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Adjustment of the Crystal-Diffraction Measurement of the Pion Mass

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The measurement of the negative-pion mass by crystal diffraction of pionic x rays is adjusted for the revised fine-structure constant, improved radiative energy-level calculations, and more precise values of the calibration lines. We find that the result quoted earlier should be reduced by 80 ppm (parts per million) to 139.566 ± 0.010 MeV (± 73 ppm).

I. INTRODUCTION

In 1965 we reported[1,2] an estimate of the negative-pion mass by measuring the Ca and Ti $4f$-$3d$ pionic x-ray energies with a 7.7-m bent crystal spectrometer.[3] Since that time there has been considerable improvement in the accuracy of the fine-structure constant and the energies of the calibration γ rays. Also, recent interest in the radiative corrections to pionic and muonic atom energy levels, stimulated by discrepancies between additional precision x-ray measurements and theory, has allowed more precise energy-level calculations for pionic transitions. We therefore present a reevaluation of the pion mass measurement.

II. ENERGY-LEVEL CALCULATIONS

The original energy-level calculations for the Ca and Ti $4f$-$3d$ transitions were carried out by this author[1,2] in 1965 using perturbation-theory corrections to the unperturbed Klein-Gordon wave equation. In recent years calculational techniques have improved considerably, namely, by numerically integrating the Klein-Gordon equation after adding the finite-nuclear-size and vacuum-polarization corrections to the Coulomb field for a point nucleus. In 1969 Fricke[4] published an extensive theoretical evaluation of radiative corrections in pionic and muonic atoms. Based on this paper, Backenstoss *et al.*[5] observed that the original perturbation theory calculations did not include a finite-size correction to the lowest-order vacuum-polarization term, and estimated the effect to cause a 200-ppm shift in the energy-level calculation. As perturbation theory indicated the effect to be approximately 5 ppm,[6] the discrepancy was quickly traced to Fricke's paper. Also, the discrepancies between muonic atom measurements and theory[7,8] lead to finding additional errors in the same reference.[9,10]

Using numerical integration techniques and Blomqvist's results,[10] Tauscher[11] has recently reevaluated the original perturbation theory calculation, and finds that the scale factors (the ratio of the pion mass to the transition energies) for the Ca and Ti $4f$-$3d$ transitions should be reduced by approximately 46 ppm (41 ppm of this is due to the 1969 reevaluation[12] of the fine structure constant). Tauscher's results, which reduce the theoretical uncertainties by about a factor of 4, are used in this reevaluation, and are presented in Table I.

III. BENT CRYSTAL SPECTROMETER CALIBRATION

The bent crystal spectrometer was calibrated with the 84-keV γ rays and the 52-keV Yb $K\alpha_1$ x rays emitted in the decay of Tm^{170}. Since the original calibration, additional measurements have been made on the energy of the 84 keV γ ray,[13-17] resulting in a more accurate value. As these measurements were referenced to the energies of either the W or Yb $K\alpha$ x rays, which depend on the 1969 evaluation of the voltage-wavelength conversion product,[12] the adjusted

TABLE I. Revised evaluation of the pion mass from measurement of the $4f$-$3d$ transitions in pionic Ca and Ti. All errors are shown in parts per million.

Quantity	πCa $4f$-$3d$	πTi $4f$-$3d$	Units
Diffraction angle [a] ($\sin\theta$)	0.072 612 0 (119 ppm)	0.059 938 8 (84 ppm)	...
Wavelength	0.171 365 (121 ppm)	0.141 456 (86 ppm)	Å*
Energy	72.350 (121 ppm)	87.648 (86 ppm)	keV
Scale factor [b]	1929.130 (14 ppm)	1592.299 (11 ppm)	...
$M_\pi c^2$	139.573 (122 ppm)	139.562 (87 ppm)	MeV

[a] Table II of Ref. 2.
[b] Reference 11.

values are shown. We include a measurement of the γ-ray energy carried out on the spectrometer used for the pion mass measurement.[17] The weighted mean is found to be 84.258± 0.003 keV, the error representing external rather than internal consistency, as the distribution of measured values is somewhat larger than the quoted internal errors would seem to indicate.

Using a wavelength of 0.147 147 Å* (±39 ppm) (where 1 Å* = 1.000 020 Å±6 ppm) for the γ ray, a wavelength of 0.236 655 Å* (±13 ppm) for the Yb $K\alpha_1$ x ray,[18] and the measured diffraction angles for these lines, we find a quartz (310) lattice spacing of d = 1.180 004 Å* (± 15 ppm) for the spectrometer calibration. In addition, each measured wavelength should include an error of $\pm 2\times10^{-6}$ Å* to allow for possible nonlinearities in the sine-screw mechanism.

IV. RESULTS

Table I shows the revised evaluation of the pion mass measurement. The error matrix for the two measurements is

$$V_{\text{tot}} = \begin{pmatrix} (122\text{ ppm})^2 & (25\text{ ppm})^2 \\ (25\text{ ppm})^2 & (87\text{ ppm})^2 \end{pmatrix},$$

where V_{11} and V_{22} represent the variance of the Ca and Ti measurements, respectively. The covariances are due entirely to spectrometer calibration and energy-level calculations.

The resulting revised estimate for the π^- mass is

$$M_\pi c^2 = 139.566 \pm 0.010 \text{ MeV } (\pm 73 \text{ ppm}),$$

in agreement with the recent revised results of Backenstoss *et al.*[19]

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where a is the rms nuclear radius and $R(r)$ is the radial wave function. Private communication, R. Shafer to G. Backenstoss (1971).
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