mu\gamma^5\psi A_\mu - 2g(\sigma\partial^\mu\pi - \pi\partial^\mu\sigma)A_\mu + 2g^2(\sigma^2+\pi^2)A^2 - G\bar{\psi}(\gamma^5\pi+\sigma)\psi , \qquad (3.12)$$

where the various couplings are chosen to be consistent with the gauge principle

$$\delta\psi = \gamma^5\psi\theta, \quad \delta\bar{\psi} = \bar{\psi}\gamma^5\theta , \quad \delta\pi = -2\sigma\theta, \quad \delta\sigma = 2\pi\theta , \quad \delta A^\mu = -\frac{1}{g}\partial^\mu\theta . \qquad (3.13)$$

However, the phenomenological Lagrangian relevant to our theory should possess only one spin-zero field, since we are ignoring the possibility of higher-mass bound states. Thus we set $\sigma^2+\pi^2 = \mu^2/4g^2$, and (3.12) becomes

$$\mathcal{L} = i\bar{\psi}\not\partial\psi - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}\mu^2A^2 + \tfrac{1}{2}\frac{\mu^2}{4g^2\sigma^2}\partial_\mu\pi\partial^\mu\pi + ig\bar{\psi}\gamma^\mu\gamma^5\psi A_\mu - G\bar{\psi}(\gamma^5\pi+\sigma)\psi - \frac{\mu^2}{2g\sigma}A^\mu\partial_\mu\pi ,$$
$$\sigma = \left(\frac{\mu^2}{4g^2} - \pi^2\right)^{1/2} . \qquad (3.14)$$

Upon redefining the field π by

$$\pi = \frac{\mu}{2g}\sin\frac{2g}{\mu}\phi , \quad \sigma = \frac{\mu}{2g}\cos\frac{2g}{\mu}\phi , \qquad (3.15)$$

we again get (3.11), with $m=G\mu/2g$.

It should be possible to derive the phenomenological Lagrangian by yet another method, which would exhibit the multiple fermion–massless-boson couplings. In this approach, which we have not studied, one would analyze the Ward identities of the underlying physical theory for n currents and two Fermi fields. The phenomenological description of the massless boson dynamics corresponds to keeping only the pole terms in these amplitudes.

It is not difficult to see that the massless field ϕ, present in (3.11), in fact decouples.[19] Change variables in (3.11):

$$\psi \to \exp\left(-\gamma^5\frac{g}{\mu}\phi\right)\psi , \quad \bar{\psi} \to \bar{\psi}\exp\left(-\gamma^5\frac{g}{\mu}\phi\right), \quad A^\mu \to A^\mu + \frac{1}{\mu}\partial^\mu\phi . \qquad (3.16)$$

Then

$$\mathcal{L} = i\bar{\psi}\not\partial\psi - m\bar{\psi}\psi - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}\mu^2A^2 + ig\bar{\psi}\gamma^\mu\gamma^5\psi A_\mu . \qquad (3.17)$$

This Lagrangian exhibits the physical spectrum of excitations: A massive fermion and vector meson interacting through the axial-vector current. Since (3.17) represents a choice of gauge, one may no longer set $\partial_\mu A^\mu$ to zero. Therefore the current $J_5^\mu = i\bar{\psi}\gamma^\mu\gamma^5\psi$ is no longer conserved. There does exist a conserved axial vector in the theory: $\mu^2 A^\mu + ig\bar{\psi}\gamma^\mu\gamma^5\psi$. However, since the time component of this object is equal to $\partial_i F^{0i}$, the "charge" vanishes, and no symmetry is generated.

The phenomenological description of our theory does not lead to a renormalizable Lagrangian. This is seen either from (3.11), where the nonrenormalizability resides in the nonpolynomial fermion-pseudoscalar interaction, or from (3.17), where it is the nonconservation of $i\bar{\psi}\gamma^\mu\gamma^5\psi$ that is responsible for the divergent, presumably unphysical, high-energy behavior of the theory. We do not consider the rapid growth at high energy as a defect of the fundamental theory. The phenomenological description is not meant to extend to high energies; in particular one does not expect to use the Lagrangians (3.11) or (3.17) for higher-order calculations.

Indeed it is quite possible that a more accurate phenomenological description of the fundamental theory can be given in terms of a renormalizable Lagrangian. For example, if there is another bound state in theory — a massive scalar — which together with the massless excitation forms a chiral multiplet, then the appropriate phenomenological Lagrangian would be (3.12) (supplemented with various chirally invariant σ and π terms) without the nonlinear constraint relating σ to π. That theory is renormalizable; it is an example of the conventional Higgs mechanism.

It is well known that the nonlinear σ model is the limiting form of a renormalizable linear σ model, where the mass of the scalar field tends to infinity. Thus we envision a hierarchy of phenomenological Lagrangians: In the low-energy domain where only the massless excitation is considered, and all other bound states are ignored, the description is in terms of (3.11). If other bound states are present, and if they should be included, then the description is as in (3.12). Renormalizability, on the phenomenological level, does not appear to be a fundamental requirement. Of course the construction of this hierarchy of phenomenological Lagrangians presupposes the possibility of solving the fundamental theory and exhibiting its complete excitation spectrum. Needless to say, at the present time, this is not possible.

In spite of our abandonment of renormalizability, the form of the phenomenological Lagrangian is nevertheless somewhat limited. For example, a nonrenormalizable Lagrangian which exhibits the Higgs mechanism could include a fermion–scalar-meson interaction of the form $G\bar{\psi}\psi(\sigma^2+\pi^2)$. Yet it does not appear that such a term can arise in a bound-state model.

It is easy to set up a correspondence between the couplings encountered in our fundamental theory and those exhibited in the phenomenological theory (3.11). In the former, the J_5^μ-ϕ transition is characterized by the strength λ, and hence the A^μ-ϕ interaction is $g\lambda$. Examining the last term in (3.11), we find an old relation $g\lambda = \mu$. Similarly, the fundamental, fermion-ϕ interaction is described by $2f$. Comparison with the next-to-last term in (3.11) gives $f = mg/\mu$, again an old result.

IV. CONCLUSION

The interesting aspect of the present investigation is the demonstration that vector particles can acquire a mass from a bound-state mechanism, rather than from a vacuum expectation value of a canonical scalar field.

The present work should be extended to include internal symmetry. More importantly, an effective computational method should be developed which bypasses ordinary perturbation theory, yet maintains renormalizability.

The physical relevance of this mechanism is not apparent at the present time. Clearly a non-Abelian version of our theory may be used for weak-interaction model building. More interesting is the possibility that the Abelian model may be relevant to pure strong interactions. A world with massless quarks and gluons which acquire their mass spontaneously without Goldstone bosons is attractive for its economy. Moreover, we see in this picture the possibility of avoiding the problem of too much symmetry in the quark model. If chiral U(3) is spontaneously broken in the fashion described here, then the troublesome ninth Goldstone boson disappears from the theory.

ACKNOWLEDGMENT

We are grateful for the interest that Steven Weinberg has shown in this work.

APPENDIX

In this appendix we return to some of the Schwinger-Dyson equations which we have encountered in the main body of the text. Our purpose is to show how they are to be written in finite, renormalized form. Also we solve them in the lowest-order Bethe-Salpeter approximation; a spontaneously generated mass is found for the fermion and vector meson.

Let us return to the equation satisfied by the proper vertex function, (2.6), and reexpress it in Bethe-Salpeter form; see Eq. (A1) of Fig. 13. In that formula K' is the Bethe-Salpeter kernel, with one-vector-meson lines deleted [compare (2.7)]. Evidently Γ_5^μ is multiplicatively renormalizable, and in the usual fashion we learn from the Ward identity (2.3) that Γ_5^μ may be renormalized by the same constant that renormalizes $G^{-1}(p)$. (The triangle anomaly is being ignored.)

Decomposing Γ_5^μ into its regular and pole parts, we have, according to (2.9c), the relation given by Eq. (A2) in Fig. 13. If (A2) is substituted into (A1), we find, by equating pole terms, that at $q^2=0$ P satisfies the homogeneous Bethe-Salpeter equation (A3) of Fig. 13. Consequently a formula for $\tilde{\Gamma}_5^\mu$ which follows from (A1)–(A3) can be found. It is Eq. (A4) of Fig. 13. The central hypothesis of this paper is that (A3) has a nontrivial solution, $P(p,p)\neq 0$. According to (2.5), (2.9), and (2.10), this allows the fermion to acquire a mass spontaneously. We shall show below that to lowest order in the coupling (A3) does indeed have a nontrivial solution.

Next we consider the equation for λ. The formula for λ^2 in terms of the vacuum polarization tensors, (2.22b), suffers from well-known overlapping divergences. Similarly the defining equation (2.9b) involves overlaps. It is our purpose to remove these overlaps and to express λ in terms of renormalized quantities.

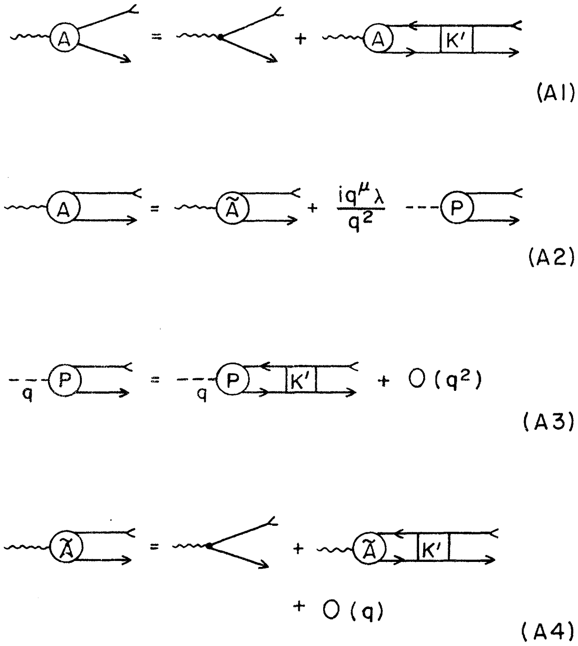


FIG. 13. Diagrammatic representation of Eqs. (A1)–(A4).

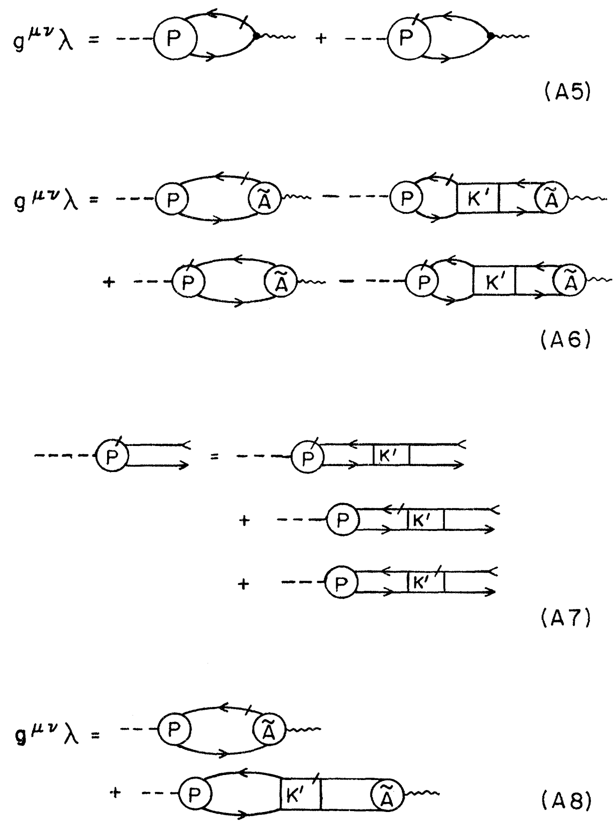


FIG. 14. Diagrammatic representation of Eqs. (A5)–(A8).

We represent (2.9b) or (2.19b) by Eq. (A5) in Fig. 14. The slash indicates differentiation with respect to q (the external momentum) and evaluation at $q=0$. The external momentum is routed through the upper fermion line. The bare vertex in (A5) may be evaluated from (A4) of Fig. 13. Hence Eq. (A6) is obtained; see Fig. 14. [The undetermined part of (A4) does not contribute, since $q=0$.] Also from (A3) of Fig. 13 we find a formula for the derivative of P. It is Eq. (A7) of Fig. 14. (Again the undetermined part disappears at $q=0$.) Upon substituting (A7) into the third term of (A6), one finds that many terms cancel and λ becomes determined by Eq. (A8) of Fig. 14.

Since P and Γ_5^μ each renormalize like G^{-1} and K' renormalizes like G^{-2}, the formula (A8) for λ is invariant under renormalization. Therefore finite, renormalized quantities P, $\tilde{\Gamma}_5^\mu$, and G can be used in evaluating λ. Furthermore, $P(r,r)$ is proportional to $\{\gamma^5,\Sigma(r)\}$; we expect it to go to zero with large r, since the bare mass is zero. Then the integrations in (A8) converge, and λ is well defined and finite. Of course if no symmetry-breaking solution exists, $P=0$ and λ vanishes.

Next we examine the lowest-order Bethe-Salpeter equation for P. Equation (A3), written explicitly, is

$$P_{ab}(p,p+q)=\int\frac{d^4r}{(2\pi)^4}\,G_{cd}(r)P_{de}(r,r+q)\times G_{ef}(r+q)K_{ac,fb}(p,r;q)+O(q^2)\,. \tag{A9}$$

(the subscripts are spinor indices). With pure axial-vector coupling, the lowest-order kernel is

$$K_{ac,fb}(p,r;q)=-g^2(i\gamma^\mu\gamma^5)_{fb}\,D_{\mu\nu}(p-r)(i\gamma^\nu\gamma^5)_{ac}\,.$$

For reasons that will emerge presently, we also imagine that there exists in the theory a massive vector meson, with vector coupling. (Recall that we found in Sec. II D that vector couplings are needed to eliminate explicitly undesirable additional symmetries.) Thus, with obvious notation, the lowest-order kernel is taken to be

$$K_{ac,fb}(p,r;q)=-g_a^{\,2}(i\gamma^\mu\gamma^5)_{fb}D^A_{\mu\nu}(p-r)(i\gamma^\nu\gamma^5)_{ac}-g_V^{\,2}(\gamma^\mu)_{fb}D^V_{\mu\nu}(p-r)(\gamma^\nu)_{ac}\,. \tag{A10}$$

For the propagators in (A9) and (A10) we take the lowest-order expressions. However, as we shall see, the spontaneously induced fermion mass is independent of coupling, while the vector-meson mass will not influence the result. Hence we keep all mass terms.

$$G(p)=\frac{i}{\not p-m}\,,$$
$$D^A_{\mu\nu}(k)=-i\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{k^2}\right)\frac{1}{k^2-\mu_A^{\,2}}\,, \tag{A11}$$
$$D^V_{\mu\nu}(k)=-i\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{k^2}\right)\frac{1}{k^2-\mu_V^{\,2}}\,.$$

We need only determine $P(p,p)$ which is set equal to $\gamma^5\mathcal{P}(p^2)$. The Bethe-Salpeter equation (A9) at $q=0$ becomes

$$\mathcal{P}(p^2)=-3i\int\frac{d^4r}{(2\pi)^4}\frac{\mathcal{P}(r^2)}{r^2-m^2}\times\left[\frac{g_V^{\,2}}{(p-r)^2-\mu_V^{\,2}}-\frac{g_A^{\,2}}{(p-r)^2-\mu_A^{\,2}}\right]. \tag{A12}$$

The solution to (A12) for large p^2, where $\mu_V^{\,2}$ and $\mu_A^{\,2}$ may be ignored, is well known.[17] A solution is obtained only if $g_V^{\,2}>g_A^{\,2}$, in which case

$$\mathcal{P}(p^2)\underset{-p^2\to\infty}{\propto}\left(\frac{-p^2}{m^2}\right)^{-\epsilon},\qquad \epsilon=\frac{3}{16\pi^2}\left[g_V^{\,2}-g_A^{\,2}\right]. \tag{A13}$$

In the case $g_A^{\,2}>g_V^{\,2}$, $\mathcal{P}(p^2)$ would increase with p^2, and the integral equation (A12) could not be satisfied. The reason for our insistence on additional vector coupling is now clear: For pure axial coupling no solution is found.

Since $\lambda P(p,p)=\lambda\gamma^5\mathcal{P}(p^2)=\{\gamma^5,\Sigma(p)\}$, it is consistent to set $\Sigma(p)$ equal to $m(|p^2|/m^2)^{-\epsilon}$, which is the formula we used in the text (Sec. II F).

According to (2.31), the axial-vector-meson mass becomes in this approximation

$$\mu_A^{\,2}=\lambda^2g_A^{\,2}=\tfrac{4}{3}m^2\frac{1}{g_V^{\,2}/g_A^{\,2}-1} \tag{A14}$$

The mass μ_A is independent of coupling strength (in the sense that it depends on the ratio of $g_V^2/g_A^{\,2}$), and can become arbitrarily large as $g_V^2/g_A^{\,2}\to1$.

Of course the present considerations do not hold for strong coupling, where the lowest order Bethe-Salpeter approximation is not applicable. In particular, we do not wish to imply that pure axial-vector coupling would necessarily fail in that case.

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*Work supported in part through funds provided by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-3069.

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PHYSICAL REVIEW D VOLUME 8, NUMBER 8 15 OCTOBER 1973

Weak and Electromagnetic Forces as a Consequence of the Self-Interaction of the γ Field

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A model is proposed which geometrizes the electromagnetic and the weak forces. The absence of electromagnetic properties of the neutrino is understood in terms of a symmetry principle.

I. INTRODUCTION

The idea that all processes in nature can be linked to geometry is at least as old as Einstein's theory.[1] However, the Eötvos experiment that gave a good reason to use the geometrical method to describe gravitational (G) interaction has no counterpart in the rest of physics. Nevertheless, the fact that all known elementary particles have the same coupling constant with the electromagnetic (em) field could also suggest that em properties can be described by means of a modification of the structure of space-time. One important distinction between em and G forces is the universality of the gravitational process and the lack of electromagnetic properties in some particles, the neutrino for instance.

So, if one considers the geometric approach, one should be faced with the first crucial question: Why does the neutrino not have an electric charge? The second question is: What modification on the Riemannian nature of space-time could electromagnetic or even weak forces make? It is our purpose in this paper to show that these two questions can be answered. We will prove it, in a naive model, in the following.

II. THE FUNDAMENTAL OBJECTS OF SPACE-TIME

As in Ref. 2, we will assume that the fundamental objects of space-time are the generalized γ's that are linked to the metric tensor by the anticommutation relation

$$\{\gamma_{\mu}(x), \gamma_{\nu}(x)\} = 2g_{\mu\nu}(x)\underline{1}. \qquad (1)$$

The choice of a set of elements of a Clifford (C) algebra as fundamental has been considered in the literature many times. The real motivation for doing this rests on the fact that going from the $g_{\alpha\beta}(x)$ metric to the $\gamma_{\alpha}(x)$ submetric the additional degrees of freedom could be used to introduce new features in the theory. It seems to us, however, that this program was not fully realized, and that is the reason to come back to it here.

Expression (1) shows that besides the manifold