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PHYSICAL REVIEW D VOLUME 8, NUMBER 7 1 OCTOBER 1973

New Approach to Relations Between Deep-Inelastic Structure Functions

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(Received 14 May 1973)

It is argued that symmetry properties available from experiments outside deep-inelastic physics can provide guidance for understanding the structure functions in the deep-inelastic region. In particular, it is suggested that the component of the current that transforms like the "ϕ" is more weakly coupled to nonstrange hadrons than the components which transform like the "ρ" or "ω". This leads to a stringent upper bound for the sum of the electromagnetic structure functions, $F_2^{\gamma p} + F_2^{\gamma n}$, which can be tested by experiments.

The theoretical descriptions used to obtain relations between deep-inelastic structure functions are generally not very restrictive, because they are either too general or too specific. General treatments place weak bounds on structure functions that at the moment are in no danger of violation by experiment. Specific models with detailed assumptions give predictions whose experimental violation can always be explained.

The general models do not exclude pathological cases like a quark-parton model of the nucleon with three valence quarks and an infinite sea of *strange* quark-antiquark pairs. This strange-quark sea can dominate the electromagnetic functions. Thus these models cannot give upper bounds on the ratio of electromagnetic to neutrino structure functions. They only give lower bounds which turn out to be rather trivial.

The general models also tend to disregard information already available from experiments outside deep-inelastic physics, such as the SU(3) properties of the electromagnetic current. The ratio 9 : 1 : 2 for the strengths of the components of the photon which transform like the vector mesons ρ, ω, and ϕ is predicted[1] by the classification of the photon as the U-spin scalar component of an octet, and the canonical ω-ϕ mixing angle. This ratio is very sensitive to the presence of a possible SU(3) singlet component. In the Sakata model which has such a singlet component, the ratio is changed from 9:1:2 to 1:1:0, which is far outside experimental limits, from experiments of e^+e^- annihilation into vector mesons[2] and vector-meson photoproduction.[3] Yet some general treatments of deep-inelastic processes give predictions[4] with coefficients depending "on the parton charge" and quote values for the Sakata model. They do not note that such variations in parton charge imply large singlet components in the electromagnetic current, which are inconsistent with e^+e^- annihilation results.

On the other hand there are specific models[5] which exclude the pathological strange-quark sea at the price of extreme detailed assumptions. One example is the quark-parton model with three valence quarks in an SU(6)-symmetric wave function and a sea of pairs with the quantum numbers of the vacuum. Such models make definite predictions which seem to disagree with experiment. However, discrepancies are easily explained by SU(6)-symmetry breaking and polarization of the sea.

The SU(6) prediction $F_1^{\gamma n}/F_1^{\gamma p} \geq 2/3$ is easily fixed by noting that SU(6) couples the isospins of all valence quarks symmetrically. It is reasonable to break the symmetry when one quark is near $x=1$. Its isospin should not be strongly coupled to the isospin of two valence quarks at $x=0$. The latter pair should be in the most strongly bound state, shown by the SU(6) breaking in Λ-Σ and N-Δ mass differences to have $I=0$. The quark at $x=1$ thus carries the full isospin of the nucleon and gives the reasonable prediction that $F_1^{\gamma n}/F_1^{\gamma p} \to 1/4$ as $x \to 1$.

Similarly a strictly isoscalar sea contains exactly equal numbers of $\mathcal{P}\overline{\mathcal{P}}$ and $\mathcal{N}\overline{\mathcal{N}}$ quark-antiquark pairs. However, the presence of a valence quark could polarize this sea to change the numbers of $\mathcal{P}\overline{\mathcal{P}}$ and $\mathcal{N}\overline{\mathcal{N}}$ pairs to $N-\epsilon$ and $N+\epsilon$. If $\epsilon=\frac{1}{3}$ a discrepancy of a factor of 2 is introduced into some of the predictions based on an isoscalar sea.

We now show how a simple and reasonable assumption can eliminate the pathological strange-quark sea and lead to relations between structure functions which can provide a significant experimental test of theoretical models. This assumption in quark language is simply that the number of strange quarks and antiquarks present in the nucleon should not be greater than the number of corresponding nonstrange quarks and antiquarks.[6] It can also be formulated without reference to quarks by using the known transformation properties of the photon. In this description the equivalent of the pathological strange-quark model is "ϕ dominance" of the electromagnetic structure function. All experience shows that the coupling to nonstrange hadrons of objects which transform like the ϕ are suppressed. It is reasonable to assume that the scattering on a nucleon of the ϕ component of the photon is not stronger than that of the ρ or ω components. This assumption is sufficient to give the same relation between electromagnetic and neutrino structure functions obtained in the quark description by restricting the contributions of the strange-quark sea.

Consider bounds on the quantity

$$R^0_{\gamma\nu} = \frac{F_1^{\gamma p} + F_1^{\gamma n}}{F_1^{\nu p} + F_1^{\nu n}} \quad . \tag{1}$$

The quark-parton model (QPM) gives a lower bound on this ratio,[7]

$$R^0_{\gamma\nu}(\text{QPM}) \geq \tfrac{5}{18} \quad . \tag{2a}$$

The same bound is obtained from the light-cone algebra. However, an experimentally indistinguishable lower bound has been obtained from much weaker assumptions. The isovector electromagnetic contribution is related by conservation of vector current (CVC) to the weak vector contribution and the chiral symmetry present in all models requires equality of vector and axial-vector contributions.[8] This gives

$$R^0_{\gamma\nu}(V=A) \geq R^0_{\gamma\nu}(I_\gamma=1;\ V=A) = \tfrac{1}{4} \quad , \tag{2b}$$

where $V=A$ denotes the assumption of equal vector and axial-vector contributions to the weak structure functions and $I_\gamma=1$ denotes that only the isovector contribution from the photon is considered,[8] the isoscalar is neglected. The inequality follows because the isoscalar contribution is positive definite and the interference between the isoscalar and isovector components of the photon drops out when structure functions are averaged over an isospin multiplet. The equality follows from CVC.

These bounds are thus not very useful. A violation which throws out chiral symmetry or CVC would be very exciting, but is not expected. The possibility of a reliable experimental value intermediate between the bounds (2a) and (2b) can be discounted. An *upper* bound on $R^0_{\gamma\nu}$ would be interesting. This requires an upper bound on the contribution of the isoscalar part of the photon. Here trouble arises from the pathological model with the large strange-quark sea. Strange quarks scatter isoscalar photons, but do not scatter strangeness-conserving weak currents nor isovector photons. Thus any version of the quark-parton model which does not place an upper limit on the strange-quark contribution cannot put an upper bound on $R^0_{\gamma\nu}$.

We now show how useful upper bounds can be obtained. We use the notation of Llewellyn Smith[4] and express the structure functions in terms of six positive definite functions of x, denoted by $U_{\mathcal{P}}$, $U_{\mathcal{N}}$, U_λ, $U_{\overline{\mathcal{P}}}$, $U_{\overline{\mathcal{N}}}$, and $U_{\overline{\lambda}}$. These are interpreted in the quark-parton model as the densities of the six quark and antiquark states in the target, but the light-cone approach obtains the same parametrization without any assumptions about quark densities. The quark-density interpretation is used as a guide to the intuition in making additional assumptions, which are then defined in a general way which does not require the quark-parton interpretation. We first note that strange quarks contribute equally to neutron and proton electromagnetic structure functions. Therefore we can bound the relative importance of strange quarks in regions where the neutron and proton structure functions are different. Thus the appropriate linear combinations of relations quoted by Llewellyn Smith, namely

$$\frac{1}{6}\left|\frac{R_{np}+1}{R_{np}-1}\right| \geq R^0_{\gamma\nu}(\text{QPM}) \geq \frac{5}{18} , \tag{3a}$$

can give an upper bound on $R^0_{\gamma\nu}$. Here

$$R_{np} = F_1^{\gamma n}/F_1^{\gamma p} , \tag{3b}$$

and QPM denotes that we are using the standard expressions from the quark-parton model in terms of the above six parameters, without any additional assumptions. Note that when $R_{np}=\frac{1}{4}$ the upper and lower bounds in Eq. (3a) become equal and the relation becomes an equality, while for $R_{np}=3$ the upper bound is $\frac{1}{3}$, which is not very far from $\frac{5}{18}$. For $R_{np}\leq 1$ the two bounds can be combined into the approximate equality

$$R^0_{\gamma\nu}(\text{QPM}) = \tfrac{1}{18}(4-R_{np})(1-R_{np})(1+\eta) , \tag{4a}$$

where η satisfies the inequality

$$|\eta| \leq \frac{1}{5}\left(\frac{4R_{np}-1}{4-R_{np}}\right) \text{ if } R_{np}\leq 1 . \tag{4b}$$

Note that $\eta=0$ when $R_{np}=\frac{1}{4}$ and $|\eta|\leq\frac{1}{11}$ when $R_{np}=\frac{1}{3}$. Thus in the region $R_{np}\leq\frac{1}{3}$ the approximate equality (4a) is good to better than 10%. Since experimental data indicate that R_{np} approaches this region as $x\to 1$, the approximate equality may be useful in this region.

When $R_{np}\sim 1$, the upper bound (3a) and the approximate equality (4a) become useless. This is to be expected in view of the strange-quark pathology, which can occur in these models when $R_{np}\sim 1$.

We now consider an additional assumption to limit the strange-quark density. The obvious ansatz is[9]

$$U_\lambda + U_{\bar\lambda} \leq U_{\mathcal{P}} + U_{\bar{\mathcal{P}}} . \tag{5}$$

This simply requires the strange-quark densities to be less than the corresponding nonstrange-quark densities. This is true in almost any reasonable quark-parton model for the nucleon. When the ansatz (5) is substituted into the standard quark-parton formulas we obtain the upper bound

$$\tfrac{1}{3} \geq R^0_{\gamma\nu}(\text{A5}) \geq \tfrac{5}{18} , \tag{6a}$$

where A5 denotes that we have used the ansatz (5) in addition to the standard assumptions of the quark-parton model. These bounds can be combined into the approximate equality

$$R^0_{\gamma\nu}(\text{A5}) = \tfrac{11}{36}(1+\epsilon) , \tag{6b}$$

where ϵ satisfies the inequality

$$|\epsilon| \leq \tfrac{1}{11} . \tag{6c}$$

The averaging over protons and neutrons in $R^0_{\gamma\nu}$ is not essential, because in the deep-inelastic region it is safe to assume that the current scatters incoherently from individual protons and neutrons, so that slightly weaker bounds can be obtained. For a stable nucleus with Z protons and $A-Z$ neutrons we define

$$R_{\gamma\nu}(Z,A) = \frac{ZF_1^{\gamma p} + (A-Z)F_1^{\gamma n}}{ZF_1^{\nu p} + (A-Z)F_1^{\nu n}}$$

and obtain the bounds

$$\frac{A-Z}{Z} R^0_{\gamma\nu} \geq R_{\gamma\nu}(Z,A) \geq \frac{Z}{A-Z} R^0_{\gamma\nu} .$$

An alternative derivation of the relation (6) is obtained by considering the individual contributions of the isovector and isoscalar components of the electromagnetic current. From the transformation properties of the photon under SU(3),

$$|\gamma\rangle = \tfrac{1}{2}\sqrt{3}\,|\text{“}\rho^0\text{”}\rangle + \tfrac{1}{2}\,|\text{“}\omega_8\text{”}\rangle , \tag{7a}$$

$$|\gamma\rangle = \tfrac{1}{2}\sqrt{3}\,(|\text{“}\rho^0\text{”}\rangle + \tfrac{1}{3}\,|\text{“}\omega\text{”}\rangle + \tfrac{1}{3}\sqrt{2}\,|\text{“}\phi\text{”}\rangle) , \tag{7b}$$

where the notation “V” denotes a state having the transformation properties of the vector meson V under SU(3) but does not assume vector dominance for any relation between “V” and physical vector mesons. The relation (2b) has been obtained by considering only the isovector first term on the right-hand side of (7). Relation (6) is obtained from the following additional assumption, introduced in order to bound the contribution of the second term:

$$F_1\text{“}\rho\text{”}p \geq F_1\text{“}\omega_8\text{”}p . \tag{8}$$

This assumption cannot be rigorously justified, but is supported by the plausibility argument that any state which transforms like the vector meson ϕ is more weakly coupled to nonstrange hadrons than corresponding states which transform like the ρ or ω. The isoscalar octet state ω_8 is $\frac{2}{3}\phi$ and only $\frac{1}{3}\omega$. Thus the inequality (8) should hold if the suppression of the ϕ component occurs in deep-inelastic scattering and the ω contribution is not anomalously large. Note that the ω component is only $\frac{1}{12}$ of the photon, as indicated by Eq. (7b), and it would take a very large anomalous contribution to cause a serious violation of the inequality (6).

The relation between these two derivations of Eq. (6) illustrates the connection between the parton and light-cone approaches. In the quark-parton model there are two independent contributions to the structure function for isoscalar photons, the scattering by the nonstrange quarks in the hadron and the scattering by the strange quarks. The isoscalar photon structure function can also be divided into two independent contributions by separating the isoscalar photon into a component which transforms like the ω and one which transforms like the ϕ. These two separations turn out to be equivalent. The ω component of the photon gives the scattering by the nonstrange quarks; the ϕ component gives the scattering by the strange quarks. This equivalence of the two formulations, one assuming a quarklike structure for the currents and the other assuming a quarklike structure for the hadrons, is characteristic of the relation between the light-cone and parton approaches.

There are already experimental determinations for two ratios relevant to this discussion[10]:

$$\frac{\int (F_2^{\gamma p} + F_2^{\gamma n})\, dx}{\int (F_2^{\nu p} + F_2^{\nu n})\, dx} = 0.30 \pm 0.06$$

and an approximate estimate[11]

$$0.23 \lesssim \frac{\int x(F_2^{\gamma p} + F_2^{\gamma n})\, dx}{\int x(F_2^{\nu p} + F_2^{\nu n})\, dx} \lesssim 0.32\ .$$

Both of them are consistent with the bounds discussed. The test of higher moments is desirable, but even more desirable are tests at small values of x. The above integrals are not sensitive to small values of x.

We now see general reasons for the validity of the approximate equality (6). Since the photon is only $\frac{1}{4}$ isoscalar, the isovector contribution by itself already gives a good approximation to the total structure function as well as a lower bound unless the isoscalar contributions are anomalously large. In almost any model the structure function for normalized pure isoscalar photons should be smaller than that for normalized pure isovector photons, because the isoscalar photons have some contribution associated with strange quarks, which is expected to be suppressed. Thus a good upper limit for the structure function for a physical photon is obtained by assuming that the structure function for the normalized isoscalar photon is less than or equal to the structure function for a normalized isovector photon. These bounds can be combined to give the approximate equality

$$R^0_{\gamma\nu} = \tfrac{7}{24}(1 \pm \delta)\ , \tag{9a}$$

$$|\delta| \leq \tfrac{1}{7}\ . \tag{9b}$$

This approximate equality is already good to better than 15%. *Any model in which the isoscalar part of the photon does not have an anomalously large contribution must satisfy the approximate equality (9).*

Note that the relations (6) and (9) predict values for $R^0_{\gamma\nu}$ which are good to better than 10% or 15%, respectively, and which are constant independent of x. That the ratio of electromagnetic to neutrino structure functions should be constant independent of x within an error of 10–15% is an interesting prediction which can be tested experimentally and also has a very simple physical interpretation. It implies that in the nucleon the density of partons which scatter photons is approximately proportional to the density of partons which scatter neutrinos. Experimental tests of this prediction will thus give a definite answer to the question of whether these two densities are the same or different.

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*Operated by Universities Research Association Inc. under contract with the United States Atomic Energy Commission.

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