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[2]We denote by the indices $+, -, 0$, differential cross sections $\sigma_{+,-,0}$ and polarizations $P_{+,-,0}$ of the reactions $\pi^+p \to p\pi^+$, $\pi^-p \to p\pi^-$, and $\pi^-p \to n\pi^0$, respectively.

[3]D. P. Owen, F. C. Peterson, J. Orear, A. L. Read, D. G. Ryan, D. H. White, A. Ashmore, C. J. S. Damerell, W. R. Frisken, and R. Rubinstein, Phys. Rev. 181, 1794 (1969).

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[5]$u = u_0$ corresponds to $\cos\theta_{\text{c.m.}} = -1$.

[6]In terms of the invariant amplitudes A and B, the fact that $M^{3/2}_{++}$ has a factor $\alpha_\Delta - \frac{1}{2}$ whereas $M^{3/2}_{+-}$ has not means that only B contains such a factor (see in the Appendix the relations between M_{++} and A, B).

[7]H. Aoi, N. Booth, C. Caverzasio, L. Dick, A. Gonidec, A. Janout, K. Kuroda, A. Michalowicz, M. Poulet, D. Sillou, C. Spencer, and W. Williams, Phys. Lett. 35B, 90 (1971); report (unpublished).

[8]If, at small $|u|$, $\text{Re}M^{1/2}_{++}$ had the sign given by a Regge N_α pole, our result implies that $\text{Re}M^{1/2}_{++}$ has a simple zero in the dip region, thus ruling out WSN zero.

[9]W. F. Baker, K. Berkelman, P. J. Carlson, G. P. Fisher, P. Fleury, D. Hartill, R. Kalbach, A. Lundby, S. Mukhin, R. Nierhaus, K. P. Pretzl, and J. Woulds, Nucl. Phys. B25, 385 (1971).

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[11]While finishing this work, we received a report [CERN-TH 1490, 1972 (unpublished)] from C. Ferro Fontan on πN backward scattering at 6 GeV/c. The author arrives at the same conclusion as we do concerning $\text{Re}M^{1/2}_{+-}$. He also considers $M^{1/2}_{+-}$ and $M^{3/2}_{++}$ as Regge-pole amplitudes, without specifying, however, the parametrization used, and in particular, mentions only briefly the question of the presence of a $\alpha_\Delta - \frac{1}{2}$ factor in the Δ_δ residue. We disagree with his $\text{Im}M^{1/2}_{++}$ having a zero at $u = -0.6$ rather than at $\alpha_N = -\frac{1}{2}$, which is in conflict with our interpretation, and that of Ref. 1, of point (a) in Sec. II A. We also arrive at a somewhat different conclusion for $M^{3/2}_{+-}$. Moreover Ferro Fontan does not present any model for the $M^{1/2}_{++}$ and $M^{3/2}_{+-}$ amplitudes he extracts from the data.

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Multichannel-Dispersion-Theory Calculation of Photopion Production

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We have performed a multichannel-dispersion-relation calculation of low-energy ($E_\gamma \lesssim 450$ MeV) pion photoproduction. We are able to fit the present γp data along with the less-well-known $\gamma n \to \pi^- p$ data without the introduction of an $I=2$ electromagnetic current. Predictions for the reaction $\gamma n \to \pi^0 n$ are made. Parameters are introduced to describe the photoproduction Born terms for the inelastic hadronic channels. We find that through the rescattering integrals, the inelastic effects strongly influence the $\gamma N \to \pi N$ amplitudes even at low energy (in particular, in the E_{0+} and M_{1-} multipoles).

I. INTRODUCTION

Considerable interest in low-energy pion-photoproduction experiments and phenomenology has been recently stimulated by (a) the suggestion that an $I=2$ electromagnetic current might exist[1] and (b) the possibility that time-reversal invariance might be violated in electromagnetic interactions.[2] If one can obtain as accurate measurements of reactions

$$\gamma + n \to \pi^0 + n, \tag{1a}$$

$$\gamma + n \to \pi^- + p \tag{1b}$$

as have been done for the reactions

$$\gamma + p \to \pi^0 + p, \tag{2a}$$

$$\gamma + p \to \pi^+ + n, \tag{2b}$$

then the question of the need for $I=2$ currents can be settled unambiguously. If the capture reaction

$$\pi^- + p \to \gamma + n \tag{3}$$

and its inverse (1b) are accurately known, the possibility of T violation in the region of the $\Delta(1236)$ can be answered.

At present only the γ-proton reactions (2) are accurately known. Although the π inverse and direct reactions seem to differ in the region of the Δ, the reactions (1b) and (3) are not well known. We think that it is premature to consider T violation a necessary ingredient in a phenomenological model of photoproduction.

Experiments to study (1a) are just under way[3] so that no direct test for an $I=2$ current can be made. However, interesting but model-dependent evidence for this current has been presented by Donnachie and Shaw[4] (DS).

The purpose of this paper is to present a conventional, theoretically based phenomenological model of photoproduction. We think that it is premature to include T violation or $I=2$ currents in this model. Any difficulty encountered in fitting our model to the data may be some indication of the existence of $I=2$ currents.

Theoretical models of photoproduction are basically quite simple. The interaction of the photon with a nucleon to produce a pion-nucleon system is given by various Born terms; the strong interactions then provide an important "rescattering" correction. Given a set of Born terms and the hadronic scattering amplitude in the ND^{-1} form, it is simply a matter of quadrature to obtain unitary photoproduction amplitudes. Beginning with Chew, Goldberger, Low, and Nambu[5] (CGLN) to the present time, the main obstacle to a complete theoretical description of photoproduction has been the lack of a theoretical model of π-N scattering or even a phenomenological treatment in which the amplitudes are separated in N and D functions. Recently this difficulty has been solved in part by the work of Ball, Garg, and Shaw[6] (BGS), and it is their phenomenological treatment of π-N scattering that will provide the basis of our model for photoproduction.

The crucial ingredient of the BGS approach was the explicit consideration of inelastic effects in a multichannel formalism. This complicates the generalization to photoproduction in that Born terms for a photon interacting with a nucleon to produce hadronic channels other than π-N must also be included.

In Sec. II we will discuss the formalization which will generate unitary photoproduction amplitudes. In Sec. III we will discuss the Born terms to be employed and the method in which the "phenomenological" inelastic channels introduced by BGS are used to induce parameters which may be used to fit the data. In Sec. IV we present our numerical results, the fits to the data, and our conclusions. Predictions for the reaction $\gamma n \to \pi^0 n$ are presented.

We are able to obtain a good fit to the present low-energy ($E_\gamma \lesssim 400$ MeV) photoproduction data compiled by DS. Our fit to the proton reactions (2) is very close to that obtained by DS, with ours having a somewhat better χ^2. The shape of our $\gamma n \to \pi^- p$ angular distribution is quite different from the DS fit but has essentially the same χ^2 for this less-well-determined reaction

We found that in our fits to the data the inelastic channels through the rescattering integrals strongly influence the $\gamma N \to \pi N$ amplitudes even at low energy. In particular, our E_{0+} and M_{1-} multipoles are quite different from those of previous calculations[7] which ignore inelastic effects.

We present our results as representing a first step in finding a complete description of low-energy photoproduction which is consistent with the elastic and inelastic π-N data.

II. FORMALISM

We will represent the partial-wave production amplitudes for photon plus nucleon going to n hadronic two-body channels by the column vector M_α. These amplitudes M_α have definite values of J, total angular momentum, parity, and total isospin, and correspond to transitions initiated by either electric or magnetic radiation via isoscalar- or isovector-photon interactions; all of these quantum numbers we summarize with the index α. For the purposes of this discussion we will assume that the correct discontinuities across the unphysical singularities of these amplitude, "left-hand cuts," are given by the Born approximation. The discontinuities across the physical cut, "right-hand cut," are given by unitarity as follows:

$$\operatorname{Im} M_\alpha = f_\alpha^\dagger \rho_\alpha M_\alpha , \tag{4}$$

where f_α is the $n\times n$ hadronic-channel scattering matrix, and ρ_α is the phase-space factor for this partial wave. The f_α are expressed in terms of the $N_\alpha D_\alpha{}^{-1}$ equations as

$$f_\alpha = N_\alpha D_\alpha{}^{-1} = \rho_\alpha{}^{-1/2} \frac{(S_\alpha - 1)}{2i} \rho_\alpha{}^{-1/2} . \tag{5}$$

If we now denote the Born approximation to M_α by B_α^γ, we can write the following expression for M_α (see Ref. 6):

$$M_\alpha = B_\alpha^\gamma + \tilde{D}_\alpha{}^{-1} \frac{1}{\pi} \int \frac{dW' \tilde{N}_\alpha(W') \rho_\alpha(W') B_\alpha^\gamma(W')}{W' - W} , \tag{6}$$

TABLE I. The values γ of the input $(B^{\gamma})_{\gamma N \to \mathrm{P.C.}} = \gamma/(W-W_p)$ (where W_p is the pole position for the strong $B_{\mathrm{P.C.}\to\mathrm{P.C.}}$), and the parameters for the hadronic ND^{-1} solutions. The momenta $q^2(m_1, m_2) = [W^2-(m_1-m_2)^2][W^2-(m_1+m_2)^2]/4W^2$. Only the 6 γ's corresponding to the $E_{0+}^{(0,1)}$, $M_{1+}^{(3)}$, $E_{1+}^{(3)}$, and $M_{1-}^{(0,1)}$ were varied in our χ^2 fit to the $\gamma N \to \pi N$. For the P_{11} strong solution, the direct-channel nucleon pole (included in the hadronic B) is

$$\mathrm{N.P.} = \begin{pmatrix} -10.85 & -3.95 \\ -3.95 & -1.44 \end{pmatrix} \Big/ (W-6.7) .$$

The residue of the ϵN channel was taken to correspond to estimates of the $g_{\epsilon NN}$ coupling constant. The photoproduction N.P. was included for $(B^{\gamma})_{\gamma N \to \epsilon N}$ in the static approximation using this $g_{\epsilon NN}$. Units are $\hbar = c = m_\pi = 1$.

		Hadronic two-channel ND^{-1} solution					
Multipole	γ	$l_{2I,2J}$	$\rho_{\pi N}$	P.C.	$l_{\mathrm{P.C.}}$	$\rho_{\mathrm{P.C.}}$	B
$E_{0+}^{(0)}$	0.048 96	S_{11}	$\frac{q}{W}$	ηN	0	$\frac{q(m_\eta, m_N)}{W}$	$\begin{pmatrix} 0.5 & 13.6 \\ 13.6 & 17.9 \end{pmatrix} \Big/ (W-5.0)$
$E_{0+}^{(1)}$	−0.013 02						
$M_{1+}^{(3)}$	0.1486	P_{33}	$\left(\frac{q}{W}\right)^3$	bN, $m_b = 4.0$	1	$\left[\frac{q(m_b, m_N)}{W}\right]^3$	$\begin{pmatrix} 76.77 & 258.24 \\ 258.24 & 968.65 \end{pmatrix} \Big/ (W-6.085)$
$E_{1+}^{(3)}$	0.00152						
$M_{1-}^{(0)}$	0.019 42 + N.P.	P_{11}	$\frac{q^3}{W^2}$	ϵN, $m_\epsilon = 4.3$	1	$\int_2^{W-m_N} \frac{[q(m, m_N)\Gamma/2]dm}{(m_\epsilon - m)^2 + (\Gamma/2)^2}$	$\mathrm{N.P.} + \begin{pmatrix} 22.58 & 3.45 \\ 3.45 & 2.93 \end{pmatrix} \Big/ (W-3.0)$
$M_{1-}^{(1)}$	0.072 47 + N.P.					$\Gamma = 2.9 \frac{(m^2-4)^{1/2}}{(m_\epsilon^2 - 4)^{1/2}}$	
E_{0+}^{3}	−0.001 98	S_{31}	q	$\pi\Delta$, $m_\Delta = 8.8$, $\Gamma = 0.86$	2	$\int_{m_N}^{W-1} \left(\frac{q(1,m)}{W}\right)^5 \frac{(W\Gamma^2/4)dm}{(m_\Delta - m)^2 + \Gamma^2/4}$	$\begin{pmatrix} 0.72 & 5.48 \\ 5.48 & 140 \end{pmatrix} \Big/ (W-6.7) - \frac{10.0}{W-2}$
M_{1-}^{3}	0.303 57	P_{31}	$\frac{q^3}{W^2}$	bN, $m_b = 4.0$	1	$\frac{q^3(m_b, m_N)}{W^2}$	$\begin{pmatrix} -6.0973 & 0.2016 \\ 0.2016 & 52.17 \end{pmatrix} \Big/ (W-6.7)$
$M_{2-}^{(0)}$	0.001 48	D_{13}	$\frac{q^5}{W^4}$	bN, $m_b = 3.1$	2	$\frac{q^5(m_b, m_N)}{W^4}$	$\begin{pmatrix} 13.5 & 463.0 \\ 463.0 & 939.0 \end{pmatrix} \Big/ (W-6.7)$
$E_{2-}^{(0)}$	0.001 67						
$M_{2-}^{(1)}$	0.011 29						
$E_{2-}^{(1)}$	0.0291						

where the integral runs over the right-hand cut, where W, the total energy of the γN system, is greater than the appropriate hadronic threshold.

The multipole amplitudes M_α and B_α which appear here have the threshold behavior removed. The relation between these multipole amplitudes for $\gamma N \to \pi N$ and those of CGLN ($M_{\mathrm{CGLN}} \equiv E_{l\pm}, M_{l\pm}$) (see Ref. 5) is

$$M_{\mathrm{CGLN}} = \frac{q^l k}{W^l} (M_\alpha)_{\gamma N \to \pi N}, \qquad (7)$$

where k and q are the photon and pion center-of-mass momenta. Thus we have ensured the correct threshold behavior when we calculate the CGLN multipoles from (6) and (7).

The analysis of BGS fitted the elastic πN phase-shift analyses for δ and η ($S = \eta e^{2i\delta}$) as a function of W using the ND^{-1} formalism. For each partial wave, we found it adequate to describe all the inelastic effects through one phenomenological higher-mass channel (P.C.). The nature of the P.C. was separately chosen for each partial wave, e.g., ϵN for the P_{11}. Simple poles were used to represent the 2×2 symmetric input potential B for the strong interaction. In each case we needed only one input potential pole for the P.C. The natures of the P.C., the ρ_α, and the input B we used to fit the elastic πN phase shifts are given in Table I. The N_α and D_α matrices needed in Eq. (6) are found by solving Eqs. (1)–(3) of BGS using the parameters in Table I.

While it is true that the scattering into inelastic channels is small in the low-energy region, it should be noted that the integral in Eq. (6) includes high energies. Thus inelastic effects can strongly influence the pion photoproduction amplitude even at low energies.

The only remaining step required to obtain unitary photoproduction amplitudes from Eq. (6) is determining reasonable photoproduction Born terms B^γ for the input.

III. MULTIPOLE BORN APPROXIMATION AND KINEMATIC EFFECTS

The fact that the BGS model we used to describe the πN scattering contains a phenomenological inelastic channel in each partial wave, in addition to the elastic pion-nucleon channel, forces us to consider the photoproduction of these channels in our model.

The Born terms for $\gamma N \to \pi N$ have a clear theoretical origin and are taken to be given by the diagrams in Fig. 1. They are known functions given in terms of the pion-nucleon coupling constant, charges, and nucleon magnetic moments. Explicit formulas for the multipole projections of the pole

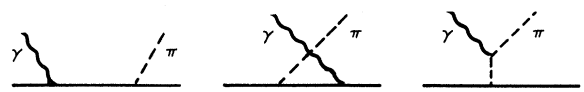


FIG. 1. Born approximation for $\gamma N \to \pi N$.

terms in Fig. 1 have been presented by a number of authors.[8] In transitions leading to the P_{11} πN partial wave, it is necessary to include the contribution of $\Delta(1236)$ exchange. We used the Δ exchange in the static approximation as given by Donnachie and Shaw.[9] Note again that our multipole inputs B^γ in (6) are simply related to these CGLN multipoles Born terms by the kinematical factor in (7).

We have unitarized all the multipoles which lead to πN final states having appreciable phase shifts at $E_\gamma < 500$ MeV or $W \lesssim 1350$ MeV. We solved (6) for the 12 multipoles leading to the following six πN final states[10]:

$E_{0+}^{(0,1)}$ to S_{11};

$E_{1+}^{(3)}$ and $M_{1+}^{(3)}$ to P_{33};

$M_{1-}^{(0,1)}$ to P_{11};

$E_{0+}^{(3)}$ to S_{31};

$M_{1-}^{(3)}$ to P_{31};

$M_{2-}^{(0,1)}$ and $E_{2-}^{(0,1)}$ to D_{13}.

The Born approximation for all other multipoles was included by simply using the complete diagrams in Fig. 1, subtracting out the multipole projections for the above 12 waves, and then adding back in their unitarized expressions obtained from Eqs. (6) and (7). This procedure guarantees that the pole singularities are correctly included in our final result.

We treated the photoproduction Born terms in (6) for the phenomenological channels in the simplest possible manner: The Born term for $\gamma N \to$ P.C. is taken to be a pole in the W plane at the position of the pole used in the BGS πN solution for that partial wave with a residue γ used as an adjustable parameter. This results in 1 parameter per multipole or 12 adjustable parameters for all the above multipoles.

By far the most prominent feature of low-energy photoproduction is the 3-3 resonance. We found that slight changes in the parameters of this resonance can have substantial effects on the quality of a fit to the data. The $\pi^+ p$ scattering is extremely well determined experimentally, and our two-channel ND^{-1} model provides a precise fit[11] to the data for the Δ^{++}. However, we need to know the parameters for the Δ^+ into the $\pi^+ n$ and $\pi^0 p$ channels. Clearly there is one S-matrix pole for the Δ^+ with its position and residue[11,12] slightly shifted from that of the Δ^{++}. We approximated these unknown shifts along with the kinematical differences between π^+n and π^0p as well as possible mass-difference-induced effects in the B^γ for this wave by treating the resonance energy as a free parameter for π^+n and for π^0p. Since the π^-p data are less accurate, we constrained its resonance energy to be the same as for the π^+n data. This introduces two additional parameters bringing the total to 14.[13]

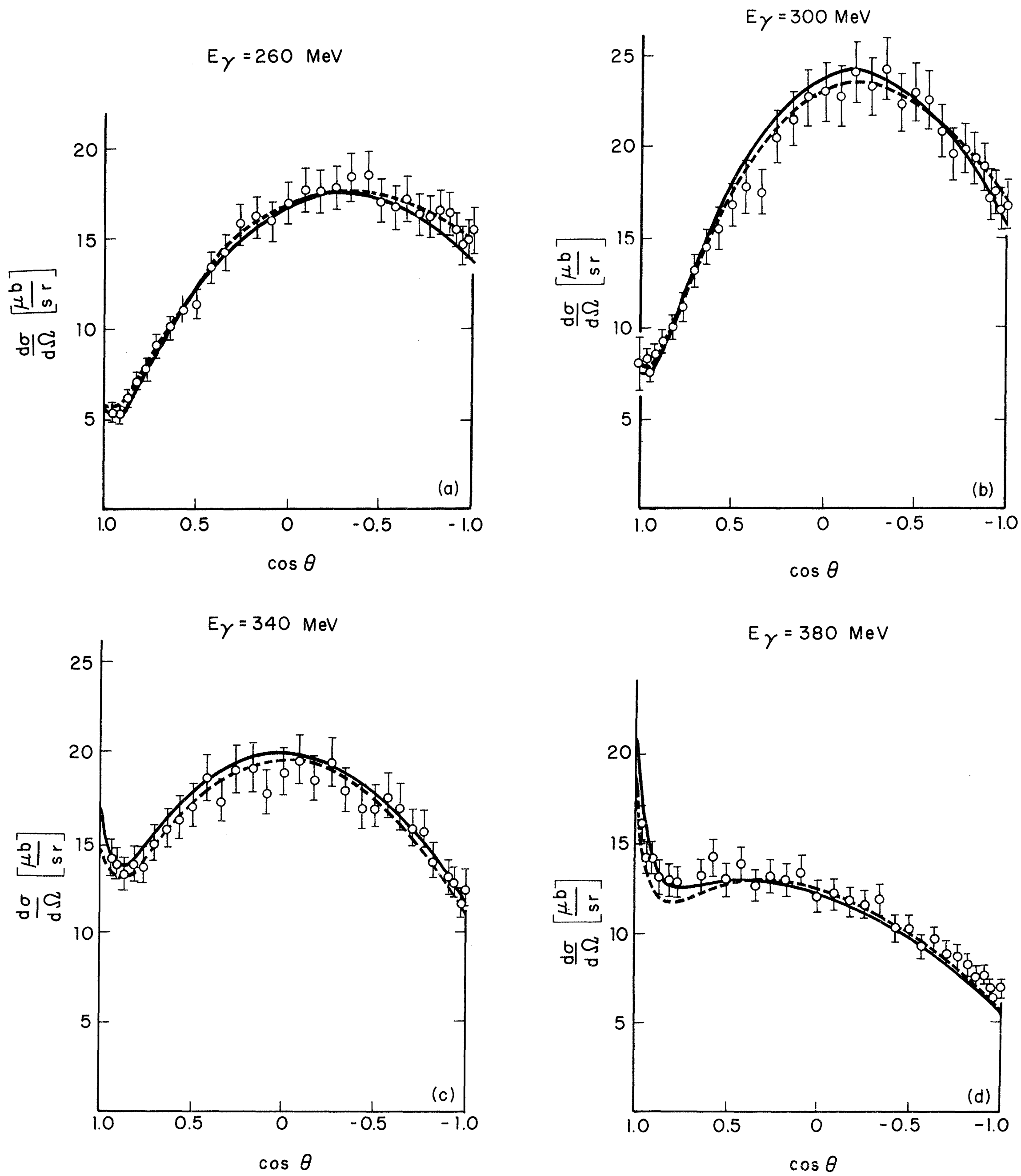
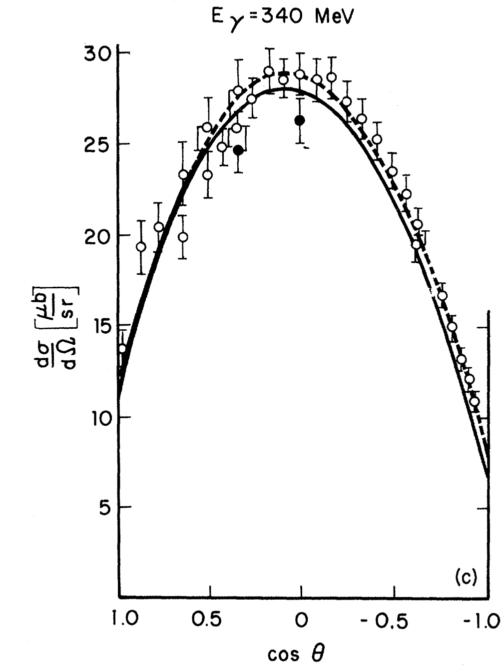


FIG. 2. Fit to the π^+ photoproduction differential cross-section data. Our fit is the solid line, the fit of DS is the dashed line. The open circles represent data from Fischer *et al.*, and the closed circles are from Betourni *et al.* (see Ref. 15).

IV. FITTING PROCEDURE, RESULTS, AND CONCLUSIONS

We obtained the initial values of our 12 parameters γ for the residues of the input multipole Born terms for the phenomenological channel by fitting the imaginary parts of our multipole to those of Walker[14] or those of Ref. 9. We found that the χ^2 was insensitive to changes in the γ's for $E_{0+}^{(3)}$, $M_{1-}^{(3)}$, $E_{2-}^{(0)}$, $M_{2-}^{(0)}$, $E_{2-}^{(1)}$, and $M_{2-}^{(1)}$. These parameters were then fixed at the starting values obtained above and a least-χ^2 fit was performed, varying the remaining 8 parameters (6 γ's + 2 mass shifts) to a portion of the photoproduction data[15] compiled by DS. In fitting these data we handled the errors by simply assigning a fixed error of 1 μb for the π^+ and π^0 data points and 3 μb for the π^-. Our final χ^2 was 1.3/data point. The values of our parameters are given in Table I.



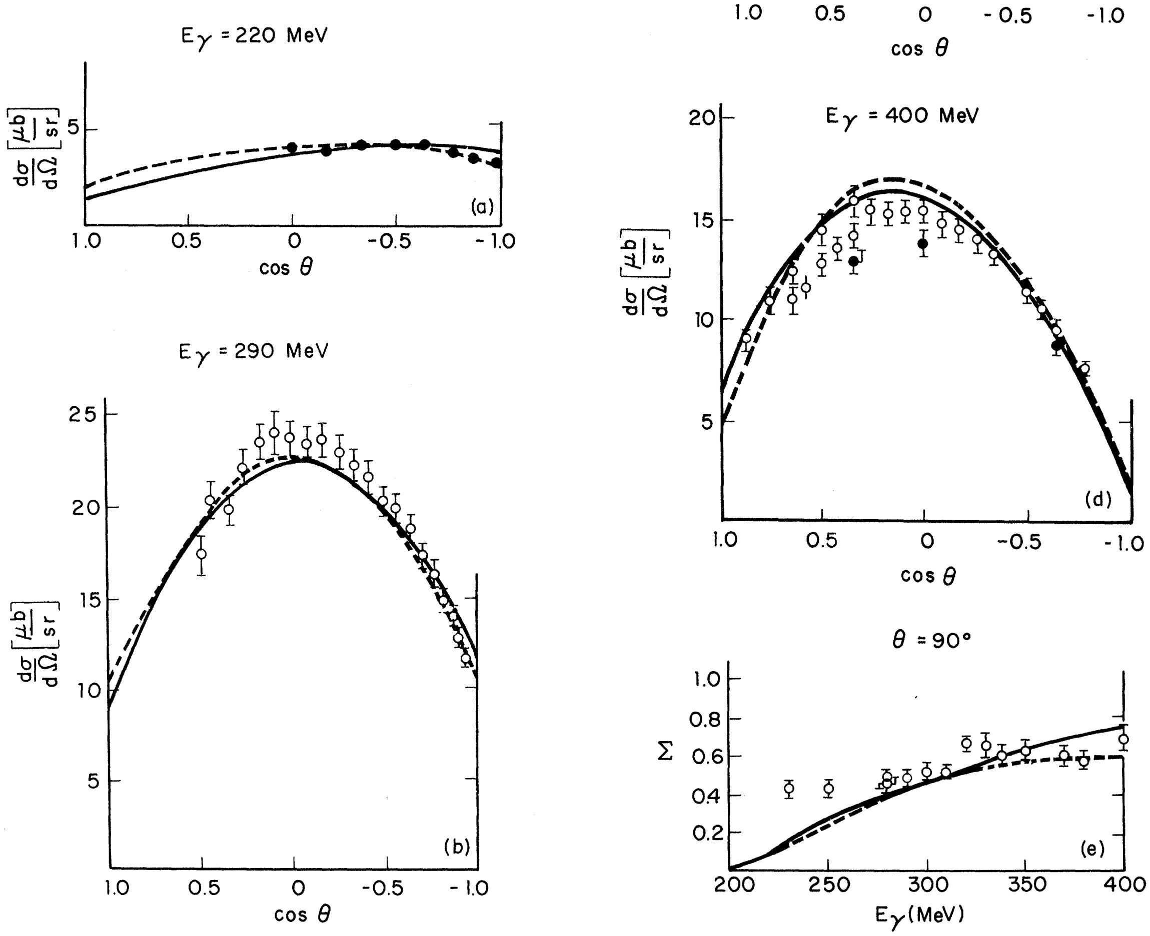


FIG. 3. Fit to the π^0 photoproduction differential cross-section and asymmetry data. Our fit is the solid line, and the fit of DS is the dashed line. The open circles represent data from Fischer *et al.*, and the closed circles are from Hilger *et al.*, and Morand *et al.* (see Ref. 15). See Fig. 10 of DS for references to the polarized-photon asymmetry data.

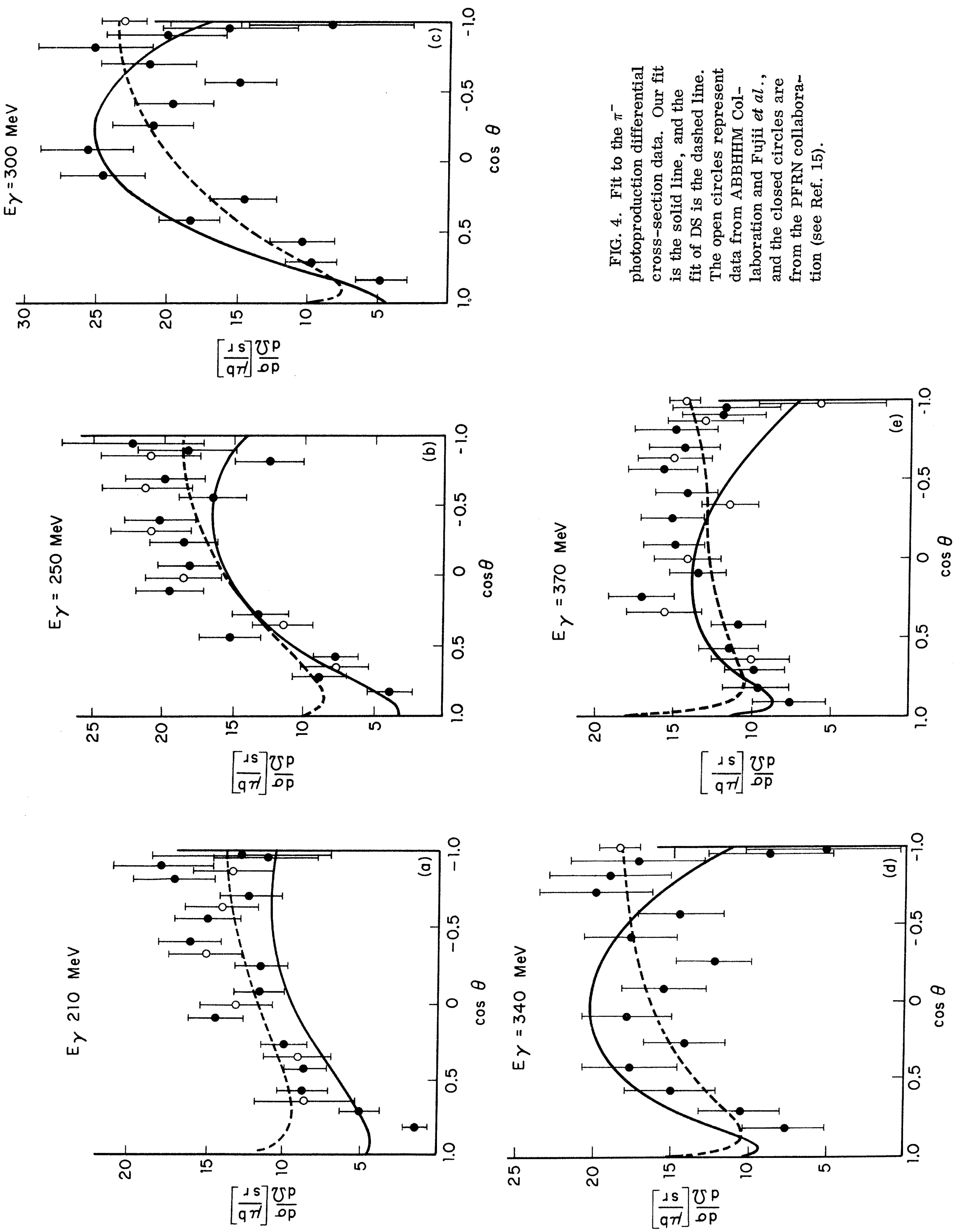


FIG. 4. Fit to the π^- photoproduction differential cross-section data. Our fit is the solid line, and the fit of DS is the dashed line. The open circles represent data from ABBHHM Collaboration and Fujii *et al.*, and the closed circles are from the PFRN collaboration (see Ref. 15).

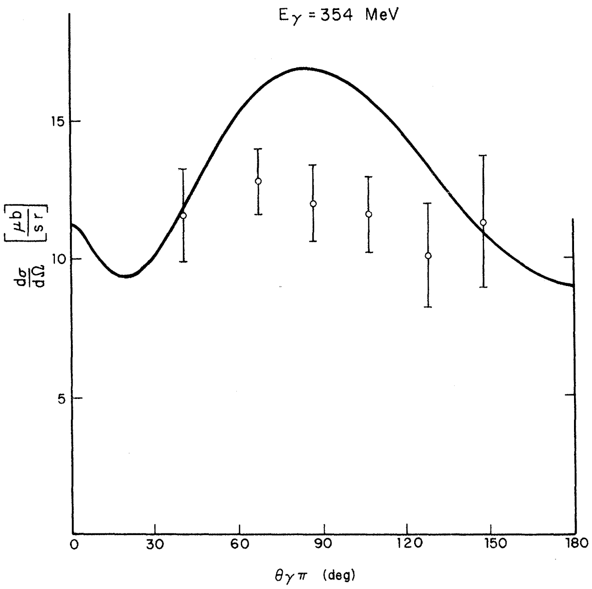


FIG. 5. Fit to the π^- capture data of Berardo *et al.* (Ref. 16).

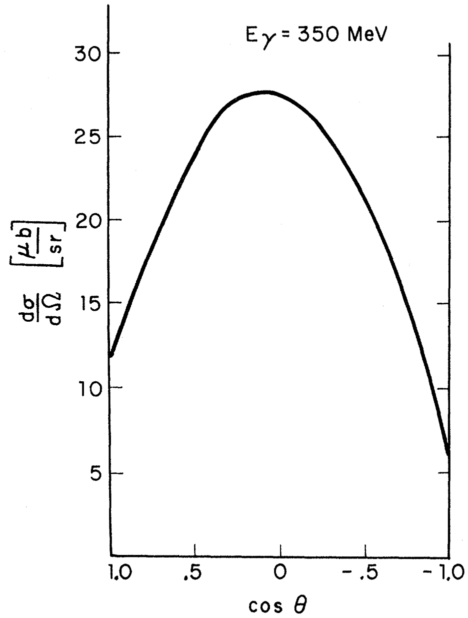


FIG. 6. Prediction for $\gamma + n \rightarrow \pi^0 + n$ differential cross section at 350 MeV.

Our fits to some of the data are shown in Figs. 2, 3, and 4. In Fig. 5 our results for π^- production at 354 MeV are shown together with the capture data of Berardo *et al.*[16] We have calculated all multipoles and all cross sections at 10-MeV intervals up to $E_\gamma = 450$ MeV, but have only presented a representative sample[17] as we view this as a preliminary calculation which will be revised as new data become available.[3]

Our calculated values for the E_{0+}, M_{1-}, and E_{1+} are quite different from previous single-channel calculations.[7] Thus we list these multipoles in Table II.

The effective shift of the P_{33} resonance which we obtained was 1.5 MeV lower for the $\pi^+ n$ channel and 7.4 MeV down for the $\pi^0 p$ channel as compared to the fit[11] to the Δ^{++}. It is reassuring that the difference between these channels which both contain Δ^+ is of the same magnitude as the difference in threshold energies.

We are able to obtain a good fit to the present low-energy ($E_\gamma \lesssim 400$ MeV) photoproduction data compiled by DS. Our fit to the proton reactions is very close to that obtained by DS, with ours having a somewhat better χ^2. The shape of our $\gamma n \rightarrow \pi^- p$ angular distribution is quite different from the DS fit, but has essentially the same χ^2. That this difference is possible is attributable to the data which are poorly determined in the backward direction. It should be emphasized that our model contains no $I = 2$ current and the only freedom is the result of inelastic hadronic channels, a freedom that necessarily exists in any model of photoproduction.

In Fig. 6 we present our predictions for the reaction $\gamma n \rightarrow \pi^0 n$ at $E_\gamma = 350$ MeV.

TABLE II. Some of our unitarized multipoles as a function of E_γ.

E_γ	$10^5 E_{0+}^{(0)}$		$10^4 E_{0+}^{(1)}$		$10^5 M_{1-}^{(0)}$		$10^5 M_{1-}^{(1)}$		$10^5 E_{1+}^{(3)}$	
(MeV)	Re	Im	Re	Im	Re	Im	Re	Im	Re	Im
220	81	6	219	16	−70	2	252	−6	−234	−42
240	101	9	194	16	−79	2	253	−7	−237	−74
260	123	12	171	16	−87	3	252	−7	−226	−110
280	145	15	150	16	−94	2	252	−7	−191	−148
300	169	19	132	15	−99	2	258	−5	−125	−170
320	194	24	114	14	−104	1	271	−2	−48	−145
340	220	29	99	13	−106	−0	300	7	−2	−82
360	284	35	84	12	−107	−0	346	31	4	−18
380	277	42	70	10	−106	1	405	73	−12	28
400	308	49	57	9	−104	4	477	141	−35	57

Our calculations presented in this paper represent a first step in obtaining a complete description of low-energy photoproduction consistent with the elastic and inelastic π-N data. Newer and more accurate data should become available in the near future[3] which will put our present model to a more stringent test.[13]

Current work at Berkeley[18] indicates that it should be possible to replace the phenomenological channels used by BGS by quasi-two-body channels obtained by direct analysis of experimental data for $\pi N \to \pi\pi N$ and extend the present type of analysis up to higher energy ($E_\gamma \sim 1500$ MeV). We also hope to be able to look at photoproduction of other hadronic channels with the goal of using some of the predictions of this model which we have left without interpretation.

ACKNOWLEDGMENT

We would like to thank Professor A. Donnachie for several very helpful discussions.

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[7]See, e.g., F. A. Berends, A. Donnachie, and D. L. Weaver [Nucl. Phys. B4, 54 (1967)] for typical single-channel calculations.

[8]See, e.g., F. A. Berends, A. Donnachie, and D. L. Weaver [Nucl. Phys. B4, 1 (1967)] for explicit formulas for the $\gamma N \to \pi N$ Born terms and multipole projections.

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[10]Superscripts on the multipoles are the standard notation; (0) denoting the isoscalar transition to the πN $I=\frac{1}{2}$ state, (1) the isovector transition to $I=\frac{1}{2}$, and (3) the isovector transition to $I=\frac{3}{2}$.

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