(and therefore the condition that a_0 and a_2 are relatively real; see Ref. 4), one has

$$\eta_{+-} = \eta_{00} = \epsilon \equiv \frac{\langle 2\pi, I=0|T|K_L\rangle}{\langle 2\pi, I=0|T|K_S\rangle}.$$

The phase of ϵ to a good approximation is $(2\Delta m/\Gamma_S)$ due to an argument of Wolfenstein; see, for example, his Erice Lecture note in *Theory and Phenomenology in Particle Physics,* Proceedings of the School of Physics "Ettore Majorana," 1968, edited by A. Zichichi (Academic, New York, 1969), p. 218. In the present scheme, the approximation amounts to dropping primarily the $|3\pi\rangle$-*real* intermediate state compared to the $|2\pi\rangle$-real intermediate state in the evaluation of the ratio of the off-diagonal and diagonal elements of the K_1-K_2-width matrix. In the present case, this is not expected to lead to an error of more than a few percent.

[7]L. Wolfenstein, Phys. Rev. Lett. 13, 562 (1964).

[8]If the $|\Delta S|=1$ part of the nonleptonic Hamiltonian in a theory could be expressed in the form (3), such that Eq. (5) were satisfied not only by $P^{(-)}$ and $P^{(+)}$, *but also* by $S^{(-)}$ and $S^{(+)}$ [with the substitution $(P^{(-)}, P^{(+)}) \to (S^{(-)}, S^{(+)})$], then one may show that the mixing parameter ρ would conspire with R, so that $\rho=-R$, and $\eta_{+-}=\eta_{00}=0$. In fact, there would be *no effective CP violation* in the theory, which of course does not happen in the present case.

PHYSICAL REVIEW D VOLUME 8, NUMBER 7 1 OCTOBER 1973

Mode and Scaling in Charged Multiplicity Distributions*

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A quasinormal expansion is used to examine a possibility for scaling of charged multiplicity distributions in *pp* collisions.

It has been pointed out by several authors that the charged multiplicity cross sections σ_n in *pp* collisions[1] with incident energies 50–300 GeV are well represented by normal[2-5] or approximately normal[6,7] distributions. The Koba-Nielsen-Olesen (KNO) scaling[8]

$$P_n \equiv \frac{\sigma_n}{\sigma_{\text{inel}}} = \frac{1}{\langle n\rangle}\,\psi\!\left(\frac{n}{\langle n\rangle}\right) \tag{1}$$

also seems not inconsistent with experiment at the present energy, where ψ is approximately Gaussian.[4,6]

A mathematical basis which leads us to obtain a quasinormal distribution was discussed in Refs. 7, 9, and 10. It is an analog of the central-limit theorem and can be stated in the following way: The asymptotic expansion at the mode[11] m,

$$P_n = \frac{1}{\sqrt{2\pi}\,\beta}\exp\left[-\frac{1}{2}\left(\frac{n-m}{\gamma}\right)^2\right]\left[1+\sum_{k=3}^{\infty} a_k\left(\frac{n-m}{\gamma}\right)^k\right] \tag{2a}$$

$$= \frac{1}{\sqrt{2\pi}\,\beta}\exp\left[-\frac{1}{2}\left(\frac{n-m}{\gamma}\right)^2 + \sum_{k=3}^{\infty} b_k\left(\frac{n-m}{\gamma}\right)^k\right], \tag{2b}$$

is valid provided that

$$\kappa_2 \to \infty \quad \text{as } s\to\infty \tag{3}$$

and that the condition

$$|(n-m)/\gamma^2| < \pi \tag{4}$$

is satisfied. The parameters β, m, γ, a_k, and b_k can be expressed in terms of moments, deviants,[10] or cumulants.[12,13] If correlations of the produced particles are *temperate*[7,9] in the sense that higher cumulants κ_k satisfy the condition

$$\kappa_k/\kappa_2^{k/2} = O(\epsilon^{k-2}), \quad k\geq 3 \tag{5}$$

with

$$\epsilon \ll 1, \tag{6}$$

then we have

$$a_{3l-4,3l-2,3l} = O(\epsilon^l), \quad l\geq 1 \tag{7}$$

and

$$b_k = O(\epsilon^{k-2}), \quad k\geq 3. \tag{8}$$

Therefore, only a few terms in expansion (4) are important.

If, moreover, the limits[7]

$$\lim_{s\to\infty}\frac{\beta}{m} = \lim_{s\to\infty}\frac{\sqrt{\kappa_2}}{\kappa_1-\frac{1}{2}(\kappa_3/\kappa_2)}[1+O(\epsilon^2)] = b, \tag{9}$$

$$\lim_{s\to\infty}\frac{\gamma}{m} = \lim_{s\to\infty}\frac{\sqrt{\kappa_2}}{\kappa_1-\frac{1}{2}(\kappa_3/\kappa_2)}[1+O(\epsilon^2)] = d \tag{10}$$

are nonvanishing, Eq. (2) reduces to

TABLE I. Values of the parameters for the best fit [N(degrees of freedom) = 42] (A) with scaling function, Eq. (11a); (B) with scaling function, Eq. (11b). The prime indicates the case where the errors due to $\sigma_{\rm inel}$ are neglected. The number in the parenthesis that follows m represents energy in the units GeV.

	A	A′	B	B′
$\beta/m = b$	1.02 ± 0.01	1.03 ± 0.01	0.97 ± 0.02	0.98 ± 0.01
$\gamma/m = d$	1.10 ± 0.01	1.11 ± 0.01	1.05 ± 0.02	1.08 ± 0.01
a_3	0.071 ± 0.006	0.081 ± 0.005	0.036 ± 0.072	0.037 ± 0.002
m(50)	1.25 ± 0.03	1.23 ± 0.01	1.31 ± 0.03	1.29 ± 0.01
m(69)	1.44 ± 0.01	1.41 ± 0.01	1.51 ± 0.02	1.48 ± 0.01
m(102)	1.59 ± 0.04	1.56 ± 0.03	1.68 ± 0.04	1.63 ± 0.02
m(205)	1.99 ± 0.03	1.96 ± 0.01	2.10 ± 0.04	2.06 ± 0.03
m(303)	2.33 ± 0.05	2.28 ± 0.03	2.46 ± 0.05	2.39 ± 0.03
χ^2	35.6	60.4	35.0	62.4

$$m\frac{\sigma_n}{\sigma_{\rm inel}} = \frac{1}{\sqrt{2\pi}\,b}\exp\left[-\frac{1}{2d^2}\left(\frac{n}{m}-1\right)^2\right]\times\left[1+\frac{a_3}{d^3}\left(\frac{n}{m}-1\right)^3+\cdots\right] \tag{11a}$$

$$= \frac{1}{\sqrt{2\pi}\,b}\exp\left[-\frac{1}{2d^2}\left(\frac{n}{m}-1\right)^2+\frac{a_3}{d^3}\left(\frac{n}{m}-1\right)^3+\cdots\right]. \tag{11b}$$

This is a scaling in terms of the variable n/m, while the KNO scaling is expressed in terms of the variable $n/\langle n\rangle$ [Eq. (1)]. Both scaling laws coincide at infinite energy provided that the limit

$$\lim_{s\to\infty}\frac{m}{\langle n\rangle} = \lim_{s\to\infty}\left\{1-\frac{\kappa_3}{2\kappa_1\kappa_2}\left[1+O(\epsilon^2)\right]\right\} \tag{12}$$

exists.[14] Certainly that is the case if

$$\kappa_k \propto (\ln s)^k\,. \tag{13}$$

The advantages of Eq. (11) over the KNO scaling (1) are that (i) the approximation is best around the mode, and (ii) the scaling function is proved to be quasinormal.

From this point of view, we analyze the pp collision experimental data based on Eq. (11). The results are shown in Figs. 1(a) and 1(b) and in columns A and B of Table I. A few remarks are given below.

(1) With the accuracy and the energy range of the present experiment, we do not see an essential difference between the two forms of Eq. (11). Any improvement of the accuracy or increase of the energy of the experiment might differentiate the two forms with respect to the effectiveness of representing the experimental data. Although the two forms of Eq. (11) are mathematically equivalent when one takes an infinite number of terms, there is a difference in practice when only a finite number of terms are considered.

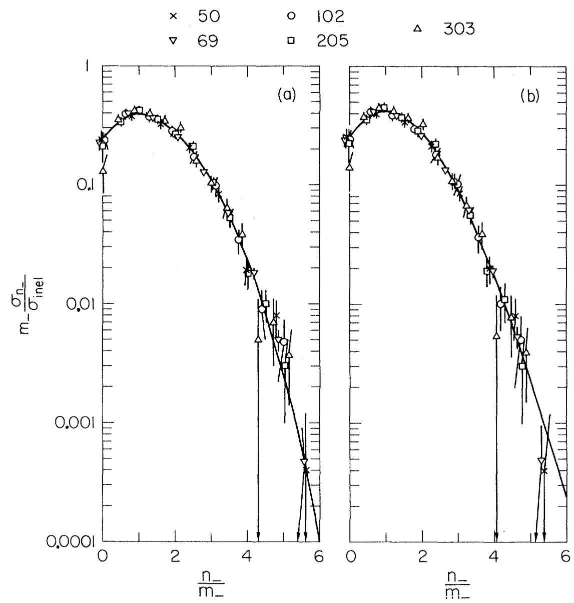

FIG. 1. Scaling of negative-charge multiplicity distributions in pp collisions (a) with scaling function, Eq. (11a); (b) with scaling function, Eq. (11b).

(2) The necessity of the a_3 term is clearly exhibited. We repeated the analysis neglecting the errors due to $\sigma_{\rm inel}$, since their inclusion forces us to give more weight to the events of high multiplicities. The results are shown in columns A′ and B′ of Table I, in which the real χ^2 value should be obtained from the listed value divided by a factor 1.5–2. In this way, the relative weight of the experimental data around the mode is increased, which is a reasonable procedure because of the nature of expansion (11). We do not see a big difference in the results, however.

In conclusion, the scaling based on Eq. (11) is consistent with the present experimental data.

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[11] Mode is the value of the multiplicity for which the distribution function is maximum.

[12] M. G. Kendall and A. Stuart, *The Advanced Theory of Statistics* (Griffin, London, 1963), Vol. 1.

[13] Cumulants are defined by

$$\phi(t)=\sum_{n=0}^{\infty} e^{itn}P_n = \exp\left[\sum_{k=1}^{\infty}\frac{\kappa_k (it)^k}{k!}\right],$$

e.g., $\kappa_1=\langle n\rangle$, $\kappa_2=\langle(n-\langle n\rangle)^2\rangle$, $\kappa_3=\langle(n-\langle n\rangle)^3\rangle$, etc.

[14] In this case, Eq. (11b) gives an explanation of the scaling function used in Ref. 6.

PHYSICAL REVIEW D VOLUME 8, NUMBER 7 1 OCTOBER 1973

Comment on the Approach to Factorization and Scaling in Inclusive Reactions*

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A previous compilation of data pertaining to the question of factorization in the central region of pion production is updated and extended to include K_S^0 and Λ^0 production.

In a recent letter we provided evidence for the factorization hypothesis in the central region of pion production.[1] It was also indicated at that time that the data were consistent with an approach to limiting behavior expected on the basis of the Mueller-Regge formalism. In this note we wish to update the data presented for the reactions[2]:

$$p+p\to\pi^-+\text{anything}, \tag{1}$$

$$\pi^++p\to\pi^-+\text{anything}, \tag{2}$$

$$\gamma+p\to\pi^-+\text{anything}, \tag{3}$$

$$K^++p\to\pi^-+\text{anything}, \tag{4}$$

$$\pi^++p\to\pi^++\text{anything}, \tag{5}$$

and examine new data for the reaction[2,3]:

$$p+p\to\pi^++\text{anything} \tag{6}$$

as well as data for the K_S^0 channels[4]:

$$p+p\to K_S^0+\text{anything}, \tag{7}$$

$$K^-+p\to K_S^0+\text{anything}, \tag{8}$$

$$K^++p\to K_S^0+\text{anything}, \tag{9}$$

$$\pi^++p\to K_S^0+\text{anything}, \tag{10}$$

and for the Λ^0 production reactions[5]:

$$p+p\to\Lambda^0+\text{anything}, \tag{11}$$

$$K^-+p\to\Lambda^0+\text{anything}, \tag{12}$$

$$K^++p\to\Lambda^0+\text{anything}, \tag{13}$$

$$\pi^++p\to\Lambda^0+\text{anything}. \tag{14}$$

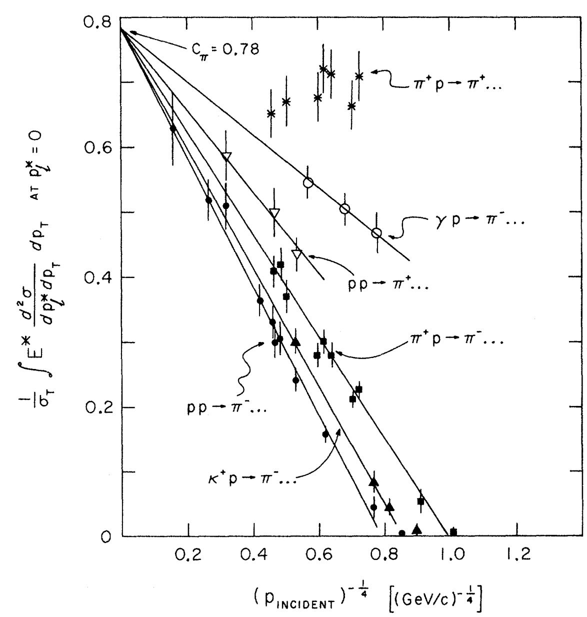

FIG. 1. Normalized invariant cross section at $p_l^*=0$ for pion production.