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[7]See Ref. 1. The data analyzed in that paper include the following: D. B. Smith *et al.*, Phys. Rev. Lett. 23, 1064 (1968); R. S. Panvini *et al.*, Phys. Lett. 38B, 55 (1972); H. Bøggild *et al.*, in *Proceedings of the Amsterdam International Conference on Elementary Particles,* edited by A. G. Tenner and M. Veltman (North-Holland, Amsterdam, 1971); L. Day *et al.*, Phys. Rev. Lett. 18, 1055 (1969); Phys. Rev. D 3, 645 (1971).

[8]Saclay-Strasbourg Collaboration and British-Scandinavian Collaboration, in *Proceedings of the XVI International Conference on High Energy Physics* (Ref. 5), Vol. 1, p. 308. See also Phys. Lett. 44B, 521 (1973) and 44B, 537 (1973).

[9]A. Bertin, P. Capiluppi, M. d'Agostini-Bruno, R. J. Ellis, G. Giacomelli, A. M. Rossi, G. Vannini, A. Bussière, and R. T. Poe, Phys. Lett. 42B, 493 (1972).

[10]R. Hagedorn, Nuovo Cimento Suppl. 3, 147 (1965).

[11]We have also analyzed the P_T distribution at 90° of π^0 presented by the CERN-Columbia-Rockefeller Collaboration at the Vanderbilt Conference, 1973 (unpublished). We are unable to obtain a satisfactory fit with the modified Bose distribution (2).

[12]See A. Jabs, Z. Phys. 222, 12 (1969). We thank Dr. Jabs for calling our attention to this point and sending us his other papers.

[13]Wing-Yin Yu, Phys. Rev. D (to be published).

[14]This Lorentz transformation of temperature was first discussed by A. Einstein [Jahrb. Radioakt. Elektron. 4, 411 (1907)] and M. Planck [Ann. Phys. (Leipz.) 26, 1 (1908)].

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CP Violation Through Phase Angles in Weak Currents and the Relation $\eta_{+-} = \eta_{00}$ *

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It is observed that in a theory where *CP* violation is introduced through phase angles between vector and axial-vector currents the relation $\eta_{+-} = \eta_{00}$ is exact if $\phi = -\xi$, *without* the assumption of soft pions. ϕ and ξ are the phase angles for the strangeness-preserving and strangeness-changing currents, respectively.

It had been noted[1] some time ago that in a theory in which *CP* violation is attributed to phase angles[2] between the weak-vector and axial-vector currents, the $|\Delta I| = \frac{1}{2}$ rule and hence the relation $\eta_{+-} = \eta_{00}$ follows in the *double-soft-pion limit* for the $K_{L,S} \to 2\pi$ decay amplitudes, provided $\phi = -\xi$, where ϕ and ξ are the phase angles for the strangeness-conserving and the strangeness-changing weak currents, respectively. It was also noted[1] that the choice $\phi = \pm\xi$ is necessary to preserve the familiar current-algebra applications to other (*CP-conserving*) nonleptonic decays. For $K \to 2\pi$ decay, since the soft-pion limit involves a rather large extrapolation from the physical point (of order m_K^2) in the relevant Mandelstam variables,[3] one may question the validity of the above result for real pions. The purpose of this note is to remark that *if* $\phi = -\xi$, the relation $\eta_{+-} = \eta_{00}$ *holds without the soft-pion approximation* for the $K \to 2\pi$ amplitudes, even though the $|\Delta I| = \frac{1}{2}$ rule may not.

To see this, write the nonleptonic weak Hamiltonian in the current-current form

$$H_W = \frac{G}{\sqrt{2}}\,(J_\mu J_\mu^\dagger + J_\mu^\dagger J_\mu)\,, \tag{1}$$

where

$$J_\mu = \cos\theta(V_\mu^{1+i2} + e^{i\phi}A_\mu^{1+i2}) + \sin\theta(V_\mu^{4+i5} + e^{i\xi}A_\mu^{4+i5})\,. \tag{2}$$

The $|\Delta S| = 1$ part of H_W for $\phi = -\xi$ is given by

$$H_W^1 = \frac{G}{\sqrt{2}}\cos\theta\sin\theta[S^{(+)} + S^{(-)} + P^{(+)} + P^{(-)}]\,, \tag{3}$$

where

$$S^{(+)}=(V_\mu^{1+i2}V_\mu^{4-i5}+\cos 2\phi A_\mu^{1+i2}A_\mu^{4-i5})+\text{H.c.},$$
$$S^{(-)}=i\sin 2\phi(A_\mu^{1+i2}A_\mu^{4-i5}-A_\mu^{1-i2}A_\mu^{4+i5}),$$
$$P^{(+)}=\cos\phi(V_\mu^{1+i2}A_\mu^{4-i5}+V_\mu^{1-i2}A_\mu^{4+i5}+V_\mu^{4+i5}A_\mu^{1-i2}+V_\mu^{4-i5}A_\mu^{1+i2})+\text{H.c.},\quad(4)$$
$$P^{(-)}=i\sin\phi(V_\mu^{1+i2}A_\mu^{4-i5}-V_\mu^{1-i2}A_\mu^{4+i5}-V_\mu^{4+i5}A_\mu^{1-i2}+V_\mu^{4-i5}A_\mu^{1+i2})+\text{H.c.}.$$

$S^{(\pm)}$ and $P^{(\pm)}$ are parity-conserving and parity-violating parts of H_W^1, respectively; the superscripts + and − correspond to CP-even and CP-odd operators, respectively. It follows from Eq. (4) and the *isospin transformation* property of the currents V_μ^i and A_μ^i that

$$[I_3, P^{(-)}]=+\tfrac{1}{2}i\tan\phi P^{(+)},\quad(5)$$

where I_3 is the third component of $\vec{\text{I}}$-spin generator. Taking the matrix element of both sides of Eq. (5) between $|K_1\rangle$ and $\langle\pi^i\pi^j|$, where $(i,j)=(+,-)$ or $(0,0)$ and K_1 and K_2 are the CP-even and CP-odd eigenstates, respectively, and noting that $I_3|\pi^i\pi^j\rangle=0$ and $I_3|K_1\rangle=-\frac{1}{2}|K_2\rangle$ we have

$$\langle\pi^i\pi^j|P^{(-)}|K_2\rangle=i\tan\phi\langle\pi^i\pi^j|P^{(+)}|K_1\rangle.\quad(6)$$

Thus the ratio of $K_2\to\pi^i\pi^j$ and $K_1\to\pi^i\pi^j$ amplitudes is given by

$$R\equiv\frac{M(K_2\to\pi^i\pi^j)}{M(K_1\to\pi^i\pi^j)}=i\tan\phi,\quad(7)$$

which is *independent of* (i,j) *and is purely imaginary.*[4] It then follows that

$$\eta_{+-}=\eta_{00}=(R+\rho)/(1+\rho R),\quad(8)$$

where

$$\eta_{ij}\equiv\frac{M(K_L\to\pi^i\pi^j)}{M(K_S\to\pi^i\pi^j)}\quad(9)$$

and ρ is the CP-even mixing parameter in K_L, i.e.,

$$K_L=\frac{K_2+\rho K_1}{1+|\rho|^2}.\quad(10)$$

For a given phase angle ϕ, the magnitude and phase of η_{+-} still depend upon the magnitude and phase of ρ, which in general is a complex number.[5] Without calculating ρ, it may be shown that to a very good approximation,[6] the phase of η_{+-} is given by

$$\phi_{+-}\simeq\tan^{-1}\frac{2\Delta m}{\Gamma_S},\quad(11)$$

where $\Delta m=m_L-m_S$, and Γ_S is the width of the short-lived kaon. As is well known, Eqs. (8) and (11) are exact predictions of the superweak theory.[7] In the present case (8) is exact, while (11) should hold to a very good approximation.[6] The two schemes can be distinguished most notably by a measurement of the electric dipole moment of the neutron.

We should remark that in the present model, there does not exist[8] any simple relationship between the parity-conserving operators $S^{(+)}$ and $S^{(-)}$ analogous to that between the parity-violating operators $P^{(+)}$ and $P^{(-)}$ given by Eq. (5). Thus, one does not expect any simple relationship between $K_1\to 3\pi$ and $K_2\to 3\pi$ amplitudes analogous to that between $K_{1,2}\to 2\pi$ amplitudes [see Eq. (7)]. This is another distinction from the superweak model.

In summary, if CP violation is introduced through phase angles in weak currents, the choice $\phi=-\xi$ leads to $\eta_{+-}=\eta_{00}$ as an exact relation without the hypothesis of current algebra, PCAC, and soft pion approximation. The latter is only relevant in yielding[1] a $\Delta I=\frac{1}{2}$ rule (in the soft-pion limit).

Added Note: After this note was written, Professor L. Wolfenstein kindly informed us that he is aware of this result; it can alternatively be deduced on the basis of phase transformation argument following his Erice lectures (Ref. 6).

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[1]J. C. Pati, Phys. Rev. Lett. 20, 812 (1968); B. R. Holstein, Phys. Rev. 171, 1668 (1968).

[2]S. L. Glashow, Phys. Rev. Lett. 14, 35 (1964); W. Alles, Phys. Lett. 15, 348 (1965).

[3]One may introduce the spurion (S) to denote the weak Hamiltonian add allow it to carry energy and momentum for off-shell amplitudes. Thus one may define the s,t,u variables for $K+$"S"$\to\pi+\pi$.

[4]In this case, one may verify that $(\text{Im}a_2)/\text{Re}a_2=(\text{Im}a_0)/\text{Re}a_0$, where a_0 and a_2 denote the $I=0$ and $I=2$, $K^0\to 2\pi$ amplitudes defined by amplitude $[K^0\to(2\pi)_{I=n}]\equiv a_n e^{i\delta_n}$ (δ_n is the $I=n$, $\pi\pi$ s-wave phase shift at invariant mass m_K). *If* one would accept the customary double-soft-pion result $K\to 2\pi$ amplitude, then a_2 would vanish (for $\phi=-\xi$) (Ref. 1), which leads to $\eta_{+-}=\eta_{00}$. Here we are showing that even if $a_2\neq 0$, it must have the *same phase* as a_0 for $\phi=-\xi$, which guarantees $\eta_{+-}=\eta_{00}$.

[5]Note that both $S^{(-)}$ and $P^{(-)}$ contribute [in conjunction with $S^{(+)}$ and $P^{(+)}$] to the CP-violating off-diagonal element of the K^0-$\bar{K}^0$ mass matrix and therefore to ρ.

[6]This is because, in the present scheme, due to Eq. (7)

(and therefore the condition that a_0 and a_2 are relatively real; see Ref. 4), one has

$$\eta_{+-} = \eta_{00} = \epsilon \equiv \frac{\langle 2\pi, I=0|T|K_L\rangle}{\langle 2\pi, I=0|T|K_S\rangle}.$$

The phase of ϵ to a good approximation is $(2\Delta m/\Gamma_S)$ due to an argument of Wolfenstein; see, for example, his Erice Lecture note in *Theory and Phenomenology in Particle Physics*, Proceedings of the School of Physics "Ettore Majorana," 1968, edited by A. Zichichi (Academic, New York, 1969), p. 218. In the present scheme, the approximation amounts to dropping primarily the $|3\pi\rangle$-*real* intermediate state compared to the $|2\pi\rangle$-real intermediate state in the evaluation of the ratio of the off-diagonal and diagonal elements of the K_1-K_2-width matrix. In the present case, this is not expected to lead to an error of more than a few percent.

[7]L. Wolfenstein, Phys. Rev. Lett. 13, 562 (1964).

[8]If the $|\Delta S|=1$ part of the nonleptonic Hamiltonian in a theory could be expressed in the form (3), such that Eq. (5) were satisfied not only by $P^{(-)}$ and $P^{(+)}$, *but also* by $S^{(-)}$ and $S^{(+)}$ [with the substitution $(P^{(-)}, P^{(+)}) \to (S^{(-)}, S^{(+)})$], then one may show that the mixing parameter ρ would conspire with R, so that $\rho = -R$, and $\eta_{+-} = \eta_{00} = 0$. In fact, there would be *no effective CP violation* in the theory, which of course does not happen in the present case.

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Mode and Scaling in Charged Multiplicity Distributions*

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A quasinormal expansion is used to examine a possibility for scaling of charged multiplicity distributions in *pp* collisions.

It has been pointed out by several authors that the charged multiplicity cross sections σ_n in *pp* collisions[1] with incident energies 50–300 GeV are well represented by normal[2-5] or approximately normal[6,7] distributions. The Koba-Nielsen-Olesen (KNO) scaling[8]

$$P_n \equiv \frac{\sigma_n}{\sigma_{\text{inel}}} = \frac{1}{\langle n\rangle}\,\psi\!\left(\frac{n}{\langle n\rangle}\right) \tag{1}$$

also seems not inconsistent with experiment at the present energy, where ψ is approximately Gaussian.[4,6]

A mathematical basis which leads us to obtain a quasinormal distribution was discussed in Refs. 7, 9, and 10. It is an analog of the central-limit theorem and can be stated in the following way: The asymptotic expansion at the mode[11] m,

$$P_n = \frac{1}{\sqrt{2\pi}\,\beta}\exp\left[-\frac{1}{2}\left(\frac{n-m}{\gamma}\right)^2\right]\left[1+\sum_{k=3}^{\infty} a_k\left(\frac{n-m}{\gamma}\right)^k\right] \tag{2a}$$

$$= \frac{1}{\sqrt{2\pi}\,\beta}\exp\left[-\frac{1}{2}\left(\frac{n-m}{\gamma}\right)^2+\sum_{k=3}^{\infty} b_k\left(\frac{n-m}{\gamma}\right)^k\right], \tag{2b}$$

is valid provided that

$$\kappa_2 \to \infty \quad \text{as } s\to\infty \tag{3}$$

and that the condition

$$|(n-m)/\gamma^2| < \pi \tag{4}$$

is satisfied. The parameters β, m, γ, a_k, and b_k can be expressed in terms of moments, deviants,[10] or cumulants.[12,13] If correlations of the produced particles are *temperate*[7,9] in the sense that higher cumulants κ_k satisfy the condition

$$\kappa_k/\kappa_2^{k/2} = O(\epsilon^{k-2}), \quad k\ge 3 \tag{5}$$

with

$$\epsilon \ll 1, \tag{6}$$

then we have

$$a_{3l-4,3l-2,3l} = O(\epsilon^l), \quad l\ge 1 \tag{7}$$

and

$$b_k = O(\epsilon^{k-2}), \quad k\ge 3. \tag{8}$$

Therefore, only a few terms in expansion (4) are important.

If, moreover, the limits[7]

$$\lim_{s\to\infty}\frac{\beta}{m} = \lim_{s\to\infty}\frac{\sqrt{\kappa_2}}{\kappa_1-\frac{1}{2}(\kappa_3/\kappa_2)}[1+O(\epsilon^2)] = b, \tag{9}$$

$$\lim_{s\to\infty}\frac{\gamma}{m} = \lim_{s\to\infty}\frac{\sqrt{\kappa_2}}{\kappa_1-\frac{1}{2}(\kappa_3/\kappa_2)}[1+O(\epsilon^2)] = d \tag{10}$$

are nonvanishing, Eq. (2) reduces to