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Tachyons, Causality, and Rotational Invariance

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We extend the previously developed one-dimensional causal theory of tachyons to three dimensions. The result is a three-dimensional theory of interacting tachyons in which coordinates in reference frames with subluminal relative velocity are related by the Lorentz transformations, and in which no paradoxes involving causal loops can arise. The resulting theory involves a preferred spatial direction and preferred velocity perpendicular to that direction, so that physical laws governing tachyons are not invariant under rotations or proper Lorentz transformations. This lack of invariance should manifest itself even in processes involving only bradyons, to the extent that coupling to virtual tachyons is important. We discuss the limits which experimental evidence on the validity of rotational invariance places on tachyon couplings in the theory and possible additional experiments for searching for lack of such invariance. In general the limits which existing evidence for rotational invariance places on tachyon couplings in our theory are much less stringent than the very low limits on tachyon coupling strengths which were obtained in a recent experimental analysis of Danburg and Kalbfleisch; we propose a possible mechanism which might allow tachyon couplings of a reasonable magnitude without producing observable effects in the experiments considered by these authors.

Experimental searches for the tachyons predicted by the theory of Bilaniuk, Deshpande, and Sudarshan[1] have yielded consistently negative results.[2] One recent experimental analysis[3] seems to place particularly drastic limits on the coupling of tachyons to ordinary matter, and must cause considerable pessimism as to the existence of tachyons with couplings of sufficient strength to allow their detection. On the other hand, some other recent experiments seem to suggest, or at least allow an interpretation in terms of the existence of tachyonic celestial bodies.[4,5] The most striking example is in the study of the quasar 3C-279 made by means of very-long-baseline interferometry which yields results whose most direct interpretation is that the object contains components which are flying apart at several times the speed of light.[4] Also Weber's observations on gravitational waves[5] can be interpreted in terms of gravitational radiation by dense aggregates of tachyonic matter.[6] The above, coupled with the intrinsic interest of the problem, suggest that it might be worthwhile to attempt a new approach to a theory of tachyons.

One such approach is the extension of the Lorentz transformations to inertial coordinate systems having relative speeds greater than that of light.[7-10] This approach can be developed within the framework of special relativity only for a one-dimensional space. Some attempts have been made to extend this model to three dimensions[11-13] but the situation as regards these results is far from clear.[14] We do not wish to discuss these attempts in detail here, except to observe that one either appears to encounter problems of consistency and physical interpretation,[11,12,14] or else one obtains a situation in which tachyons and bradyons can interact only by the exchange of internal quantum numbers and not 4-momentum.[13] We note that the latter situation would offer no solution to the problems discussed in Refs. 4–6. In this paper we propose a procedure, which we believe is the only feasible approach, for extending the results of Refs. 7–10 to three dimensions.

The results yield a theory of tachyons with the following properties: The space-time coordinates of events in reference frames moving with subluminal relative velocities are connected by the

Lorentz transformations; tachyons and bradyons can interact with exchange of 4-momentum; no paradoxes involving causal loops can arise. The results of the present paper are, perhaps, of interest simply in demonstrating the existence of a theory with these three properties, which to our belief has not hitherto been done. The "price" one pays is that in the theory presented here certain physical laws (those involving tachyons) are not invariant under either rotations or proper Lorentz transformations.[15] (The theory also violates invariance under space inversion, although this is perhaps less alarming to one's sensibilities.) Whether this lack of invariance rules out the present theory on experimental grounds depends essentially on the (unknown) strength of tachyon-bradyon couplings. We will discuss this question somewhat further below.

In the interest of clarity for the three-dimensional case discussed here, we first summarize the argument which shows that causal loops cannot arise in the one-dimensional model discussed in I and II. There are two crucial observations. First, if the separation in time between two events on the world line of a particle is positive in some reference frame, then the generalized Lorentz transformations imply that the separation in space between the two events will always be positive (or always negative, depending on the choice of direction of the spatial axis) in a reference frame having superluminal velocity ($\beta > 1$) with respect to the original frame. Secondly, if a particle is a tachyon in one reference frame, it is a bradyon in any reference frame moving with speed greater than c relative to the first frame. From these observations it follows that if a particle always travels forward in time in a reference frame where it is a bradyon (e.g., its rest frame) then it always travels forward in space (although perhaps backwards in time) in reference frames in which it is a tachyon. It is then easy to show that it is impossible to send a signal around a loop by using any combination of tachyons and bradyons in such a way that the signal returns to the same point in space at a time earlier than it was sent, and hence the type of paradoxical behavior in which an event occurs if and only if it does not occur is avoided.

We should perhaps clarify what is meant by saying that a particle travels in a certain sense along its world line. The assumption underlying I and II, as well as most other discussions of possible causal anomalies,[16] is that there is a unique direction along the world line of a particle in which information flows, which we might call the causal direction. For example, if there exist particles with world lines connecting two pieces of apparatus, say A and A', then one of these, say A, can be called the source and the other, say A', the detector (or A the cause and A' the effect) by observing that if A is turned off from time to time, say by the output of a random number generator, then there are corresponding time intervals during which particles never appear at A', while no correlation will exist between turning off A' and the appearance of particles at A. (This formulation of the distinction between cause and effect is essentially that of Newton.)[17] The causal direction along the particle world lines is then from their appearance at A to their appearance at A'. We then make the assumption, based on our everyday experience, that the causal direction along the world line of a particle is in the direction of increasing time in reference frames in which it is subluminal. When we say that a particle travels in in a certain direction along its world line, we mean the causal direction. Presumably a tachyon whose causal direction is backward in time, and forward in space, which will have negative energy,[9,10] will *appear* to be a positive-energy tachyon traveling forward in time and backward in space in accordance with the reinterpretation principle.[1] Thus there would *appear* to be tachyons traveling in both directions in space, although in fact, according to the theory in I and II, all tachyons "really" travel in the positive spatial direction, in the sense that that is the causal direction along their world lines.

To generalize the considerations of I and II to three dimensions, we suppose that for some preferred direction in space there is a class of coordinate systems, which we call preferred coordinate systems, whose relative velocities are along that direction (but may have magnitude greater or less than c) such that the coordinates of events as seen in two of these systems, say S and S', are connected by the extended Lorentz transformation equations of II, namely:

$$x = \mu\gamma(x' + \beta t') , \tag{1a}$$

$$t = \mu\gamma(t' + \beta x') , \tag{1b}$$

where we have taken the x and x' axes to lie along the preferred direction and β is the velocity (in units with $c = 1$) of S' relative to S. μ and γ are given by

$$\gamma = |1 - \beta^2|^{-1/2} \tag{2}$$

and

$$\mu = \begin{cases} 1 & \text{for } \beta < 1 \\ \dfrac{\beta}{|\beta|} & \text{for } \beta > 1 \end{cases} \tag{3}$$

To Eqs. (1a) and (1b) we adjoin the usual transformation equations for the coordinates in the directions transverse to the relative velocity of the two reference frames,

$$y = y', \tag{1c}$$

$$z = z'. \tag{1d}$$

We refer to the preferred direction as the "tachyon corridor." For $\beta < 1$, Eqs. (1) are, of course, just the ordinary Lorentz transformations.

If we now let S' move with superluminal velocity along the tachyon corridor relative to S, we may define the complete set of reference frames admissable in our theory. (This is the same as the set of possible velocities for particles with rest mass $\neq 0$, since one assumes that the set of admissable reference frames are defined by the set of possible particle rest frames.) These consist of the set of all reference frames having any possible velocity of magnitude <1 in S, which we will call reference frames of class I, and those having any possible velocity of magnitude <1 in S', which we call class II. The two classes are well defined, since, from II, one knows that if the x component of the velocity of a particle in S' is less than unity, then the x component of its velocity (and therefore, of course, the magnitude of its velocity) in S will be greater than unity; this clearly holds also in the three-dimensional case, since the transformation equations for x and t are same as in the one-dimensional model.

We take the transformation equations connecting the coordinates of an event in two reference frames of the same class to be the Lorentz transformations, which agrees with Eqs. (1) for the special case that the relative velocity of the two reference frames is along the tachyon corridor. We take the procedure for transforming from a general reference frame of class I, say S_1, to a general reference frame in class II, say S_2, to be given by the following prescription. Transform from S_1 to some one of the preferred reference frames, say S, belonging to class I using the ordinary Lorentz transformation. Then transform from S to one of the preferred references frames in class II, say S', by an extended Lorentz transformation along the tachyon corridor, using Eqs. (1). Finally, transform from S' to S_2. The theory not only introduces a preferred direction in space, but it introduces a preferred velocity, namely that of the preferred reference frames, in the plane perpendicular to the tachyon corridor. That is to say, simply, that the set of preferred reference frames are, in fact, preferred; reference frames with a nonzero transverse velocity relative to the preferred frames enter the theory on a different footing. On the other hand, there is no preferred speed along the tachyon corridor in the theory. Equations (1) do not distinguish any particular value of the relative velocity along the tachyon corridor (except, of course, $|\beta| = 1$ is not allowed), so that the theory is invariant under proper Lorentz transformation in that direction.

The recipe for transforming to a reference frame with superluminal velocity is consistent in the sense that it does not depend on the particular choice of which of the preferred reference frames one uses in the intermediate steps. This follows from the group property of the ordinary Lorentz transformations with $\beta < 1$, and of the extended Lorentz transformations of II for the case of a single spatial dimension. The result of this prescription is *not*, as may easily be seen, the same as using an extended Lorentz transformation along the direction of the relative velocity of S_1 and S_2. One cannot construct a consistent theory using the latter procedure, since the group property is not obeyed even after spatial rotations are included; it is this fact which forces the introduction of the tachyon corridor as the preferred direction along which one can use the extended transformations.

Since Eqs. (1a) and (1b) are the same as in the one-dimensional theory, it follows by the same arguments as in the one-dimensional case that the three-dimensional theory is free of causal loops. We suppose that the time component of an interval along the world line of a particle in the causal direction is always positive in reference frames, say those of class II, in which the particle is a bradyon. Then it follows from (1a) and (1b) that, in reference frames (those of class I) in which the particle is a tachyon, the component of the interval along the tachyon corridor (the x axis) will always be positive. The tachyon corridor plays the same role for tachyons as the time axis does for bradyons. In the same way that the component of the world line of a bradyon along the time axis is always increasing, the component of the world line of a tachyon along the tachyon corridor is constantly increasing. Hence the same arguments as in I and II guarantee that no combination of tachyons and bradyons can be used to send a signal around a path in space-time so that it returns to its starting point (or, indeed, to any point with the same x coordinate as its starting point) at a time before it was sent.

The foregoing remarks indicate that we have, indeed, a consistent theory containing interacting tachyons, in which coordinate frames with subluminal relative velocity are connected by the Lorentz transformations, and in which causal loops cannot arise. We now examine several experimental consequences of the theory and consider briefly whether it can be excluded on the basis of experimental considerations.

We first note that it follows from Eqs. (1) that, in the preferred coordinate systems, the angle between the velocity vector of a tachyon and the tachyon corridor is less than 45°. This is easily shown. Suppose a particle is a bradyon in S' with speed $v'<1$. Let $v_\parallel'$ and $v_\perp'$ be the components of its velocity parallel and perpendicular to the tachyon corridor. If $v_\parallel$ and $v_\perp$ are the corresponding components in S, where the particle is a tachyon, Eqs. (1) yield immediately

$$v_\parallel = \frac{v_\parallel' + \beta}{1 + v_\parallel' \beta}, \tag{4}$$

$$v_\perp = \mu \frac{v_\perp' \,|1-\beta^2|^{1/2}}{1 + v_\parallel' \beta}, \tag{5}$$

where β is the velocity of S' relative to S. It is then easy to show that

$$v_\parallel{}^2 - v_\perp{}^2 = \frac{(1-v')^2(\beta^2-1) + (1+\beta v_\parallel')^2}{(1+\beta v_\parallel')^2} \tag{6}$$

whence, remembering that $v'<1$ and $\beta>1$, it fol-at once that $v_\parallel{}^2 > v_\perp{}^2$.

In a reference frame having a velocity u in a direction perpendicular to the tachyon corridor relative to a preferred reference frame, the velocity vectors of tachyons are confined to a cone whose opening angle is of order $\tan^{-1}(1-u^2)^{-1/2}$. It is amusing to note that it would be possible, depending on the velocity of the earth relative to the tachyon corridor and on what the orientation of experimental apparatus relative to the preferred direction happened to be, that a particular experimental arrangement for a tachyon production experiment might be incapable of resulting in the detection of tachyons, even if their production were possible.

Next we observe that the transformation equations (1) do not leave invariant the magnitude of the scalar product $a_0b_0 - a_1b_1 - a_2b_2 - a_3b_3$ of two four-vectors "a" and "b" under a transformation to a coordinate system with superluminal velocity. This is true because, as discussed in II, for $\beta>1$ Eqs. (1a) and (1b) give

$$x^2 - t^2 = t'^2 - x'^2 . \tag{7}$$

Thus Eqs. (1) result in

$$a_0b_0 - a_1b_1 - a_2b_2 - a_3b_3 = a_1'b_1' - a_0'b_0' - a_2'b_2' - a_3'b_3', \tag{8}$$

where we have taken the "1" axis to be along the tachyon corridor.

The noninvariance of the four-vector scalar product has two consequences. First, taking $a=b=(t,\vec{r})$, a lightlike space-time interval with $t^2-r^2=0$, one sees from (8) that $t'^2-r'^2=0$ only if $\vec{r}=(x_1,0,0)$, i.e., if $\vec{r}$ is along the tachyon corridor. Thus a light signal in class-I coordinate systems will have speed c in class-II systems only if, as measured by a preferred observer, it happens to be moving along the tachyon corridor. Let us take "our" coordinate systems, i.e., those with subluminal velocity relative to the earth, as those of class I. Clearly there is no direct conflict with experiment in a prediction that the speed of "our" photons, i.e., quanta of the fields which obey Maxwell's equations in our coordinate systems, do not in general have speed c in class-II coordinate systems, since we do not, after all, have any data on the speed of light taken with apparatus at rest in a superluminal reference frame. This result does have some implications, however. The transformation equations (1) are completely symmetric between coordinate systems of classes I and II. In particular c plays a special role for both classes of reference frames, in that an object moving with speed c in one reference frame moves with speed c in every reference frame *of the same class*; this is implied, of course, by the fact that the extended Lorentz transformations are, for subluminal velocities, simply the usual ones, constructed to make the speed of light invariant. It is thus most natural and pleasing to assume that physics is at least to some extent the same in the two classes of reference frames. In particular, it seems very natural to suppose that there are fields which obey Maxwell's equations in reference frames of class II—we might call these "their" electromagnetic fields.[18] However, if our proposed extended Lorentz transformation equations were to be correct, "their" electromagnetic fields would not be "our" electromagnetic fields (and vice versa) since their field quanta do not, in general, propagate with speed c, and hence the fields do not obey Maxwell's equations, in our reference frames. Hence, there is no reason to suppose that the coupling strength of their photons to our charged particles is given by the electric charge e; for that matter, the coupling between a class-II photon and a class-I particle might not even have anything to do with the particular property which we call electric charge which governs the coupling of class-I particles to the fields obeying Maxwell's equations in class-I coordinate systems. Thus even if there are "charged" tachyons, i.e., tachyons which behave in class-II reference frames the way ordinary charged particles behave in our reference frames, the present theory would imply nothing about the way in which they would couple to ordinary matter; in particular, their coupling to ordinary matter might be arbitrarily weak, so that no minimum rates for the production of tachyons would be implied. This is, of course, a two-edged aspect of the theory. On the one hand it means that the negative results obtained thus far in experimental searches for tachyons[2] certainly have no absolute implications about their nonexistence. On the other hand, it could well be that even if tachyons exist, their coupling to ordinary matter would be so weak as to make their observation, for all practical purposes, impossible.

The second consequence of the noninvariance of the magnitude of the four-vector scalar product, as expressed by Eq. (8), is that one will observe a continuum of "rest masses" for tachyons. Equation (8) implies that the energy and momentum of a tachyon in a preferred reference frame obey

$$E^2 - p^2 = -m_0'^2 - 2p_\perp^2 , \tag{9}$$

where m_0' is its rest mass in class-II reference frames where it is subluminal, and $p_\perp$ is the component of its momentum perpendicular to the tachyon corridor. We assume, as implied by the extended principle of relativity, that m_0' has a definite value, just as the rest mass of a bradyon has a definite value in class-I reference frames. Then, from (9), it follows that the value of the "rest mass," i.e., $E^2 - p^2$, of a particular kind of tachyon will have a unique value in class-I reference frames only for those particles having the same transverse momentum in the preferred reference frames. There is, of course, no violation of symmetry between the two classes of coordinate system involved here. In class-II coordinate systems, which include all tachyon rest frames, tachyon rest masses have well-defined real values, and class-I bradyons (bradyons in class-I systems) would be observed to have a continuum of negative values for $E'^2 - p'^2$. One further observation may be made. The one-dimensional theories involving extended Lorentz transformations to superluminal reference frames[8,10] kinematically resemble the previous tachyon theories.[8] It is clear from (9) that the present three-dimensional generalization does not, except along the tachyon corridor.

The most startling aspect of the present theory is the lack of invariance under rotations and under pure Lorentz transformations perpendicular to the tachyon corridor. If tachyons couple to ordinary matter, one will observe a lack of invariance even in processes involving only bradyons because of virtual tachyon effects. Since one is still supposing that reference frames with subluminal relative velocities are connected by the usual Lorentz transformations, the experimental predictions which provide the principal support for our belief in special relativity, namely the validity of relativistic kinematics and the invariance of the speed of light, can hold equally well for the present theory, so that the success of these predictions provides no evidence against the tachyon corridor idea. In principle the lack of invariance under proper Lorentz transformations would appear, e.g., as a difference in the values obtained for the pp cross section in two measurements taken with the same beam 12 hours apart, when the earth's rotation will have caused the direction in space of the incident beam to reverse. The two experimental situations differ only by the fact that the center-of-mass frames in the two cases are connected by a proper Lorentz transformation along the beam direction. It is not clear what the experimental limits on such a variation are, since it has probably not been looked for explicitly when such cross sections have been measured. In any event, as we will see, experimental tests of rotational invariance put sufficiently small limits on the magnitude of tachyon-bradyon couplings that any effects of the type we are discussing due to tachyons would be unobservable. We remark in passing that, on general grounds, a careful search for variation in the pp cross section with the earth's rotation, i.e., with the velocity of the center-of-mass system would, to the extent that systematic effects could be eliminated, be useful for establishing limits on (or detecting) a noninvariance of the strong interactions under proper Lorentz transformations, something on which there does not seem to be much in the way of existing experimental evidence.

The lack of rotational invariance (RI) in the present theory, and the consequent violation of conservation of angular momentum, might be expected to manifest themselves more readily. The best experimental limits on the violation of RI would appear to come from the validity of selection rules derived from angular momentum conservation in electromagnetic decays of nuclei. Some very long lived nuclear levels are known which decay primarily by the emission of $E5$ multipole radiation, although the reduced decay rates for these transitions are down by factors of the order of 10^{18} compared to the numerous measured rates for $E2$ (electric quadrupole) radiation.[19] (Presumably the $E5$ reduced rates are even smaller compared with those for electric dipole transitions, but rather little data are available on the latter.) The factor of 10^{18}, as well as the rates for other multipole transitions, are in rough agreement with the general theoretical expectation that the rate for emission of multipole order l is proportional to $(R/\lambda)^{2l+1}$, where R is the nuclear radius and λ the wavelength of the emitted radiation, so that the low rates for the $E5$ transitions are certainly due primarily to the high multipole order, and not to any details of the nuclear physics. The fact that the transition occurs by the emission of such a high multipole is due, of course, to the large difference in the angular momentum of the initial and final states. The fact, then, that these transitions occur with such extremely slow rates implies that the eigenstates of the nuclear Hamiltonian correspond to definite values, J_i, of the angular momentum with a maximum admixture of other values being of the order of 10^{-9} in amplitude. (If the admixture consisted almost entirely of states with angular momenta differing from the J_i by ± 1, the limits on the admixture would be only about 10^{-3} in amplitude. There seems to be, however, no obvious reason why a violation of RI of the type we are considering here would lead exclusively to admixtures of states differing by only ± 1 in angular momentum.) This result then implies a bound on the square of the coupling constant between nucleons and virtual tachyons of about 10^{-9} times the square of typical strong coupling constants, or 10^{-6} times the fine-structure constant. Similar arguments for the rotational invariance of the nuclear Hamiltonian arise from the validity of selection rules in nuclear β decay, although the implied limits in that case, while comparable, appear somewhat less stringent. The β-decay selection rules imply an additional kind of restriction on tachyon couplings. Namely, if there exists a tachyon state which couples both to a proton and antineutron, and to a lepton pair, i.e., an electron and antineutrino, with coupling constants $G_{p\bar{n}t}$ and $G_{e\bar{\nu}t}$, respectively, then one must have $G_{p\bar{n}t}\ G_{e\bar{\nu}t} \lesssim 10^{-9}G$, where G is the usual coupling constant in the current-current theory of weak interactions; that is, the product $G_{p\bar{n}t}\, G_{e\bar{\nu}t}$ must be less than about 10^{-20} times the square of typical strong-interaction coupling constants. If this were not so, tachyon exchange could compete with the conventional weak interactions to an extent that would violate the observed validity of β-decay angular momentum selection rules, not because the initial and final states do not correspond to definite values of J, but because the interaction Hamiltonian would contain a piece which would not transform as a scalar under rotations.

We also mention briefly experimental limits on the violation of RI in various other situations. The best limits in processes involving coupling to electrons would again appear to follow from the validity of angular momentum selection rules (this time for atoms). Since one is able to observe only first-order forbidden atomic transitions, because of collisional deexcitation, whose decay rates are down by factors of the order of 10^6 compared with the rate for normal transitions, one obtains a limit of about 10^{-3} of the fine-structure constant on the square of the coupling constant between electrons and virtual tachyons. We also note that the availability of vast amounts of accurate astronomical data implies that any effects which violate RI arising from a long-range force affecting all mass equally must be very small compared with the usual gravitational interaction.

There is comparatively little evidence for rotational invariance in strangeness-changing weak decays, and one might hope that these would afford the opportunity of observing the RI-violating effects prediced by the present theory. Suppose that tachyon couplings are such that tachyons couple to baryons with strangeness change of either 0 or 1, in the way that the weak vector boson is assumed to do; that is, suppose, for example, that there exists a tachyon which can couple both to a proton and antiproton, and to a neutron and $\bar{\Lambda}$. Then in the present theory, if the magnitude of the tachyon couplings is comparable to usual weak-interaction couplings, tachyon exchange processes can compete with those involving W exchange and could be expected to lead to such effects as anisotropy in the decay $K^+ \to \pi^+ + \pi^0$ in the center-of-mass system. It is not clear to us what the experimental limits on such an anisotropy are, but we suspect they are not very severe. This is especially true since, depending on the times at which data were taken and the orientations of the tachyon corridor, the earth's rotation, unless corrected for, may wash out the effect, since a fixed direction in space will not correspond to a fixed direction with respect to the experimental apparatus. The presence of such anisotropies could provide a method of detecting the effects of tachyons obeying our theory if tachyon-bradyon couplings are of the magnitude of the usual weak couplings. (It could, of course, be that tachyon couplings which contribute to weak processes are correspondingly weaker than those which contribute to processes which proceed by strong or electromagnetic interactions among bradyons, in which case one would gain nothing by searching for tachyonic effects in weak processes. There is, however, no *a priori* reason why tachyon couplings should, for example, respect conservation of strangeness, so it is possible that their relative importance could be much greater in a process such as $K \to 2\pi$.) However, the Fermi constant, G, is of order 10^{-6} in units of the K mass, i.e., about 10^{-7} times typical strong-coupling constants. Thus virtual tachyons even if coupled with the maximum strength consistent with observations on the validity of RI, could lead to anisotropies in K decay of only 1% or less, which would be difficult to detect.

From the above limits on the ratio of tachyon couplings to the fine-structure constant, one would estimate an upper limit on tachyon production cross sections of perhaps 10^{-6} times photoproduction cross sections; the latter are typically of the order of 100 μb. The upper limit of about 10^{-4} μb on tachyon production cross sections which is thus suggested if the present theory is not to conflict with experimental evidence on RI is substantially below the sensitivity of any of the experimental searches for tachyon production, and is small enough that even if tachyons described by the present theory were to exist, and their couplings to ordinary matter to have the maximum values consistent with evidence for RI, the detection of tachyon production would be very difficult. In addition, of course, the small limits on tachyon couplings in our theory mean that their direct observation, even after having been produced, would be exceedingly difficult. In particular we have seen that any coupling they have must be much weaker than the usual electromagnetic couplings, so that those experiments which look for Čerenkov radiation from tachyons would not be expected to be successful, leaving experiments of the missing-mass type, which seek to observe systems of negative "rest mass" squared, as the only hope for the direct observation of individual tachyons; the feasibility of such experiments with cross sections as small as implied by our theory is, clearly, extremely doubtful.

Actually, the strongest experimental argument against the existence of tachyons is given by Danburg and Kalbfleisch.[3] They analyze several experiments to look for decays of protons or electrons into tachyons or tachyon-antitachyon pairs. For protons, they find no evidence of such decays in which the energy gain of the proton in the laboratory system is greater than a few MeV. This corresponds to a squared rest mass for the tachyon system, which is also the squared 4-momentum transfer to the proton (the usual Mandelstam variable t) less than about -0.01 GeV^2. The analysis of Danburg and Kalbfleisch leads to a tachyon-proton coupling constant squared of the order of 10^{-13} times the square of the graviton-proton coupling, or about 10^{-50} times the fine-structure constant. For electons, comparable but slightly less stringent limits on the strength of the coupling are obtained, and the results extend down to even smaller limits on the magnitude of t. This analysis suggests that one will never observe individual tachyons, since even if they did exist with couplings this weak, it seems unlikely that they would ever be detectable. The fact that the experimental results extend to tachyon emission with very small values of t seems to preclude any argument that tachyons have couplings with a reasonable order of magnitude but that form factors for tachyon emission fall very rapidly with t. There is one way around the conclusion that tachyon couplings obey these very stringent limits however. Consider the reaction

$$B(0)\to B(-\vec{p})+T(+\vec{p}), \qquad (10)$$

where B is a bradyon, T a tachyon or system of tachyons, and the symbols in parentheses are the three-momenta of the corresponding particles, so that we are in the rest frame of the initial bradyon. If m_t^2 is the (negative) rest mass squared of the tachyonic system, then the possible energies, E_t, of the tachyonic system are

$$E_t=\pm(p^2+m_t^2)^{1/2}. \qquad (11)$$

Of course if (10) is to occur as a real process, then conservation of energy requires that E_t have the value corresponding to choosing the minus sign in (11); it is reactions of this type which the results of Ref. 3 show can occur only with extremely small couplings. It seems conceivable, however, that the nature of the interaction between bradyons and tachyons is such that (10) can occur only with a positive-energy tachyon, i.e., with the choice of the positive sign in (11). We emphasize that this is a relativistically invariant notion. There will be reference frames in which the tachyon energy is negative, reflecting the spacelike nature of tachyon 4-momenta. However, all observers will agree on the sign of the tachyon energy *in the initial bradyon rest frame*. Likewise, some other bradyon would only be able to emit tachyons *with positive energy in its rest frame.* This assumption implies, of course, that (10) can occur only as a virtual process, but this would still allow the production of real tachyons in reactions such as $B+B\to B+B+T$. Thus, if one assumes that (10) can occur only with the plus sign in (11), it would appear that the existence of tachyons with couplings to ordinary particles of such a magnitude as to at least give hope of being able to observe them is not ruled out experimentally; if they are described by the theory developed in this paper, the limits on their couplings would appear to be those discussed above which follow from the observed validity of RI. If (11) can hold with the minus sign, then the extremely low limits on tachyon coupling strengths obtained in Ref. 3 hold, and the possibility of detecting individual tachyons seems remote. As a matter of principle, of course, tachyons described by the theory developed in this paper could exist in nature with coupling strengths obeying the limits obtained in Ref. 3, since these are many orders of magnitude too small to lead to observable violations of RI. It is conceivable in this case that massive tachyonic bodies, should they exist and couple coherently to "our" luxon or bradyon fields, might be detectable, even though individual tachyons would not.

The assumption that a particle can only emit tachyons with positive energy in its rest frame has another interesting result. It leads to the elimination of poles in scattering amplitudes in the variable t which can occur for physical values of t and hence result in infinite differential cross sections if single, stable tachyons can be exchanged, as has been pointed out, e.g., in Ref. 20. To see how this comes about, consider a bradyon reaction $A+B\to A'+B'$. Let us work in the rest frame of particle B, which we assume to be stable. Then, in notation analogous to that in Eq. (10), the reaction can be written

$$A(\vec{p})+B(0)\to A'(\vec{p}-\vec{q})+B'(\vec{q}) \; . \tag{12}$$

We consider the contribution to the scattering from the process in which particle B emits a virtual tachyon T of three momentum $-\vec{q}$, later absorbed by A, and recoils as a B'; that is, we consider the contribution in 2nd-order perturbation theory from an intermediate state containing $A(\vec{p})$, $B'(\vec{q})$, and $T(-\vec{q})$. Let ΔE be the difference in energy of the B' and the B so that

$$\Delta E=(m_{B'}{}^2+q^2)^{1/2}-m_B \; , \tag{13}$$

where the stability of B implies $m_{B'}\geqslant m_B$ so that $\Delta E>0$. [Note that it follows from Eq. (9) that $q^2>0$ provided the tachyon is not massless; there are no zero-momentum tachyons, just as there are no zero-energy bradyons, except in the case of massless particles.] Now one will get an infinite contribution to the cross section, coming from the vanishing of the perturbation-theory energy denominator, whenever the energies of the intermediate and initial states are equal, i.e., when

$$\Delta E=-E_T(-\vec{q}) \; , \tag{14}$$

where the tachyon energy E_T is given by

$$E_T(-\vec{q})=\pm[q^2+(m_T(-\vec{q})^2]^{1/2} \; , \tag{15}$$

and $m_T(-\vec{q})$ is the observed "rest mass" of the tachyon when it has momentum $-\vec{q}$; we remind the reader that, from Eq. (9), the value of $E_T{}^2-q^2$ depends on $\vec{q}$. For Eq. (14) to be satisfied, it must be true that

$$\begin{aligned} t &= (\Delta E)^2-q^2 \\ &= [m_T(-\vec{q})]^2 \; . \end{aligned} \tag{16}$$

In the case of the exchange of ordinary particles with positive m^2 the analog of (16) gives no problem, as it always leads to a pole at a value of t outside the physical region. However for tachyons $m_T{}^2<0$, and the value of t satisfying (16) can be in the physical region. However, the infinity only occurs if Eq. (14) is satisfied, and since $\Delta E>0$, that implies that one must take the minus sign in (15). If it is true that tachyons are only emitted with positive energy in the rest system of the emitting particle, then the energy of the exchanged virtual tachyon is given by (15) with the plus sign, (14) is not satisfied, and no infinity occurs.

The problem of infinities in scattering amplitudes can also be avoided as in Ref. 20, where it is postulated that only unstable tachyons are exchanged, i.e., that one exchanges systems of tachyons which can have a spread of energies for a given momentum. [Note, incidentally, that the spread in energy, or mass, corresponding to the presence of an unstable system, is different from the dependence of mass on momentum in the present theory as exhibited in Eq. (9). In the latter case, one can have tachyonic systems which have a definite mass corresponding to any particular value of their momentum, albeit the value depends on the momentum. In the case of an unstable particle, the mass can vary even when the momentum is unchanged.] The authors of Ref. 20 use such a model to explain some of the structure in momentum transfer observed in strong-interaction differential cross sections. Tachyons obeying the theory presented in the present paper could not, of course, do this even if unstable, since they must be coupled far too weakly in order to avoid observable violations of RI. However, in a theory with only unstable tachyons, which cannot propagate over long distances, it would appear that, at least as a practical matter, problems with causal loops would not arise. It was largely to avoid these problems that we were driven, in the present work, to a theory with a preferred spatial direction. Hence if one is willing to forego the possibility of stable tachyons, there is presumably no reason to abandon RI, and hence no need to assume a limit on the strength of tachyon couplings.

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[15]We emphasize, perhaps unnecessarily, the difference between the statement that coordinates in different reference frames are connected by the Lorentz transformations, and the statement that the laws of physics are invariant under such transformations. Although, by construction, the Lorentz transformations do guarantee the invariance of certain laws of physics (e.g., Maxwell's equations and the conservation of four-momentum) which we believe are valid, it is perfectly possible that the coordinates of events as seen by different observers are related by Lorentz transformations, but that some (or indeed, in principle all) laws of physics are not the same when expressed in different reference frames.
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[18]That the laws of tachyon and bradyon physics are the same in both class-I and class-II coordinate systems is plausible within the context of the three-dimensional theory, since the derivation of Eqs. (1a) and (1b) in the one-dimensional theory was based on the postulate of the principle of relativity of superluminal, as well as subluminal, transformations. In the three-dimensional theory, even though Lorentz invariance is not preserved in general, the generalized relativity principle is still valid for transformations between preferred reference frames. This suggests the possible complete equivalence of the two classes of reference frames, i.e., that is there is no experiment that an observer can perform which will tell him to which class of reference frames he belongs. This equivalence has been called the "extended principle of relativity" by Parker in Ref. 8 and the "principle of duality" in Refs. 12 and 13.
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Note on Motion in the Schwarzschild Field*

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It is often said that the speed of a freely falling test particle in the Schwarzschild field approaches the speed of light at the Schwarzschild radius. It is shown that this is not the case.

Many discussions of motion in the Schwarzschild field say that the speed of a freely falling test particle approaches the speed of light as the particle approaches the Schwarzschild radius; Refs. 1–5 indicate a sampling of such discussions. One might infer from these sources that a test particle does, in fact, cross the Schwarzschild radius with the speed of light. It is the purpose of this note to emphasize that this is not the case. (For simplicity, only radial geodesics will be discussed.)

Before giving what I consider to be the correct description, it may be instructive to examine briefly two of the "standard" treatments; other treatments may be analyzed in similar fashion. Zel'dovich and Novikov say[6] that the velocity they use "has direct physical significance. It is the velocity measured by an observer who is at rest (r, θ, ϕ constant) at the point which the particle is passing." The coordinates referred to here are the usual Schwarzschild coordinates; with a suitable choice of units, the line element may be written as

$$ds^2 = (1 - 2m/r)dt^2 - (1 - 2m/r)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)\,. \tag{1}$$

It is clear from Eq. (1) that an observer "at rest" at the Schwarzschild radius, $r = 2m$, must move with the speed of light. One might say, then, that the reason a test particle's speed approaches the speed of light is that it is measured by a family of observers whose speeds approach the speed of light. There is no reason to conclude from such measurements that the particle actually reaches