Scaling Property of Modified Bose Distribution for Inclusive p-p Interactions*

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Recent CERN Intersecting Storage Rings data on inclusive p-p collisions have been analyzed using a Bose-type distribution with two parameters: temperature T and scaling factor λ. The results of the analysis agree with the previous conclusion that the Feynman-Yang scaling takes place for incident $P_{\text{lab}} > 20$ GeV/c and that λ behaves as $\lambda \to 2/\gamma$, γ being the c.m. Lorentz factor. The temperature is found to be $T \simeq 0.120$ GeV.

In a previous paper,[1] the scaling property of

$$p+p \to \pi^{\pm} + \text{anything} \tag{1}$$

for incident proton momentum $P_{\text{lab}} < 30$ GeV/c has been investigated by means of a Bose-type distribution:

$$\frac{d\sigma}{dP_T^{\,2}dP_L} \sim \frac{1}{e^{\epsilon(\lambda)/T}-1} \tag{2}$$

where P_T and P_L are the c.m. transverse and longitudinal momenta, and

$$\epsilon(\lambda) = (P_T^{\,2} + \lambda^2 P_L^{\,2} + m^2)^{1/2}, \tag{3}$$

m being the mass of the secondary meson; we have set $c=1$. The two parameters of the distribution are T the temperature and λ the scaling factor. We recall that from the scaling laws[2,3] we should expect

$$\lambda P_{\max} \to \text{const}, \tag{4}$$

$P_{\max}$ being the maximum of the c.m. momentum of the secondary meson.[4]

Previous analyses of data for $P_{\text{lab}} = 12$ to 28 GeV/c indicate that the products $\lambda P_{\max}$ remain practically constant for $P_{\text{lab}} > 20$ GeV/c. In this note we investigate further this property using higher-energy data from recent CERN Intersecting Storage Rings (ISR) experiments.

We begin with the data of Albrow *et al.*[5] at two energies, $\sqrt{s} = 44.6$ and 53.0 GeV, which are equivalent to $P_{\text{lab}} = 1060$ and 1500 GeV/c, respectively. The parameters T (in GeV) and λ (dimensionless) are estimated by least-squares fits with (2) to their measurements of various momentum spectra for both secondary π's and K's. The results thus obtained are summarized in Table I.

We note that the λ's thus estimated are much less than 1, and that all these data here considered cannot be fitted with the Bose distribution which corresponds to $\lambda = 1$. This gives further justification for introducing this scaling parameter λ.[6]

As an illustration of our fits, we present in Fig. 1 the P_T distributions of π^- for $\sqrt{s} = 53.0$ GeV. The fit is performed by treating simultaneously the three sets of points with the same parameters T and λ and one normalization constant.

Consider first π secondaries. We neglect their mass and assume $P_{\max}$ equal to the incident proton momentum, writing

$$P_{\max} = \gamma M, \tag{5}$$

where γ is the Lorentz factor of the c.m. system with respect to the lab system, and M is the mass of the colliding protons. The values of $\lambda P_{\max}$ thus obtained are shown in Fig. 2 together with those of P_{lab} below[7] 30 GeV/c of the previous investigation.[1] A comparison of the values of $\lambda P_{\max}$ indicates that for $P_{\text{lab}} > 20$ GeV/c it is consistent to regard them as constant. This confirms our previous conclusion,[1] namely, the scaling for the inclusive reaction (1) holds for $P_{\text{lab}} > 20$ GeV/c.

If we take the averages of $\lambda P_{\max}$ for $P_{\text{lab}} > 20$ GeV/c, we find 1.80 ± 0.12 and 1.84 ± 0.04 GeV/c for π^+ and π^-, respectively. Since these two values are not significantly different, henceforth no distinction will be made for the charge state of π. Note that the average value of $\lambda P_{\max}$ is very close to $2M$. Therefore, we may write the Feynman-Yang scaling law for reaction (1) as

$$\lambda\gamma \to 2. \tag{6}$$

If we use the Feynman variable $x = P_L/\gamma M$, then the expression $\epsilon(\lambda)$ takes the form

TABLE I. Estimates of parameters T and λ. ISR data by Albrow *et al.* (Ref. 5).

c.m. energy (GeV)	Secondary	T (GeV)	λ
44.6	π^+	0.128 ± 0.015	0.072 ± 0.005
	K^+	0.139 ± 0.020	0.074 ± 0.008
53.0	π^+	0.114 ± 0.010	0.069 ± 0.008
	π^-	0.111 ± 0.014	0.065 ± 0.005
	K^+	0.116 ± 0.011	0.058 ± 0.006
	K^-	0.109 ± 0.014	0.063 ± 0.004

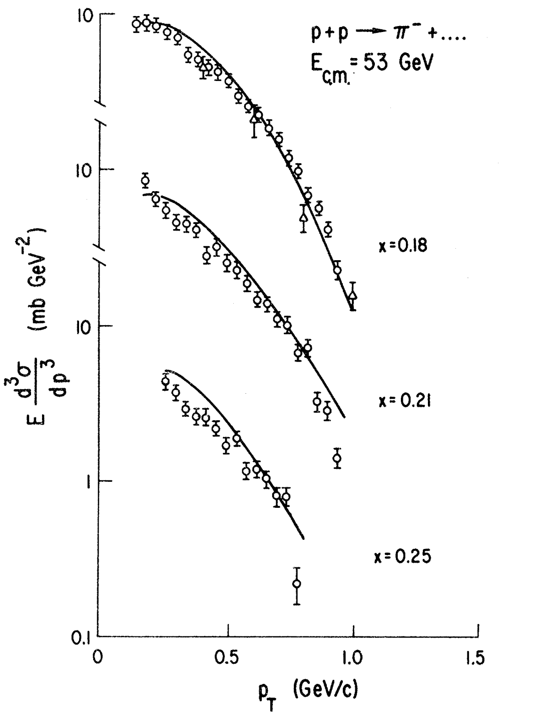

FIG. 1. P_T distribution fit with modified Bose distribution. ISR data from Albrow *et al*. (Ref. 5). The parameters T and λ, estimated by the least-squares fit, are listed in Table I.

$$\epsilon=(P_T{}^2+4x^2M^2+m^2)^{1/2}\,. \tag{7}$$

Then the cross section expressed by the modified Bose distribution (2) will be independent of the incident momentum, provided that T is a constant. Let T_c denote this temperature, which we shall estimate by means of the following data. First, the P_T distributions at $x=0$ for $\pi^\pm$ of the Saclay-Strasbourg and British-Scandinavian collaborations.[8] These experiments cover the following range: 500, 1100, and 1500 GeV/c. As their measured invariant cross sections plotted against P_T fall on the same curve, we shall fit the three energies together. We find $T_\perp=0.117\pm0.006$ GeV. Next, we consider the data from Bertin *et al.*[9] These measurements cover in addition 270 GeV/c, and are at $x=0.16$. We find $T=0.122\pm0.008$ GeV. Taking the average of these two values we get

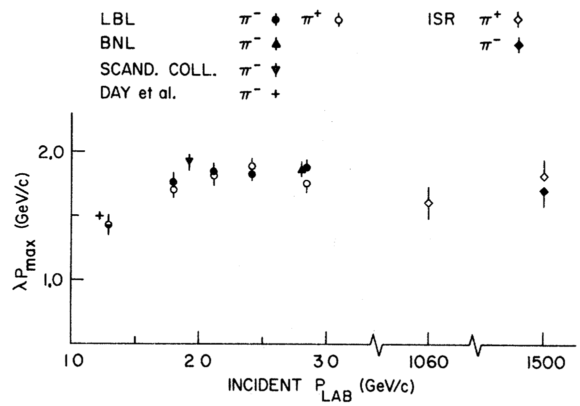

FIG. 2. Plot of $\lambda P_{\max}$ vs incident lab momentum. $P_{\max}$ is the maximum c.m. momentum of secondary π. Feynman-Yang scaling requires $\lambda P_{\max}\rightarrow$ const.

$$T_c=0.120\pm0.006\text{ GeV}\,,$$

which is consistent with the values listed in Table I. It should be mentioned that this temperature is much lower than the limit temperature postulated by Hagedorn[10] in his thermodynamical model of strong interactions.[11]

Consider next the K mesons. Referring to Table I, we notice that, within fitting errors, both parameters T and λ are comparable to those of π of the same incident energy. This property has already been discussed in the previous paper.[1]

Finally, it should be mentioned that the introduction of the scaling parameter λ in (3) to modify the Bose distribution (cf. Ref. 4) has been motivated by our attempt to account for the scaling law. It is anticipated that our parameter ought to be related to other parameters used in another similar approach, for instance the Lorentz factor γ_f of the fireball model.[12] An attempt to derive formally the relation between λ and γ_f has recently been made by Yu.[13] In his approach, Yu considers only the geometric effect due to the Lorentz contraction, while he has left aside the temperature T. As is well known, the temperature, which is a thermodynamical quantity, is not an invariant from the point of view of special relativity, but transforms as $T=T_0/\gamma_f$.[14] Consequently the neglect of its proper treatment raises questions as regards the approach presented by Yu.

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[3] J. Benecke, T. T. Chou, C. N. Yang, and E. Yen, Phys. Rev. 188, 2159 (1969).

[4] For a discussion on this property we refer to a previous paper: T. F. Hoang, Nucl. Phys. B38, 333

(1972).

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[6]The Bose distribution is often found in the literature to fit the single-particle momentum spectrum. See Morrison, in *Proceedings of the Fourth International Conference on High Energy Collisions, Oxford, 1972,* edited by J. R. Smith (Rutherford High Energy Laboratory, Chilton, Didcot, Berkshire, England, 1972), Vol. 1, p. 315.

[7]See Ref. 1. The data analyzed in that paper include the following: D. B. Smith *et al.*, Phys. Rev. Lett. 23, 1064 (1968); R. S. Panvini *et al.*, Phys. Lett. 38B, 55 (1972); H. Bøggild *et al.*, in *Proceedings of the Amsterdam International Conference on Elementary Particles,* edited by A. G. Tenner and M. Veltman (North-Holland, Amsterdam, 1971); L. Day *et al.*, Phys. Rev. Lett. 18, 1055 (1969); Phys. Rev. D 3, 645 (1971).

[8]Saclay-Strasbourg Collaboration and British-Scandinavian Collaboration, in *Proceedings of the XVI International Conference on High Energy Physics* (Ref. 5), Vol. 1, p. 308. See also Phys. Lett. 44B, 521 (1973) and 44B, 537 (1973).

[9]A. Bertin, P. Capiluppi, M. d'Agostini-Bruno, R. J. Ellis, G. Giacomelli, A. M. Rossi, G. Vannini, A. Bussière, and R. T. Poe, Phys. Lett. 42B, 493 (1972).

[10]R. Hagedorn, Nuovo Cimento Suppl. 3, 147 (1965).

[11]We have also analyzed the P_T distribution at 90° of π^0 presented by the CERN-Columbia-Rockefeller Collaboration at the Vanderbilt Conference, 1973 (unpublished). We are unable to obtain a satisfactory fit with the modified Bose distribution (2).

[12]See A. Jabs, Z. Phys. 222, 12 (1969). We thank Dr. Jabs for calling our attention to this point and sending us his other papers.

[13]Wing-Yin Yu, Phys. Rev. D (to be published).

[14]This Lorentz transformation of temperature was first discussed by A. Einstein [Jahrb. Radioakt. Elektron. 4, 411 (1907)] and M. Planck [Ann. Phys. (Leipz.) 26, 1 (1908)].

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CP Violation Through Phase Angles in Weak Currents and the Relation $\eta_{+-} = \eta_{00}$ *

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It is observed that in a theory where *CP* violation is introduced through phase angles between vector and axial-vector currents the relation $\eta_{+-} = \eta_{00}$ is exact if $\phi = -\xi$, *without* the assumption of soft pions. ϕ and ξ are the phase angles for the strangeness-preserving and strangeness-changing currents, respectively.

It had been noted[1] some time ago that in a theory in which *CP* violation is attributed to phase angles[2] between the weak-vector and axial-vector currents, the $|\Delta I| = \frac{1}{2}$ rule and hence the relation $\eta_{+-} = \eta_{00}$ follows in the *double-soft-pion limit* for the $K_{L,S} \to 2\pi$ decay amplitudes, provided $\phi = -\xi$, where ϕ and ξ are the phase angles for the strangeness-conserving and the strangeness-changing weak currents, respectively. It was also noted[1] that the choice $\phi = \pm\xi$ is necessary to preserve the familiar current-algebra applications to other (*CP-conserving*) nonleptonic decays. For $K \to 2\pi$ decay, since the soft-pion limit involves a rather large extrapolation from the physical point (of order m_K^2) in the relevant Mandelstam variables,[3] one may question the validity of the above result for real pions. The purpose of this note is to remark that *if* $\phi = -\xi$, the relation $\eta_{+-} = \eta_{00}$ *holds without the soft-pion approximation* for the $K \to 2\pi$ amplitudes, even though the $|\Delta I| = \frac{1}{2}$ rule may not.

To see this, write the nonleptonic weak Hamiltonian in the current-current form

$$H_W = \frac{G}{\sqrt{2}}\,(J_\mu J_\mu^\dagger + J_\mu^\dagger J_\mu)\,, \tag{1}$$

where

$$J_\mu = \cos\theta(V_\mu^{1+i2} + e^{i\phi}A_\mu^{1+i2}) + \sin\theta(V_\mu^{4+i5} + e^{i\xi}A_\mu^{4+i5})\,. \tag{2}$$

The $|\Delta S| = 1$ part of H_W for $\phi = -\xi$ is given by

$$H_W^1 = \frac{G}{\sqrt{2}}\cos\theta\sin\theta[S^{(+)} + S^{(-)} + P^{(+)} + P^{(-)}]\,, \tag{3}$$