Multiperipheral Model for the Nonforward M^2 Absorptive Part in the Double-Regge Region

Dennis Silverman*
Department of Physics, University of California, Irvine, Irvine, California 92664

(Received 14 February 1973)

The forward absorptive part in M^2 of $a+b+\bar{c}$ elastic scattering has been related by Mueller to the inclusive single-particle spectrum for $a+b\to c+X$. The nonforward $a+b+\bar{c}$ 3-3 M^2 absorptive part in the double-Regge region also has several physical applications. The M^2 absorptive part of this amplitude is calculated in the region of double-Regge exchange using a multiperipheral model with exponential damping in momentum transfer and Regge behavior in inclusive subenergies. The absorptive part is then extended to an analytic function in M^2.

I. INTRODUCTION

Recent investigations of inclusive spectra have given importance to the three-particle to three-particle (3-3) amplitude. This is due to the optical theorem of Mueller[1] which relates the $M^2 = (p_a + p_b + p_{\bar{c}})^2$ absorptive part of the forward 3-3 amplitude for $a+b+\bar{c}\to a+b+\bar{c}$ to the single-particle inclusive cross section for $a+b\to c+X$. The *nonforward* 3-3 amplitude also plays an important role in the analysis of inclusive reactions. In order to fully study the analytic properties of the forward 3-3 absorptive part it is necessary to examine analytic continuations in two-particle subenergies and in the trajectories by using the nonforward 3-3 amplitude. This was first done by Halliday and Parry in φ^3 theory to demonstrate the factorization of the M^2 discontinuity.[2]

The nonforward multiperipheral amplitude that we calculate here is used to study the relation between Pomeranchukon coupling strengths in inclusive and exclusive experiments.[3] The six-point amplitude with double Pomeranchukon exchange was fitted to the pionization spectra in the 3-3 region to determine the double Pomeranchukon coupling strength. The nonforward six-point amplitude was then analytically continued to the forward $2\to 4$ double Pomeranchukon exchange production region and compared with experimental data to demonstrate the need for Pomeranchukon decoupling.[3]

Another application of the nonforward 3-3 absorptive part is to inclusive experiments on nuclei as studied by Bander[4] using the eikonal approach. In the usual eikonal computation of a total cross section on a nucleus from the optical theorem, one uses the amplitude $S(b)$ for nucleon-nucleon elastic scattering at an impact parameter b, which requires a full knowledge of its nonforward transform $S(q^2)$. In the eikonal treatment of the inclusive 3-3 amplitude[4] it is similarly necessary to know it at nonforward momentum transfers and energies in order to compute the scatterings with the nucleons located at various impact parameters and parallel displacements.

The 3-3 absorptive part calculated in this paper is also evaluated at the forward point to give the pionization spectrum. The method given here is a simpler and more direct way to get this pionization spectrum for the exponentially damped multiperipheral models. This result has recently been used in generalizing to arbitrary forms of damping in momentum transfer, thereby allowing fits to be obtained for pionization spectra over all $q_\perp^2$.[5]

In this paper we compute the M^2 absorptive part of the nonforward 3-3 amplitude in the double-Regge region for a simple multiperipheral model with Regge behavior and exponential damping in momentum transfer. After computing the M^2 absorptive part we then extend it to an analytic function.

The model for the absorptive part in the inclusive central plateau region consists of the production of the observed particle c with a summation over numbers of particles emitted faster or slower than c, Fig. 1. The summation over both of these sets of particles is similar to the summation in a total cross section and is assumed to produce a Regge behavior, $s_1^{\alpha_1}$, $s_2^{\alpha_2}$ as in the multiperipheral model.[6] In order to get the observed exponential falloff in $(p_\perp^c)^2$ in the central region, we include factors for exponential damping in the internal momentum transfers $e^{\Omega_l t_l}$, $e^{\Omega_r t_r}$. This model has been formulated by Caneschi and Pignotti[7] and analytically computed by Silverman and Tan[8] and others.[9]

By squaring this amplitude and integrating over the inclusive phase space, we get a form for the single-particle spectrum or equivalently the M^2 absorptive part of the forward 3-3 amplitude in the central inclusive region.[8] In this paper we use this model to compute the nonforward 3-3 M^2 absorptive part in the double-Regge region, Fig. 2. By the double-Regge region we mean that subenergies s_l, s_l', s_r', and s_r are large enough to be in the Regge region, Fig. 3. The calculations are carried out in the nonforward region adjacent to the forward central plateau region so that $p_{\bar{c}} = -p_c$ has negative energy $p_{\bar{c}}^0 < -m_c$. The absorptive part is then used to construct the analytic nonforward 3-3 amplitude in the double-Regge region.

As a special point we will present the forward 3-3 M^2 absorptive part for double Pomeranchukon exchange $\alpha_1 = 1$, $\alpha_2 = 1$, which agrees with the previously calculated single-particle spectrum in this model.[8] The method employed in this paper gives a simpler and more direct calculation, however, as well as extending the calculation to trajectories of arbitrary intercept.

In Sec. II we formulate the model and calculate part of the intermediate-state integration as the quasi-two-body phase space for producing quasiparticles of intermediate masses squared s_1 and s_2, Fig. 2. The integration over the quasiparticle masses squared s_1 and s_2 is carried out in Sec. III in the double-Regge region. The results are presented in terms of the external invariants in Sec. IV. In Sec. V, we extend the absorptive part to an analytic function for the nonforward 3-3 amplitude in the double-Regge region.

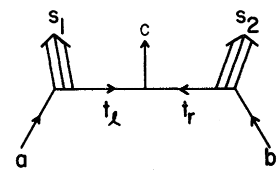

FIG. 1. Multiperipheral diagram for $a + b \rightarrow c + X$ in the central region.

II. CALCULATION OF M^2 ABSORPTIVE PART

The M^2 absorptive part for Fig. 2 can be computed by first performing the phase-space integrations over all of the particles in the sets labeled by momenta p_1 and p_2. We assume this is done (Ref. 8, Appendix) and produces Regge behavior in the momenta squared $s_1 = p_1^2$, $s_2 = p_2^2$. The remaining integrations over p_1 and p_2 can be performed by considering these sets as intermediate particles of masses squared s_1, s_2, and then doing the simple two-body phase-space integral to get a function A'. Finally, the intermediate particle masses squared s_1, s_2, are integrated over their allowed values to get the absorptive part A.

The two-body phase-space integral over the exponentially damped momentum transfers for intermediate-particle-masses squared s_1, s_2, is (Fig. 2)

$$A' = \int d^4p_1 \delta^+ (p_1^2 - s_1) \int d^4p_2 \delta^+ (p_2^2 - s_2) \delta^4(p_1 + p_2 - p_a - p_b - p_{\bar{c}}) \times \exp\{\Omega_l[(p_a' - p_1)^2 + (p_a - p_1)^2] + \Omega_r[(p_b' - p_2)^2 + (p_b - p_2)^2]\}. \quad (2.1)$$

To compute the two-body phase space we work in the c.m. frame of p_1 and p_2 where the kinematics are:

$$\begin{aligned}
&\bar{p}_1 + \bar{p}_2 = 0 = \bar{p}_a + \bar{p}_b + \bar{p}_{\bar{c}} = \bar{p}_a' + \bar{p}_b' + \bar{p}_{\bar{c}}' ,\\
&M^2 = (p_a + p_b + p_{\bar{c}})^2 = (p_1 + p_2)^2 ,\\
&p_1^0 + p_2^0 = M , \quad M \equiv \sqrt{M^2} ,\\
&P \equiv |\bar{p}_1| = |\bar{p}_2| = \frac{\Delta^{1/2}(M^2, s_1, s_2)}{2M} ,\\
&\Delta(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx ,\\
&p_1^0 = \frac{M^2 + s_1 - s_2}{2M} , \quad p_2^0 = \frac{M^2 + s_2 - s_1}{2M} .
\end{aligned} \quad (2.2)$$

The integrals give the usual two-body phase-space result of an integral over the angles of $\bar{p}_1$

$$A' = \frac{P}{4M} \int d\Omega_1 \exp\{\Omega_l[2m_a^2 + 2s_1 - 2(p_a'^0 + p_a^0)p_1^0 + 2(\bar{p}_a' + \bar{p}_a)\cdot\bar{p}_1] + \Omega_r[2m_b^2 + 2s_2 - 2(p_b'^0 + p_b^0)p_2^0 - 2(\bar{p}_b' + \bar{p}_b)\cdot\bar{p}_1]\} . \quad (2.3)$$

We define a vector which is fixed by external momenta

$$\bar{Q} = \Omega_l(\bar{p}_a' + \bar{p}_a) - \Omega_r(\bar{p}_b' + \bar{p}_b) , \quad Q \equiv |\bar{Q}| , \quad (2.4)$$

and choose the z axis along this vector. The angular integrals are then easily done. The result for the two-body phase space is

$$A' = \frac{\pi}{2MQ}\sinh(2QP)\exp\{\Omega_l[2m_a{}^2+2s_1-2(p_a'^0+p_a^0)p_1^0]+\Omega_r[2m_b{}^2+2s_2-2(p_b'^0+p_b^0)p_2^0]\}. \tag{2.5}$$

III. INTEGRATION OVER SUBENERGIES

We now integrate over the subenergies s_1, s_2 to get the nonforward M^2 absorptive part A. We assume Regge behavior for the absorptive parts of $p_a+Q_1\to p_a'+Q_1'$ and $p_b+Q_2\to p_b'+Q_2'$ and include dependence on the external momentum transfers $t_1=(p_a'-p_a)^2$, $t_2=(p_b'-p_b)^2$.

$$A=c\beta_a(t_1)\beta_b(t_2)\int ds_1 ds_2 (s_1)^{\alpha_1(t_1)}(s_2)^{\alpha_2(t_2)}A'. \tag{3.1}$$

Since the over-all factors c, β_a, β_b do not enter into the integrals we neglect them for now.

The dependence on s_1 and s_2 enters explicitly and through P, p_1^0, p_2^0 in Eq. (2.5). Defining

$$Q^0\equiv\Omega_l(p_a'^0+p_a^0)-\Omega_r(p_b'^0+p_b^0), \tag{3.2}$$

we have explicitly

$$A=\frac{\pi}{2MQ}\exp\{\Omega_l[2m_a{}^2-M(p_a'^0+p_a^0)]+\Omega_r[2m_b{}^2-M(p_b'^0+p_b^0)]\}$$
$$\times\int ds_1\int ds_2\sinh\left(\frac{Q}{M}\Delta^{1/2}(M^2,s_1,s_2)\right)s_1{}^{\alpha_1}s_2{}^{\alpha_2}\exp\left[2\Omega_l s_1+2\Omega_r s_2-\frac{Q^0}{M}(s_1-s_2)\right]. \tag{3.3}$$

The boundary is given by the condition that the c.m. momentum be positive, and that the subenergies be above threshold

$$\Delta(M^2,s_1,s_2)\geq 0,\quad s_1\geq s_1^0,\quad s_2\geq s_2^0. \tag{3.4}$$

To do the integration we substitute

$$x=\frac{1}{M}(p_1^0-P),\quad y=\frac{1}{M}(p_2^0-P) \tag{3.5}$$

which gives

$$\frac{s_1}{M^2}=x(1-y),\quad \frac{s_2}{M^2}=y(1-x),\quad ds_1ds_2=(M^2)^2(1-x-y)dxdy \tag{3.6}$$

and simplifies

$$\Delta^{1/2}(M^2,s_1,s_2)=M^2(1-x-y).$$

We then split up $\sinh[QM(1-x-y)]$ into its two exponentials. In the second exponential we substitute

$$x=1-y',\quad y=1-x',\quad 1-x-y=-(1-x'-y'). \tag{3.7}$$

Then both exponentials are identical and can be combined to give

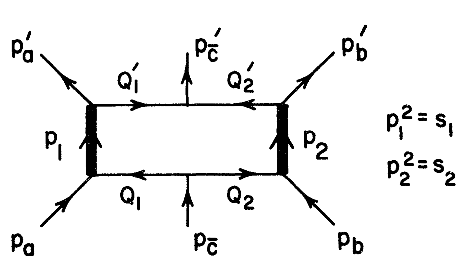

FIG. 2. Multiperipheral diagram for the M^2 absorptive part of nonforward $ab\bar{c}$ scattering.

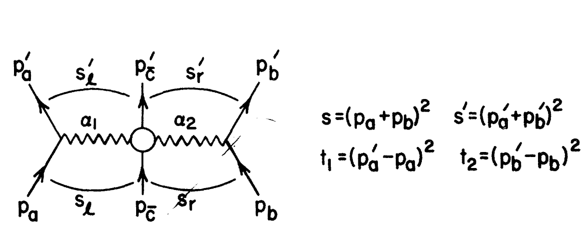

FIG. 3. External invariants for the nonforward $ab\bar{c}$ scattering.

$$A=(M^2)^{2+\alpha_1+\alpha_2}\frac{\pi}{4MQ}\exp\{\Omega_l[2m_a{}^2-M(p_a'^0+p_a^0)]+\Omega_r[2m_b{}^2-M(p_b'^0+p_b^0)]+MQ\}$$
$$\times\int_0^1 dx\int_0^1 dy(1-x-y)[x(1-y)]^{\alpha_1}[y(1-x)]^{\alpha_2}$$
$$\times\exp\{-x[M(Q+Q^0)-2\Omega_lM^2]-y[M(Q-Q^0)-2\Omega_rM^2]-2xy(\Omega_l+\Omega_r)M^2\}. \quad (3.8)$$

We have taken the limit $M^2\to\infty$ here and s_l, s_r, s_l', $s_r'\to\infty$ in order to make the thresholds reduce to the limits 0 to 1.

In Sec. IV, Eq. (4.9), we show that the coefficients of x and y in the exponential are proportional to $-|s_l+s_l'|$ and $-|s_r+s_r'|$, respectively. We evaluate the integral for very large magnitudes of these external subenergies, that is, the double-Regge region. Then x and y terms may be dropped with respect to 1 in the factors, and the limits of x and y extended to infinity. Defining

$$x'=x[M(Q+Q^0)-2\Omega_lM^2],\qquad y'=y[M(Q-Q^0)-2\Omega_rM^2], \quad (3.9)$$

the integration becomes

$$\int_0^\infty dx'\int_0^\infty dy'(x')^{\alpha_1}(y')^{\alpha_2}e^{-x'-y'-x'y'/\kappa}=\Gamma(\alpha_1+1)\Gamma(\alpha_2+1)\kappa^{\alpha_2+1}U(\alpha_2+1,\,-\alpha_1+\alpha_2+1,\,\kappa), \quad (3.10)$$

where U is the confluent hypergeometric function[10] (also called Ψ)[11] and

$$\kappa=\frac{[M(Q+Q^0)-2\Omega_lM^2][M(Q-Q^0)-2\Omega_rM^2]}{2M^2(\Omega_l+\Omega_r)}. \quad (3.11)$$

The result of the phase-space integrations in the double-Regge region is

$$A=(M^2)^{2+\alpha_1+\alpha_2}\frac{\pi}{4MQ}[M(Q+Q^0)-2\Omega_lM^2]^{-\alpha_1-1}[M(Q-Q^0)-2\Omega_rM^2]^{-\alpha_2-1}$$
$$\times\exp\{\Omega_l[2m_a{}^2-M(p_a'^0+p_a^0)]+\Omega_r[2m_b{}^2-M(p_b'^0+p_b^0)]+MQ\}\Gamma(\alpha_1+1)\Gamma(\alpha_2+1)\kappa^{\alpha_2+1}U(\alpha_2+1,\,-\alpha_1+\alpha_2+1;\,\kappa). \quad (3.12)$$

IV. KINEMATICS AND THE DOUBLE-REGGE RESULT

It remains to express the c.m. momentum components in terms of the invariants, Fig. 3,

$$s_l=(p_a+p_{\bar c})^2,\quad s_l'=(p_a'+p_{\bar c}')^2,$$
$$s_r=(p_b+p_{\bar c})^2,\quad s_r'=(p_b'+p_{\bar c}')^2, \quad (4.1)$$
$$s=(p_a+p_b)^2,\quad s'=(p_a'+p_b')^2.$$

In the nonforward $ab\bar c$ region, $p_{\bar c}=-p_c$, and $p_{\bar c}^0<-m_c$, so that the subenergies s_l, s_l', s_r, s_r' are negative. The time components in the c.m. system are found by scalar products with the vector $p_a+p_b+p_{\bar c}=(M,\vec 0)$. This gives

$$2Mp_a^0=s+s_l-m_b{}^2-m_c{}^2,$$
$$2Mp_a'^0=s'+s_l'-m_b{}^2-m_c{}^2,$$
$$2Mp_b^0=s+s_r-m_a{}^2-m_c{}^2,$$
$$2Mp_b^{0\prime}=s'+s_r'-m_a{}^2-m_c{}^2, \quad (4.2)$$
$$2MQ^0=2M^2(\Omega_l-\Omega_r)-\Omega_l(s_r+s_r')+\Omega_r(s_l+s_l')+2(\Omega_lm_a{}^2-\Omega_rm_b{}^2).$$

In the last equation we have used

$$M^2=s+s_l+s_r-m_a{}^2-m_b{}^2-m_c{}^2,$$
$$M^2=s'+s_l'+s_r'-m_a{}^2-m_b{}^2-m_c{}^2. \quad (4.3)$$

To calculate $Q\equiv|\vec Q|$, we first calculate $Q_\mu Q^\mu$, and then

$$\vec Q^2=(Q^0)^2-Q_\mu Q^\mu. \quad (4.4)$$

Using (3.2) and (2.4) along with

$$p_a'-p_a+p_b'-p_b=p_{\bar c}-p_{\bar c}',$$
$$\tau\equiv(p_{\bar c}-p_{\bar c}')^2, \quad (4.5)$$

we find

$$-Q_\mu Q^\mu=2\Omega_l\Omega_r(2M^2-s_l-s_l'-s_r-s_r')+(\Omega_lt_1+\Omega_rt_2)(\Omega_l+\Omega_r)-\Omega_l\Omega_r\tau-4\Omega_l{}^2m_a{}^2-\Omega_r{}^2m_b{}^2+4\Omega_l\Omega_rm_c{}^2. \quad (4.6)$$

Now using (4.4), (4.2), and (4.6) we compute Q in the double-Regge limit

$$M^2\to\infty,\quad |s_l+s_l'|\to\infty,\quad |s_r+s_r'|\to\infty,$$
$$t_1,\ t_2,\ \tau,\ \frac{(s_l+s_l')(s_r+s_r')}{M^2}\ \text{all fixed.} \quad (4.7)$$

The result including terms of $O(m^2)$ is

$$2MQ = 2M^2(\Omega_l + \Omega_r) - \Omega_l(s_r + s_r' - t_1 + 2m_a^2) - \Omega_r(s_l + s_l' - t_2 + 2m_b^2) - \frac{\Omega_l \Omega_r}{\Omega_l + \Omega_r}\left[\frac{(s_l + s_l')(s_r + s_r')}{M^2} + \tau - 4m_c^2\right]. \tag{4.8}$$

For the coefficients of x and y in the exponent of (3.8) and in the powers of (3.12) we have

$$[M(Q + Q^0) - 2\Omega_l M^2] = -\Omega_l(s_r + s_r') + O(m^2),$$
$$[M(Q - Q^0) - 2\Omega_r M^2] = -\Omega_r(s_l + s_l') + O(m^2). \tag{4.9}$$

For κ of Eq. (3.11) we then have

$$\kappa = \frac{2\Omega_l\Omega_r}{\Omega_l + \Omega_r}\,\frac{(s_l + s_l')(s_r + s_r')}{4M^2}. \tag{4.10}$$

Our final result from (3.12) for the 3-3 nonforward absorptive part in the double-Regge region is

$$A = c\beta_a(t_1)\beta_b(t_2)\,\frac{\pi\Gamma(\alpha_1 + 1)\Gamma(\alpha_2 + 1)}{8(\Omega_l + \Omega_r)^{\alpha_1 + \alpha_2 + 2}}[-\tfrac{1}{2}\Omega_r(s_l + s_l')]^{\alpha_1}[-\tfrac{1}{2}\Omega_l(s_r + s_r')]^{\alpha_2} \times \exp\left[-\frac{2\Omega_l\Omega_r}{\Omega_l + \Omega_r}(\tfrac{1}{4}\tau - m_c^2) + \tfrac{1}{2}(\Omega_l t_1 + \Omega_r t_2)\right]e^{-\kappa}\kappa^{-\alpha_1}U(\alpha_2 + 1,\, -\alpha_1 + \alpha_2 + 1;\, \kappa), \tag{4.11}$$

where $\alpha_1 = \alpha_1(t_1)$, $\alpha_2 = \alpha_2(t_2)$.

The result (4.11) also holds in the forward direction, where

$$p_a = p_a',\quad p_b = p_b',\quad p_{\bar c} = p_{\bar c}' = -p_c,$$
$$s_l = s_l',\quad s_r = s_r',\quad s = s', \tag{4.12}$$
$$t_1 = 0,\quad t_2 = 0,\quad \tau = 0,$$

and

$$\kappa = \frac{2\Omega_l\Omega_r}{\Omega_l + \Omega_r}\,\frac{s_l s_r}{M^2} = \frac{2\Omega_l\Omega_r}{\Omega_l + \Omega_r}[(p_\perp^c)^2 + m_c^2]. \tag{4.13}$$

For the case of double Pomeranchukon exchange $\alpha_1(0) = 1$, $\alpha_2(0) = 1$, we obtain for the single-particle spectrum or M^2 absorptive part in the central plateau region

$$A_{PP} = c\beta_a(0)\beta_b(0)\,\frac{\pi M^2}{16(\Omega_l + \Omega_r)^3} \times \exp\left(\frac{2\Omega_l\Omega_r}{\Omega_l + \Omega_r}m_c^2\right)e^{-\kappa}U(2, 1, \kappa). \tag{4.14}$$

This is the same as the earlier result[8] if we note that[3]

$$e^{-\kappa}U(2, 1, \kappa) = (1 + \kappa)E_1(\kappa) - e^{-\kappa}, \tag{4.15}$$

where $E_1(\kappa)$ is the exponential integral function.

V. EXTENSION TO ANALYTIC FUNCTION IN M^2

The absorptive part in M^2 calculated in (4.11) can be extended to an analytic function by a procedure equivalent to using a Cauchy integral or dispersion relation in M^2. Of course this does not give the entire 3-3 amplitude but only that part arising from the M^2 cut, or equivalently, from diagrams like Fig. 2.

We note that the only occurrence of M^2 in the result (4.11) is in the variable $\kappa^{(2)}$ (4.10). Keeping $(s_l + s_l')$ and $(s_r + s_r')$ fixed and negative, but very large in magnitude, the M^2 cut from threshold to infinity is mapped into a cut in κ for $0 \leq \kappa \leq \infty$. Therefore we make the analytic continuation in the variable κ. The function of κ in (4.11) can be written in terms of entire functions[10] M (also called Φ).[11]

$$\mathrm{Im} f(\kappa) \equiv e^{-\kappa}\kappa^{-\alpha_1}U(\alpha_2 + 1,\, -\alpha_1 + \alpha_2 + 1,\, \kappa) = e^{-\kappa}\frac{\pi}{\sin\pi(-\alpha_1 + \alpha_2 + 1)} \times\left[\kappa^{-\alpha_1}\frac{M(\alpha_2 + 1,\, -\alpha_1 + \alpha_2 + 1,\, \kappa)}{\Gamma(\alpha_1 + 1)\Gamma(-\alpha_1 + \alpha_2 + 1)} - \kappa^{-\alpha_2}\frac{M(\alpha_1 + 1,\, -\alpha_2 + \alpha_1 + 1,\, \kappa)}{\Gamma(\alpha_2 + 1)\Gamma(-\alpha_2 + \alpha_1 + 1)}\right]. \tag{5.1}$$

Since the M functions are entire, we can extend $\mathrm{Im}f(\kappa)$ to an analytic function with a $0 \leq \kappa < \infty$ cut by replacing $\kappa^{-\alpha_1}$ by $(-\kappa)^{-\alpha_1}/\sin\pi\alpha_1$:

$$\mathrm{Im}\left[\frac{(-\kappa - i\epsilon)^{-\alpha_1}}{\sin\pi\alpha_1}\right] = \kappa^{-\alpha_1} \tag{5.2}$$

and similarly for $\kappa^{-\alpha_2}$.

Using

$$\frac{\pi}{\sin\pi\alpha_1} = -\Gamma(1 + \alpha_1)\Gamma(-\alpha_1) \tag{5.3}$$

and the Kummer transformation[10]

$$e^{-z}M(a, b, z) = M(b - a, b, -z), \tag{5.4}$$

we have

$$f(\kappa) = \frac{-1}{\pi}\Gamma(-\alpha_1)\Gamma(-\alpha_2)\frac{\pi}{\sin\pi(-\alpha_1 + \alpha_2 + 1)} \times\left[(-\kappa)^{-\alpha_1}\frac{M(-\alpha_1,\, -\alpha_1 + \alpha_2 + 1,\, \kappa)}{\Gamma(-\alpha_2)\Gamma(-\alpha_1 + \alpha_2 + 1)} - (-\kappa)^{-\alpha_2}\frac{M(-\alpha_2,\, -\alpha_2 + \alpha_1 + 1,\, -\kappa)}{\Gamma(-\alpha_1)\Gamma(-\alpha_2 + \alpha_1 + 1)}\right]. \tag{5.5}$$

This may be recombined to give the analytic function

$$f(\kappa) = -\frac{1}{\pi}\Gamma(-\alpha_1)\Gamma(-\alpha_2)(-\kappa)^{-\alpha_1} \times U(-\alpha_1, -\alpha_1+\alpha_2+1, -\kappa). \quad (5.6)$$

The same result is obtained by performing the Cauchy integral,[12] and is unique up to an arbitrary polynomial in κ.

Including (5.6) with the remainder of (4.11) gives the analytic extension in M^2 or κ. The signature structure of the Regge exchanges α_1 and α_2 can be found by adding diagrams like Fig. 2 but with exchanges $(a \leftrightarrow \bar{a}')$, $(b \leftrightarrow \bar{b}')$, and $(a \leftrightarrow \bar{a}', b \leftrightarrow \bar{b}')$. Care must be taken to establish that the diagram of Fig. 2 and the exchanged diagrams from $(a \leftrightarrow \bar{a}')$ and $(b \leftrightarrow \bar{b}')$ are evaluated above the κ cut, but the diagram from the exchange $(a \leftrightarrow \bar{a}', b \leftrightarrow \bar{b}')$ is evaluated below the κ cut. This has been carried out in Ref. 3.

ACKNOWLEDGMENTS

We would like to express our gratitude to W. R. Frazer, C. H. Mehta, and M. Bander for helpful discussions. We also thank SLAC for its hospitality.

*Supported in part by the National Science Foundation.

[1] A. H. Mueller, Phys. Rev. D 2, 2963 (1970).
[2] I. G. Halliday and G. W. Parry, Nucl. Phys. B36, 162 (1972).
[3] C. H. Mehta and D. Silverman, Nucl. Phys. B52, 77 (1973).
[4] M. Bander, Nucl. Phys. B51, 145 (1973).
[5] R. M. Barnett and D. Silverman, Phys. Lett. 44B, 281 (1973); this issue, Phys. Rev. D 8, 2108 (1973).
[6] D. Amati, S. Fubini, and A. Stanghellini, Nuovo Cimento 26, 896 (1962).
[7] L. Caneschi and A. Pignotti, Phys. Rev. Lett. 22, 1219 (1969).
[8] D. Silverman and C.-I Tan, Nuovo Cimento 2A, 489 (1971).
[9] N. F. Bali, A. Pignotti, and D. Steele, Phys. Rev. D 3, 1167 (1971); S.-Y. Mak and C.-I Tan, *ibid.* 6, 351 (1972).
[10] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1965), pp. 504-510.
[11] *Higher Transcendental Functions* (Bateman Manuscript Project), edited by A. Erdélyi (Mc Graw-Hill, New York, 1953) Vol. I, Ch. VI.
[12] H. Buchholz, *The Confluent Hypergeometric Function* (Springer, New York, 1969), p. 126.

PHYSICAL REVIEW D VOLUME 8, NUMBER 7 1 OCTOBER 1973

A Statistical Theory of Particle Production*

D. J. Scalapino and R. L. Sugar
Department of Physics, University of California, Santa Barbara, California 93106
(Received 11 April 1973)

A statistical theory of particle production is formulated in analogy with the generalized Ginzburg-Landau theory of phase transitions in superconductors and intensity fluctuations in lasers.

I. INTRODUCTION

At present accelerator energies the number of secondaries produced in hadron-hadron collisions is large enough so that one can consider statistical theories of particle production. In this paper we shall present such a theory.[1]

Our basic approach is as follows: We do not attempt to treat in detail the fundamental dynamics underlying particle production. Instead, we imagine integrating out the microscopic degrees of freedom and representing them in terms of a small number of phenomenological parameters. We are then left with a statistical theory of the macroscopic observables.

A well-known example of this approach is the Ginzburg-Landau theory of superconductivity.[2] In its generalized form[3] it provides a statistical theory of the superconducting order parameter which depends upon a small set of experimentally determined parameters. Once a microscopic theory of superconductivity was formulated by BCS,[4] it was possible to derive the Ginzburg-Landau theory from it and to obtain expressions for the phe-