*Work supported in part by the U. S. Atomic Energy Commission.

†Permanent address.

[1]V. V. Ammosov *et al.*, Phys. Lett. 42B, 519 (1972); J. W. Chapman *et al.*, Phys. Rev. Lett. 29, 1686 (1972); G. Charlton *et al.*, *ibid.* 29, 515 (1972); T. T. Dao *et al.*, *ibid.* 29, 1627 (1972).

[2]G. D. Kaiser, Nucl. Phys. B44, 171 (1972).

[3]G. W. Parry and P. Rotelli, ICTP Trieste Report No. IC/73/3 (unpublished).

[4]P. Olesen, Phys. Lett. 41B, 602 (1972).

[5]Y. Tomozawa, Nucl. Phys. B (to be published).

[6]P. Slattery, Phys. Rev. D 7, 2073 (1973).

[7]Y. Tomozawa, this issue, Phys. Rev. D 8, 2138 (1973).

[8]Z. Koba, H. B. Nielsen, and P. Olesen, Nucl. Phys. B40, 317 (1972).

[9]J. B. S. Haldane, Biometrika 32, 294 (1942).

[10]Y. Tomozawa, SLAC Report No. SLAC-PUB-1233, 1973 (unpublished).

[11]Mode is the value of the multiplicity for which the distribution function is maximum.

[12]M. G. Kendall and A. Stuart, *The Advanced Theory of Statistics* (Griffin, London, 1963), Vol. 1.

[13]Cumulants are defined by

$$\phi(t)=\sum_{n=0}^{\infty} e^{itn}P_n=\exp\left[\sum_{k=1}^{\infty}\frac{\kappa_k(it)^k}{k!}\right],$$

e.g., $\kappa_1=\langle n\rangle$, $\kappa_2=\langle(n-\langle n\rangle)^2\rangle$, $\kappa_3=\langle(n-\langle n\rangle)^3\rangle$, etc.

[14]In this case, Eq. (11b) gives an explanation of the scaling function used in Ref. 6.

PHYSICAL REVIEW D VOLUME 8, NUMBER 7 1 OCTOBER 1973

Comment on the Approach to Factorization and Scaling in Inclusive Reactions*

T. Ferbel†

Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627

(Received 14 May 1973)

A previous compilation of data pertaining to the question of factorization in the central region of pion production is updated and extended to include K_S^0 and Λ^0 production.

In a recent letter we provided evidence for the factorization hypothesis in the central region of pion production.[1] It was also indicated at that time that the data were consistent with an approach to limiting behavior expected on the basis of the Mueller-Regge formalism. In this note we wish to update the data presented for the reactions[2]:

$$p+p\to\pi^-+\text{anything}, \tag{1}$$

$$\pi^++p\to\pi^-+\text{anything}, \tag{2}$$

$$\gamma+p\to\pi^-+\text{anything}, \tag{3}$$

$$K^++p\to\pi^-+\text{anything}, \tag{4}$$

$$\pi^++p\to\pi^++\text{anything}, \tag{5}$$

and examine new data for the reaction[2,3]:

$$p+p\to\pi^++\text{anything} \tag{6}$$

as well as data for the K_S^0 channels[4]:

$$p+p\to K_S^0+\text{anything}, \tag{7}$$

$$K^-+p\to K_S^0+\text{anything}, \tag{8}$$

$$K^++p\to K_S^0+\text{anything}, \tag{9}$$

$$\pi^++p\to K_S^0+\text{anything}, \tag{10}$$

and for the Λ^0 production reactions[5]:

$$p+p\to\Lambda^0+\text{anything}, \tag{11}$$

$$K^-+p\to\Lambda^0+\text{anything}, \tag{12}$$

$$K^++p\to\Lambda^0+\text{anything}, \tag{13}$$

$$\pi^++p\to\Lambda^0+\text{anything}. \tag{14}$$

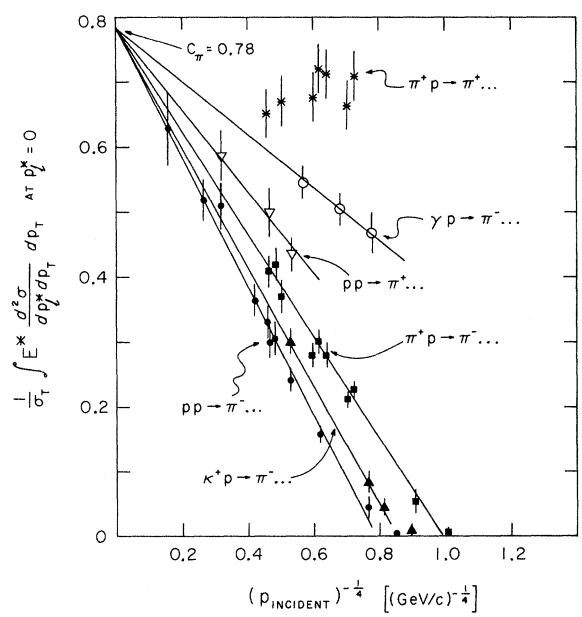

FIG. 1. Normalized invariant cross section at $p_l^*=0$ for pion production.

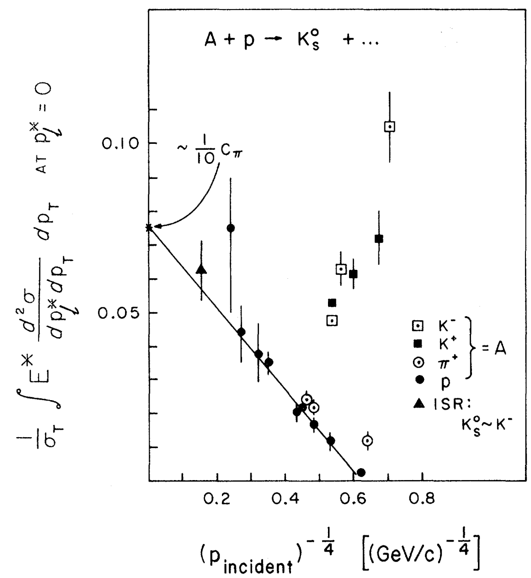

FIG. 2. Normalized invariant cross section at $p_l^* = 0$ for K_S^0 production.

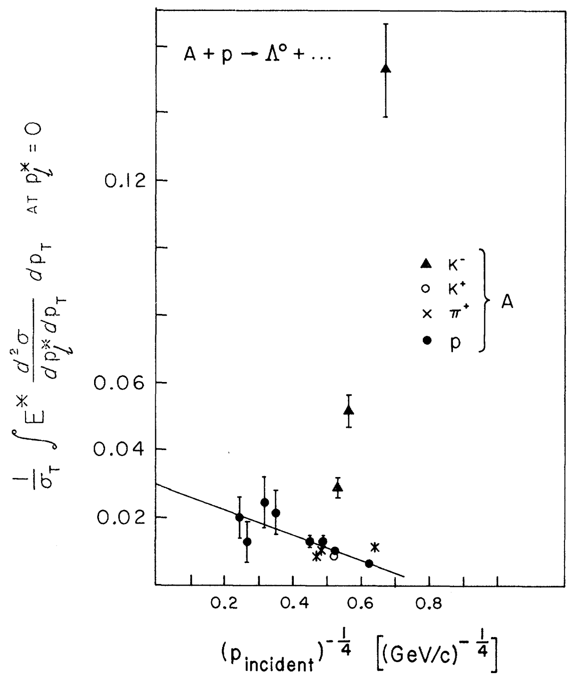

FIG. 3. Normalized invariant cross section at $p_l^* = 0$ for Λ^0 production (see Ref. 5).

Figures 1, 2, and 3 display, respectively, the normalized invariant cross sections at 90° in the center-of-mass system, integrated over transverse momenta, for reactions (1) through (6), (7) through (10), and (11) through (14); the ordinate in the graphs, the parameter c, is defined as follows:

$$c = \frac{1}{\sigma_T} \int_{\text{all } p_T} E^* \frac{d^2\sigma}{dp_l^* \, dp_T} dp_T \bigg|_{p_l^* = 0},$$

where σ_T is the asymptotic total cross section for each individual incident channel[6]; p_T, p_l^*, and E^* are the transverse momentum, the longitudinal momentum, and the energy of the produced particle in the center-of-mass frame. The abscissas on the figures are fourth roots of the inverse of the incident momentum in the laboratory.[7]

A general feature apparent in the energy dependence of the data is that those reactions which contain sizable diffractive components or other strong leading-particle effects [reactions (5), (8), (9), and (12)][8] have large cross sections for $p_l^* = 0$ at low-incident momenta. The cross sections for these channels appear to fall, initially, as the colliding energy is increased. On the other hand, cross sections for reactions which do not have substantial leading-particle contributions appear to rise almost linearly from threshold and are consistent (within sizable errors) with a dependence of the form $(p_{\text{incident}})^{-1/4}$. If all cross sections for any particular final-state particle will factorize at high enough energies (as the trend appears to indicate), then the cross sections for reactions such as reaction (5), (8), (9), and (12) will eventually also rise and approach the same limits for the parameter c.[9]

I thank P. Slattery for reading this note.

After completing this work we became aware of a similar compilation of K_S^0 and Λ^0 data in pp channels made by the French-Soviet Mirabelle collaboration.

---

*Research supported by the U. S. Atomic Energy Commission. This comment is based on a more extensive review paper presented at the conference on "Recent Advances in Particle Physics," sponsored by the New York Academy of Sciences (March 1973).

†A. P. Sloan Fellow.

[1]T. Ferbel, Phys. Rev. Lett. 29, 448 (1972).

[2]I wish to thank Anatole Shapiro for pointing out some inconsistencies in the $\pi^+ p$ data which were presented in Ref. 1. In particular, several points below 7 GeV/c were systematically too low (typically $\lesssim 10\%$ correction). The new data in Fig. 1, for reactions (1) through (6), are from the Michigan-Rochester collaboration, C. Bromberg *et al.*, Bull. Am. Phys. Soc.

18, 665 (1973), reaction (1) and (6) at 102 GeV/c; the ANL, NAL, ISU, MSU, UM collaboration, L. Hyman *et al.*, Bull. Am. Phys. Soc. 18, 666 (1973), reaction (1) at 205 GeV/c; E. L. Berger *et al.*, Phys. Rev. Lett. 29, 675 (1972), reaction (1) at 21 GeV/c; Brown University report (A. Shapiro, private communication), reaction (2) and (5) at 4.1 GeV/c; Bonn-Hamburg-Munich collaboration (V. Blobel, private communication), reaction (6) at 12 GeV/c and 24 GeV/c.

[3]We have also examined data for other pion-production reactions at high energies. In particular, the cross section for $\pi^- p \to \pi^-$ + anything has a dependence very similar to that observed for reaction (5). The other available channels, viz., $K^- p \to \pi^+$ + anything (two experiments) and $\pi^- p \to \pi^+$ + anything (three experiments) suffer from the presence of proton background in the final states and are consequently difficult to measure at $p_l^* = 0$, and the meager data that exist for these reactions may be inconsistent with each other. We note, however, that in both reactions there is one data point grossly inconsistent with the general energy trend observed for the other channels displayed in Fig. 1.

[4]The references for previous K_S^0 and Λ^0 data can be found in the review by T. Ferbel, in *Proceedings of the Third International Colloquium on Many Body Reactions, Zakopane, 1972,* edited by A. Białas *et al.* (Cracow, Poland, 1972). The new data are from the NAL-UCLA collaboration, as reported by R. Engelmann, Bull. Am. Phys. Soc. 18 (1973), reactions (7) and (11) at 303 GeV/c; G. Charlton *et al.*, Phys. Rev. Lett. 30, 574 (1973), reactions (7) and (11) at 205 GeV/c; A. Seidl *et al.*, Bull. Am. Phys. Soc. 18, 665 (1973), reactions (7) and (11) at 102 GeV/c; Soviet-French collaboration report (unpublished) reactions (7) and (11) at 69 GeV/c; Notre Dame report (unpublished) (P. Stuntebeck, private communication), reactions (10) and (14) at 18.5 GeV/c. The entry from CERN Intersecting Storage Rings (ISR) energies in Fig. 2 is estimated from the review of E. Lillethun, Bergen Report No. 47, 1972 (unpublished), assuming that K^- production is the same as K_S^0 near $p_l^* = 0$.

[5]See references given in Ref. 4. The Λ^0 inclusive cross section contains the sum of the Λ^0 and Σ^0 cross sections.

[6]The total cross sections used here are the same as given previously in T. Ferbel, Ref. 4, namely, 39.8 mb, 23.4 mb, 17.4 mb, 99 μb, and 22.9 mb, respectively, for pp, $\pi^+ p$, $K^+ p$, γp, and $K^- p$ channels.

[7]This is equivalent to $s^{-1/4}$ for high energies; see Ref. 1.

[8]In this regard, see the compilation of H. Meyer and W. Struczinsky, DESY report, 1972 (unpublished). In particular, note the low-energy behavior of $\gamma p \to \pi^+$ + anything which contains sizable contributions from vector-meson production and proton fragmentation.

[9]We note that these naive remarks assume that new phenomena which may appear at higher energies will not void all the conclusions we reach from presently measured data. The growth of the total pp cross section at ISR energies in particular, certainly raises very interesting questions concerning the meaning of all the results we have presented. See University of Rochester Report No. UR429 (to be published in Ann. N. Y. Acad. Sci.) for further comments concerning asymptotic behavior.

PHYSICAL REVIEW D VOLUME 8, NUMBER 7 1 OCTOBER 1973

Suitability of Nuclear Emulsion for Nuclear-Structure Studies at a High-Energy Muon Facility*

R. K. Shivpuri† and M. R. Cruty‡
Physics Department, Clarkson College of Technology, Potsdam, New York 13676

P. J. McNulty§
Space Physics Laboratory, Air Force Cambridge Research Laboratories, L. G. Hanscom Field, Bedford, Massachusetts 01730 and Physics Department, Clarkson College of Technology, Potsdam, New York

(Received 4 January 1972; revised manuscript received 15 November 1972)

Recently Jain and Stern reported nuclear photographic emulsion to be particularly suited for nuclear-structure studies at high-energy muon facilities. Significant discrepancies between their results and our own are presented and discussed.

In a recent report[1] published under the editorial policy announced July 20, 1964,[2] Jain and Stern claim to present a study of the giant-resonance (GR) phenomenon in interactions of high-energy muons with photographic emulsion nuclei—a phenomenon first studied in nuclear emulsion using high-energy beam muons by Kirk *et al.*[3] The conclusion of Ref. 1 is that the interactions of high-energy muons with emulsion nuclei can be used for the study of nuclear structure. Of course, the study of nuclear structure requires that the identity of the target nucleus be known. The argument that nuclear emulsion would make an ideal target-detector system rests on a method reported in Ref. 1