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Limits on the Photon Mass*

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An upper limit is found for the photon mass by exploiting the gravitational deflection of electromagnetic radiation.

Various techniques for setting limits on the photon mass have been reviewed recently by Goldhaber and Nieto.[1] These techniques varied from terrestrial measurements of c at different frequencies to observations of the earth's magnetic field. This comment describes another method for setting limits on the photon mass by exploiting the gravitational deflection of electromagnetic radiation. A better upper limit than that already published is not obtained, but the method is interesting and its presentation adds to the evidence restricting the magnitude of the photon mass.

The general theory of relativity predicts a deflection of starlight by the sun of 1.75 sec of arc. We may ask how this deflection is altered if the photon has a small rest mass μ. The answer is

$$\theta=\theta_0\left(1+\frac{\mu^2c^4}{2h^2\nu^2}\right),\qquad \theta_0=\frac{4MG}{Rc^2}, \tag{1}$$

where θ is the deflection angle for photons of finite mass, and θ_0 is the deflection angle for massless photons. R is the photon impact parameter (normally the solar radius), M the solar mass, G the gravitational constant, c the speed of light, h Planck's constant, and ν the frequency.

The correction term $\mu^2c^4/2h^2\nu^2$ represents the correction to the Einstein derivation due to a finite photon mass. If we now set this correction term equal to the difference between the measured deflection angle and the calculated deflection angle for photons of zero rest mass, a limit on the photon mass is obtained:

$$\delta\equiv\theta_{\text{measured}}-\theta_0=\theta_0\frac{\mu^2c^4}{2h^2\nu^2}.$$

Since the deflection measurements are typically only 10% accurate, the correction δ will normally be set equal to the error in the measured deflection angle. Thus, an expression setting an upper limit for the photon mass can be written in terms of δ, θ_0, and the photon energy:

$$\mu\lesssim\frac{h\nu}{c^2}\left(\frac{2\delta}{\theta_0}\right)^{1/2}. \tag{2}$$

A brief description of the derivation of Eq. (1) follows. The differential equation that described the general-relativistic motion of a particle of mass μ in the gravitational field of a star is given by[2]

$$\frac{d^2u}{d\phi^2}+u=\frac{MG}{H^2c^2}+\frac{3MGu^2}{c^2}, \tag{3}$$

where

$$u=\frac{1}{r},$$

$$H=r^2\frac{d\phi}{ds},$$

ds = Schwarzschild line element.

The terms on the right-hand side of (3) represent the general-relativistic correction to the classical equation. For massless photons $H \to \infty$. It is in this limit that one calculates the standard starlight deflection angle θ_0. For photons of finite mass the quantity H is large but finite. Its contribution leads to the desired correction term.

The solution to the differential equation is found by treating the right-hand side of (3) as a perturbation to the Newtonian trajectory. The relevant coordinate system is shown in Fig. 1. In the absence of the perturbing terms

$$\frac{MG}{H^2c^2}+\frac{3MGu^2}{c^2},$$

a solution to (3) is the equation $U=[\cos(\phi)]/R$ for a straight line which has an impact parameter R measured from the solar center. By substituting this equation into the right-hand side of (3) it is possible to obtain a second approximation for U:

$$u \approx \frac{\cos(\phi)}{R}+\frac{2MG}{c^2R^2}\sin^2(\phi)+\frac{MG}{c^2R^2}\cos^2(\phi)+\frac{MG}{H^2c^2}.$$

The angular deviation of photons is found by changing to Cartesian coordinates and calculating the slope dy/dx as $r \to \infty$:

$$\theta = 2\left|\frac{dy}{dx}\right|,$$

$$\theta=\frac{4MG}{Rc^2}\left(1+\frac{R^2}{2H^2}\right). \tag{4}$$

Only H need now be determined. Recall that $H=r^2\,d\phi/ds$ where ds^2 is

$$ds^2=-\left(1+\frac{MG}{2rc^2}\right)^4(dx^2+dy^2+dz^2)+\frac{(1-MG/2rc^2)^2}{(1+MG/2rc^2)^2}c^2(dt)^2,$$

and since $MG/Rc^2 \sim 10^{-6}$, it can be neglected; then ds^2 reduces to the special-relativistic result:

$$\gamma ds = c dt,$$

$$\gamma=(1-v^2/c^2)^{-1/2}.$$

The photon orbital angular momentum is $\gamma\mu r^2 d\phi/dt = \gamma\mu vR$, so that H becomes

$$H=\frac{\gamma vR}{c}\approx \gamma R=\frac{Rh\nu}{\mu c^2}.$$

The correction term δ is now specified in terms of the photon mass μ:

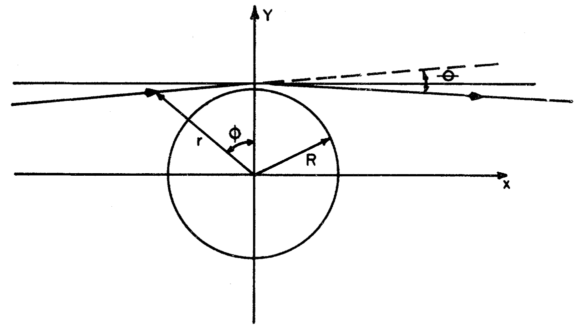

FIG. 1. Schematic of the photon path about the solar limb showing the undeflected Newtonian trajectory. The Newtonian path is $\cos(\phi)/R$, where R is the impact parameter and ϕ the angular position. The radial position of a photon is given by r, and the total angular deflection is defined as θ.

$$\delta=\frac{2MGR}{H^2c^2}=\frac{2MG\mu^2c^2}{Rh^2\nu^2}, \tag{5}$$

$$\mu \leqslant \frac{h\nu}{c^2}\left(\frac{2\delta}{\theta_0}\right)^{1/2}. \tag{6}$$

This result is essentially classical in nature and not dependent on general relativity. Our purpose in starting with the general-relativistic equation of motion was to obtain the correct magnitude for the total deflection. In Eq. (3) the term $3MGu^2/c^2$ is the general-relativistic correction to the Newtonian equations and leads to the Einstein deflection, which is twice the classical deflection. The term MG/H^2c^2 is entirely classical since the Schwarzschild line element (ds) was set equal to the flat-space result.

The classical calculation of the photon deflection is carried out by determining the impulse given to the photon perpendicular to its path as it passes by the solar limb. The result is found to be $\theta=2MG/Rv^2$. The velocity v is a function of the photon mass and energy, and is $v^2=c^2[1-(\mu c^2/h\nu)^2]$. Substituting this into the equation for the classical deflection and expanding $1/v^2$ in powers of $(\mu c^2/h\nu)$ we find the deflection

$$\theta=\frac{2MG}{Rc^2}+\frac{2MG}{Rc^2}\frac{(\mu c^2)^2}{(h\nu)^2}.$$

The leading term is off by the factor of 2, but the correction term involving the mass μ is the same as that previously obtained in Eq. (5).

Numerical results. Limits on the photon mass are now found using Eq. (6) and the data currently available on the deflection of electromagnetic radiation by the sun. θ_0 is the normal Einstein deflection angle, and depends on the distance at which photons pass the solar center. In each of the three cases which follow, we have set δ equal to the measurement uncertainty in the angular deflection.

(i) Visible light.[3]

$\nu = 5\times 10^{15}$ Hz ,

$\delta \cong 0.1$ sec of arc ,

$\mu < 1\times 10^{-33}$ g $= 0.56$ eV.

(ii) Radio source 3C 279 (Ref. 4).

$\nu = 3\times 10^{9}$ Hz ,

$\delta \cong 0.1$ sec of arc ,

$\mu < 7\times 10^{-40}$ g $= 4\times 10^{-7}$ eV .

(iii) Intercontinental baseline interferometry.[5,6] This technique promises to improve the deflection measurements at radio frequencies to at least 0.001 sec of arc. If achieved, it would give a mass limit of $\mu < 7\times 10^{-41}$ g $= 4\times 10^{-8}$ eV.

It is interesting to discuss how a better upper limit on the photon mass might be obtained using Eq. (6). First, if the deflection measurements could be made with greater accuracy the value of δ would be reduced, giving a better limit. This improvement in measurement seems unlikely since the radio telescopes are operated at their diffraction limit and in order to improve their resolution larger apertures must be obtained. Intercontinental baseline interferometry is currently being developed, and the next significant improvement would have to be an earth-moon interferometer, but such an instrument would give less than an order of magnitude improvement in the photon mass limit. Second, if deflection measurements can be obtained at longer wavelengths, while maintaining the telescope resolution, the mass limit will be improved in direct proportion to the wavelength. This possibility is intrinsically limited because as the wavelength is increased the photons are subject to increasing refraction in the solar atmosphere. This effect on the angular deflection can be calculated if one knows the electron-density profile in the solar corona. However, the electron density is not known precisely; further, as the frequency is decreased below the plasma frequency the solar atmosphere becomes a reflector for the radiation. The index of refraction (neglecting the solar magnetic field) is $N^2 = 1 - \omega_p^2/\omega^2$ where ω_p is the plasma frequency. Assuming an electron density in the corona of the Allen-Baumback model,[7] $N_e = 1.55\times 10^{8}\rho^{-6}$ electrons per cubic centimeter, where ρ is the distance measured in solar radii of the photon's closest approach to the sun's center, we calculate the plasma frequency at the solar limb to be 600 MHz. This is certainly a lower limit on the frequency and would give only an order of magnitude improvement in the mass limit.

Note added. It has been pointed out that this technique for setting limits on the photon mass is in principle worse than methods that directly measure the dispersion of light passing through interstellar space. This can be demonstrated by noting that the limit on the photon mass as determined by measuring starlight dispersion is given by

$$\mu \le \frac{h\nu}{c^2}\left(\frac{2\Delta v}{c}\right)^{1/2},$$

where Δv is the difference in velocity of two wavelengths. This equation is the same as Eq. (6) with δ/θ_0 replaced by $\Delta v/c$.

δ is essentially the angular resolution of the telescope and must be larger than λ/D where D is the telescope aperture. Therefore $\delta/\theta_0 > \lambda/D\theta_0$. Further, $\Delta v/c = c\Delta t/L$, where L is the distance to the star and Δt is the arrival-time difference between the two wavelengths considered. Since Δt must be larger than $1/\nu$ (ν is the frequency at the longer wavelength), $\Delta v/c$ must be greater than λ/L. In summary we list

$$\frac{\delta}{\theta_0} > \frac{\lambda}{D}\left(\frac{1}{\theta_0}\right),$$

$$\frac{\Delta v}{c} \ge \frac{\lambda}{L}.$$

Clearly δ/θ_0 is always much larger than $\Delta v/c$, since L can be many light years and D can at best be the earth-moon distance. Also, θ_0 is on the order of 10^{-5} rad and further increases the magnitude of δ/θ_0 in relation to $\Delta v/c$. Thus, in principle direct measurement of starlight dispersion must set a lower limit on the photon mass than measurements of starlight deflection. However, in practice these two methods give comparable results. This is because the starlight dispersion technique is limited by the interstellar plasma.

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[1]A. Goldhaber and M. Nieto, Rev. Mod. Phys. 43, 277 (1971).
[2]Richard Tolman, *Relativity, Thermodynamics, and Cosmology* (Oxford Univ. Press, New York, 1962), p. 208.
[3]W. W. Campbell and R. Trumpler, Lick Obs. Bull. 11, 41 (1923); 13, 130 (1928).
[4]R. A. Sramek, Astrophys. J. 167, L55 (1971).
[5]William Ward, Astrophys. J. 162, 345 (1970).
[6]K. Kellermann, Sci. Am. 72, (No. 2) 226 (1972).
[7]C. N. Allen, Mon. Not. R. Astron. Soc. 107, 426 (1947).

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Tachyons, Causality, and Rotational Invariance

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We extend the previously developed one-dimensional causal theory of tachyons to three dimensions. The result is a three-dimensional theory of interacting tachyons in which coordinates in reference frames with subluminal relative velocity are related by the Lorentz transformations, and in which no paradoxes involving causal loops can arise. The resulting theory involves a preferred spatial direction and preferred velocity perpendicular to that direction, so that physical laws governing tachyons are not invariant under rotations or proper Lorentz transformations. This lack of invariance should manifest itself even in processes involving only bradyons, to the extent that coupling to virtual tachyons is important. We discuss the limits which experimental evidence on the validity of rotational invariance places on tachyon couplings in the theory and possible additional experiments for searching for lack of such invariance. In general the limits which existing evidence for rotational invariance places on tachyon couplings in our theory are much less stringent than the very low limits on tachyon coupling strengths which were obtained in a recent experimental analysis of Danburg and Kalbfleisch; we propose a possible mechanism which might allow tachyon couplings of a reasonable magnitude without producing observable effects in the experiments considered by these authors.

Experimental searches for the tachyons predicted by the theory of Bilaniuk, Deshpande, and Sudarshan[1] have yielded consistently negative results.[2] One recent experimental analysis[3] seems to place particularly drastic limits on the coupling of tachyons to ordinary matter, and must cause considerable pessimism as to the existence of tachyons with couplings of sufficient strength to allow their detection. On the other hand, some other recent experiments seem to suggest, or at least allow an interpretation in terms of the existence of tachyonic celestial bodies.[4,5] The most striking example is in the study of the quasar 3C-279 made by means of very-long-baseline interferometry which yields results whose most direct interpretation is that the object contains components which are flying apart at several times the speed of light.[4] Also Weber's observations on gravitational waves[5] can be interpreted in terms of gravitational radiation by dense aggregates of tachyonic matter.[6] The above, coupled with the intrinsic interest of the problem, suggest that it might be worthwhile to attempt a new approach to a theory of tachyons.

One such approach is the extension of the Lorentz transformations to inertial coordinate systems having relative speeds greater than that of light.[7-10] This approach can be developed within the framework of special relativity only for a one-dimensional space. Some attempts have been made to extend this model to three dimensions[11-13] but the situation as regards these results is far from clear.[14] We do not wish to discuss these attempts in detail here, except to observe that one either appears to encounter problems of consistency and physical interpretation,[11,12,14] or else one obtains a situation in which tachyons and bradyons can interact only by the exchange of internal quantum numbers and not 4-momentum.[13] We note that the latter situation would offer no solution to the problems discussed in Refs. 4–6. In this paper we propose a procedure, which we believe is the only feasible approach, for extending the results of Refs. 7–10 to three dimensions.

The results yield a theory of tachyons with the following properties: The space-time coordinates of events in reference frames moving with subluminal relative velocities are connected by the