to the sum rule is negligible. This is necessitated by the fact that we have yet to write a consistent dual model with broken SU(3) couplings.

[16]The discontinuity function in this limit was first calculated by D. Gordon and G. Veneziano, Phys. Rev. D 3, 2116 (1971); for details of the applications of dual models in inclusive reactions, see Ref. 10 and references therein.

[17]In this model

$$\beta(t) = \alpha'^2(m_\pi^{\,2} - \tfrac{1}{2}t - c^2)^2\Gamma^2(-\alpha(t))\Gamma(2\alpha(t) - \alpha_0),$$

notice that in the physical region the zeros of $\sin\pi[\alpha_0 - 2\alpha(t)]$ are canceled by the poles of the Γ function. For further discussion, see Ref. 10 and references therein.

[18]K. Huang and G. Segrè, Phys. Rev. Lett. 27, 1095 (1971).

[19]

$$f(\rho, t, s) = \frac{1}{\alpha(s)} \operatorname*{Disc}_{M^2} \frac{1}{2\pi i}\beta_6,$$

$\alpha(s)$ comes from the flux factor; other constants have been taken care of in Eq. (8).

[20]Data for widths have been taken from Particle Data Group, Phys. Lett. 39B, 1 (1972).

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Comment on the Distributions of Charged Pions

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Wang's idea that the charged pions are emitted from production cells inside the nucleon is reexamined. It is shown that the over-all charge conservation is more important than the local charge conservation in the emission of charged pions from nearly independent production cells.

Some time ago Wang[1] proposed a simple and very interesting mechanism for particle production: The particles are emitted from nearly independent production cells inside the nucleon (or other hadrons). He discussed two possibilities: Either charged pairs (like $\pi^+\pi^-$) are emitted from production cells with local charge conservation, or single-charged particles (like π^+ or π^-) are emitted from production cells with over-all charge conservation. In the latter case, the emitted charged particle can actually be identified with the production cell itself. After comparing the derived multiplicity distributions with experiments, Wang concluded that the first possibility is favored over the second. In this note we wish to demonstrate that careful analysis indicates that the second possibility is actually favored over the first by experiments.

Following Wang,[1] we assume that all the reactions are mainly of the type

$$A + B \to A' + B' + \text{pions}, \tag{1}$$

i.e., we take into account the fact that most of the secondaries produced are pions.[2] In the energy region of up to 27 GeV, which is of interest to us here, the production of strange particles and antinucleons is small anyway. One further assumes that the neutral-pion production is independent from the charged-pion production. Let us denote the number of charged primaries in the initial state $(A+B)$ by α, which clearly is restricted to the values of 0, 1, and 2. Next we define a net number of emitted charged secondary pions m_c by the relation

$$m_c = n_c - \alpha, \quad \alpha = 0, 1, 2 \tag{2}$$

where n_c is the total number of charged particles in the final state. This definition makes m_c always even, since the over-all change conservation demands that

$$\begin{aligned} &n_c \text{ is even for } \alpha = 0, 2, \\ &n_c \text{ is odd for } \alpha = 1. \end{aligned} \tag{3}$$

Correspondingly, the mean multiplicities $\langle m_c\rangle$ and $\langle n_c\rangle$ are related as

$$\langle m_c\rangle = \langle n_c - \alpha\rangle, \quad \alpha = 0, 1, 2. \tag{4}$$

Next we consider two reactions which will clarify the meaning of the net number of charged secondary pions m_c. For the reaction $(\alpha = 1)$

$$\pi^+ + n \to \pi^+ + n + m_c(\text{charged pions}) + m_0(\text{neutral pions}),$$

m_c is the number of all secondary pions beyond the primary pion π^+ in the final state. However, for the reaction $(\alpha = 2)$

$$\pi^- + p \to \Lambda + K^0 + 2(\text{charged pions}) + m_c(\text{charged pions}) + m_0(\text{neutral pions}),$$

m_c is the total number of secondary charged pions less 2 secondary charged pions which, one may believe, are emitted when K^0 emerges from the nucleon "core" turning the remaining nucleon into Λ. In other words, we believe that of the total number of secondary charged pions $m_c+2=n_c$, only $n_c-2=m_c$ follows the Wang mechanism of production, i.e., it is emitted from nearly independent cells inside the nucleon.

If one now assumes that from each nearly independent production cell a pair, $\pi^+\pi^-$, is emitted, then we can write down immediately the Wang first distribution function as

$$W^{\rm I}_{n_c}=\frac{(\frac{1}{2}\langle m_c\rangle)^{m_c/2}}{(m_c/2)!}\exp(-\tfrac{1}{2}\langle m_c\rangle),$$
$$m_c=n_c-\alpha,\quad \langle m_c\rangle=\langle n_c-\alpha\rangle,$$
$$\alpha=0,1,2,\quad m_c=0,2,4,\ldots, \tag{5}$$

where formally $\frac{1}{2}\langle m_c\rangle$ represents the mean number of charged-pion pairs produced. Despite the fact that $W^{\rm I}$ was formally derived as a multiplicity distribution for charged-pion pairs, it is also a multiplicity distribution for the net number of secondary pions, as it should be; namely, if

$$\tfrac{1}{2}\langle m_c\rangle=\sum_{\frac{1}{2}m_c}\tfrac{1}{2}m_c W^{\rm I}_{n_c},$$

then clearly also

$$\langle m_c\rangle=\sum_{m_c}m_c W^{\rm I}_{n_c}.$$

Demanding that the charge conservation holds only for the whole collision system, Wang writes down the second multiplicity distribution functions as

$$W^{\rm II}_{n_c}=\frac{(\langle m_c\rangle^{m_c}/m_c!)\exp(-\langle m_c\rangle)}{\sum_{m_c}(\langle m_c\rangle^{m_c}/m_c!)\exp(-\langle m_c\rangle)},$$
$$m_c=n_c-\alpha;\quad \langle m_c\rangle=\langle n_c-\alpha\rangle;$$
$$\alpha=0,1,2;\quad m_c=0,2,4,\ldots. \tag{6}$$

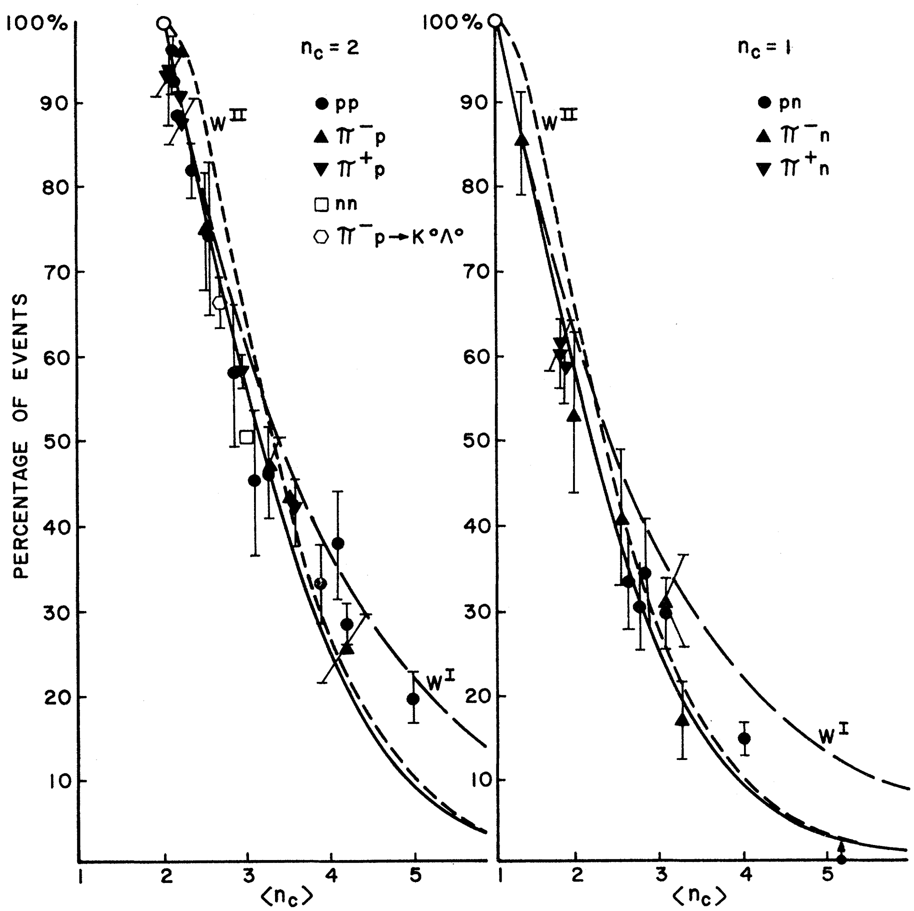

FIG. 1. Relative frequency of events against mean multiplicity $\langle n_c\rangle$ at various energies and for $m_c=0$. The data and the fits $W^{\rm I}$ and $W^{\rm II}$ are taken from Wang's paper (Ref. 1). One notices that $W^{\rm II}$ and $W^{\rm I}$ tend to miss the data at low and high $\langle n_c\rangle$, respectively. The solid curve corresponds to our distribution P [Eqs. (7a) and (7b)].

Clearly, since we only demand over-all charge conservation, one may now assume that one charged meson at a time is emitted from a production cell, the cells themselves being nearly independent. Equivalently, now one may simply identify the production cells with the emitted charged mesons. From comparison with experiments, Wang concluded that at least at low multiplicities $\langle n_c \rangle$, the mechanism in which a pion pair $\pi^+\pi^-$ is emitted from a production cell is favored over the mechanism in which a single-charged meson is emitted from a production cell, the production cells being nearly independent inside the nucleon. This conclusion is drawn from the fact that $W^{I}_{n_c}$ fits the data better than $W^{II}_{n_c}$ at low $\langle n_c \rangle$ (see Figs. 1–3). However, we disagree with this conclusion since, from the mathematical point of view, the distribution $W^{II}_{n_c}$ is valid only for very large mean multiplicities $\langle n_c \rangle$ as will be seen below. Incidentally, the sum in the denominator in (6) can be written in a closed form as $\cosh\langle m_c \rangle \times \exp(-\langle m_c \rangle)$.

In order to replace (6) with the multiplicity distribution function which is valid also for low $\langle m_c \rangle$, we note that m_c is always even and that the properly normalized distribution function is

$$P_{n_c} = \frac{1}{\cosh B} \frac{B^{m_c}}{m_c!}, \qquad m_c = n_c - \alpha; \quad \alpha = 0, 1, 2, \qquad m_c = 0, 2, 4, \ldots, \tag{7a}$$

where B is related to $\langle m_c \rangle$ as

$$\langle m_c \rangle = \langle n_c - \alpha \rangle = \sum_{m_c} m_c P_{n_c} = B \tanh B. \tag{7b}$$

For large B, $\tanh B \approx 1$ and $\langle m_c \rangle \approx B$; therefore, P_{n_c} from (7a) does not differ very much from $W^{II}_{n_c}$ from (6). As a matter of fact, we see from Figs. 1–3 that for a large $\langle n_c \rangle$, P_{n_c} (solid line) approaches asymptotically $W^{II}_{n_c}$ (dashed line). Generally, however, from (7b) we always have that $B \geq \langle m_c \rangle$, and, particularly for $0 < B \ll 1$, we may approximate $\langle m_c \rangle$ by B^2 giving

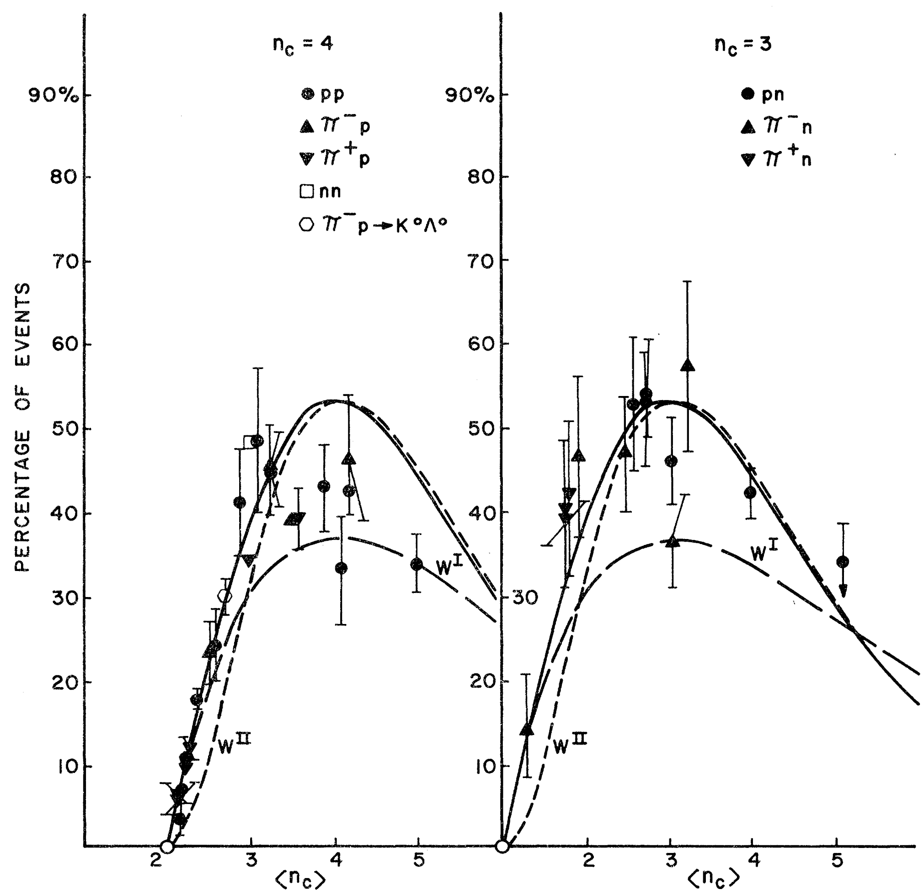

FIG. 2. The same as Fig. 1 but for $m_c = 2$.

$$P_{n_c} \approx \frac{1}{\cosh(\langle m_c\rangle)^{1/2}} \frac{\langle m_c\rangle^{m_c/2}}{m_c!},$$

which differs from $W^{II}_{n_c}$ for small $\langle m_c\rangle$.

In Figs. 1–3 we compare the predictions of Eqs. (7a) and (7b) with the data assembled by Wang.[1] He complied many $\pi^{\pm}p$, $pp(\alpha=2)$, $\pi^{\pm}n$, $pn(\alpha=1)$, and $nn(\alpha=0)$ inelastic production experiments below 27 GeV. Each figure has a definite m_c, i.e., $m_c=0$ for Fig. 1, $m_c=2$ for Fig. 2, and $m_c=4$ and 6 for Fig. 3. The nn data have $\alpha=0$, and the origin is shifted by two units to the right. For the same reason, the nn data have $n_c=0$ for Fig. 1, $n_c=2$ for Fig. 2, and $n_c=4$ for Fig. 3. It is interesting to note that for $m_c=6$ (Fig. 3) the distributions W^{II} and P do not differ very much, which comes from the fact that here $m_c!$ is very large and, being in the denominator of both distribution functions, it tends to equalize them. Therefore, not only for large $\langle n_c\rangle$, n_c finite, but also for very large n_c, $\langle n_c\rangle$ anything, P is quite well approximated by W^{II}. For small and intermediate $\langle n_c\rangle$, where W^{II} fails, P fits the data quite well. Although W^{I} fits the low-n_c and the low-$\langle n_c\rangle$ region, it fails at higher n_c and higher $\langle n_c\rangle$. So we can say that in general P fits the data better than W^{I}, suggesting that over-all charge conservation is more important than the local charge conservation in the mechanism of emission of charged pions from nearly independent production cells from inside the nucleon.

Kastrup[3] and Horn and Silver,[4] on a purely statistical basis, have derived a distribution function for charged-pion pairs which, although being formally different from our distribution function (7a) and (7b), shows a certain degree of similarity with it graphically. It would be interesting to find an interpretation of their distribution function in terms of Wang's production cells.

It is our feeling that one might actually be able to derive distribution functions (7a), (7b), and (5) by means of the PDECC (partial differential equations with respect to coupling constants) method,[5] thus enabling us to study the role of Wang's production cells inside hadrons in more detail and

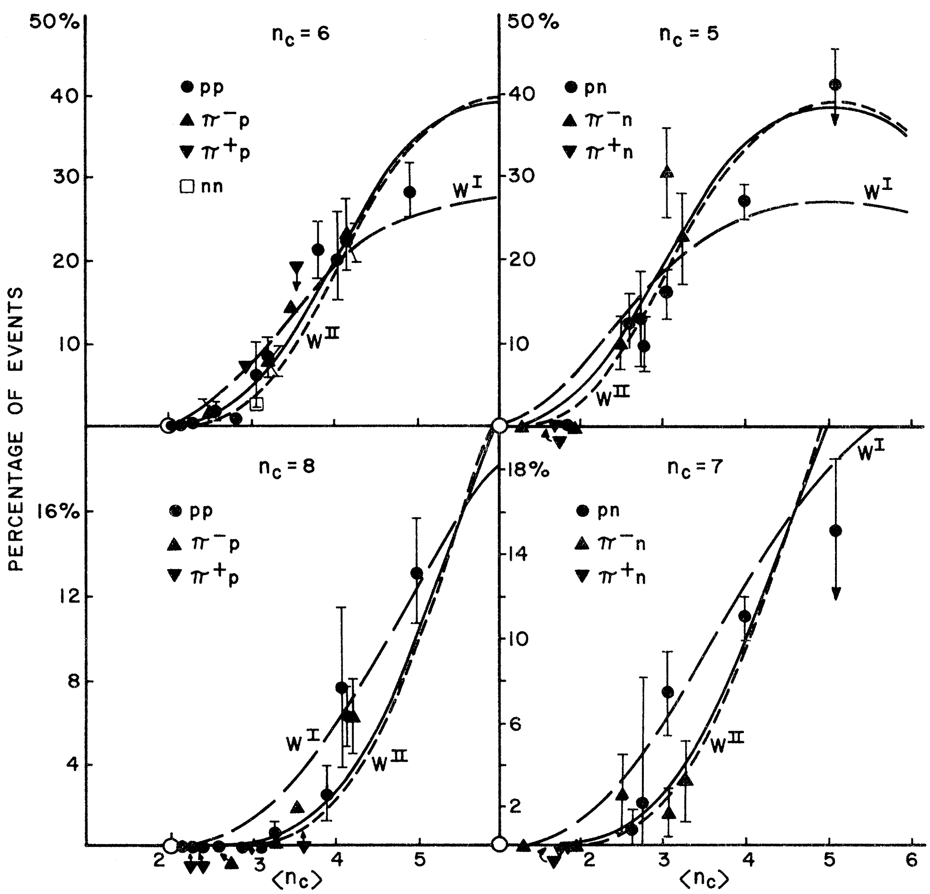

FIG. 3. The same as Fig. 1 but for $m_c = 4$ ($n_c = 6$ and 5) and $m_c = 6$ ($n_c = 8$ and 7). Here one notices that as one goes from $m_c = 4$ to $m_c = 6$, the difference between W^{II} and P becomes markedly smaller.

see how much similarity there is between them and Feynman's partons. This problem is presently being studied.

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[1]C. P. Wang, Phys. Rev. 180, 1463 (1969). See also C. P. Wang, Nuovo Cimento, 64A, 546 (1969); Phys. Lett. 30B, 115 (1969); 32B, 125 (1970).

[2]As is customary in bremsstrahlung models, in a reaction $A + B \rightarrow A' + B'$ + anything, we call the particles denoted by A and B primaries in the initial state, the particles denoted by A' and B' primaries in the final state, while everything else in the final state we simply call secondaries. [See, for example, H. Gemmel and H. A. Kastrup, Nucl. Phys. B14, 566 (1969).]

[3]H. A. Kastrup, Nucl. Phys. B1, 309 (1967).

[4]D. Horn and R. Silver, Phys. Rev. D 2, 2082 (1970).

[5]J. Šoln, Phys. Rev. D 6, 2277 (1972); 7, 1637 (1973).

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Measurement of the Σ^- Beta Decay Rate and an Improved Test of the $\Delta S = -\Delta Q$ Selection Rule, B. Sechi-Zorn and G. A. Snow [Phys. Rev. D 8, 12 (1973)]. In the last line of the abstract $\times 10^{-13}$ should read $\times 10^{-3}$.