emerging leptons produced via an exchange of a single photon (based on the parton model) is given in the last paper [Eq. (35)] of Ref. 7 and may be of some use.

ACKNOWLEDGMENT

I am sincerely grateful to Professor Norman Christ for his sponsorship of this problem.

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[2]This is to acknowledge the help of Professor L. M. Lederman in carrying out a similar calculation on radiative corrections to the decay of an indefinite-metric photon (unpublished).

[3]J. Christenson *et al.*, Phys. Rev. Lett. 25, 1523 (1970).

[4]That is, $\epsilon=\epsilon_1+\cdots$ contributions from higher-order radiative corrections to the lepton-pair production cross section.

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[8]$\epsilon^{2\gamma}$ is computed in the last paper of Ref. 7.

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Some Consequences of a Modified Kuti-Weisskopf Quark-Parton Model*

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We attempt to modify the Kuti-Weisskopf quark-parton model so as to obtain agreement with recent experimental and theoretical results. We find that, at the cost of sacrificing some simplicity, reasonable phenomenological fits can be obtained.

I. INTRODUCTION

The parton model has been very useful in helping us to guess at regularities in deep-inelastic lepton-nucleon scattering. The interesting quantities which we study are the deep-inelastic structure functions which cannot be calculated without making strong dynamical assumptions about the parton distribution functions. The SLAC data[1] require that most of the charged partons have spin $\frac{1}{2}$ so a rather natural assumption is that the charged partons are quarks.

Once this choice is made, one can derive, on the basis of a few assumptions, the quark-parton momentum distributions which, in turn, allow us to predict all that can be measured about deep-inelastic lepton processes.

Kuti and Weisskopf[2] proposed such a model which, at the time, fit the available experimental data. Recent data, however, have shown their explicit model to be incorrect, since it fails to correctly predict the observed behavior on the F^{en}-F^{ep} ratio.[3] If we alter one of the quark-parton probability distributions, according to a suggestion of Friedman, and discussed briefly by Kuti and Weisskopf, agreement with some of the data is improved. We describe in this note the results of our attempts to reconcile this simple quark-parton model with all of the recent experimental and theoretical results.

II. THE QUARK-PARTON MODEL

In a simple quark-parton model, the physical nucleon is seen to be composed of three valence quarks (qqq) which contribute all of the nucleon's quantum numbers, plus a "sea" of $\bar{q}q$ pairs and neutral gluons. The natural interpretation of such an arrangement is to suppose that the valence quarks contribute only toward peripheral scattering off the nucleon, while the core contributes only to the diffractive scattering. In Regge language, this is equivalent to assuming that the valence contributions correspond to normal Regge exchanges (P', A_2, etc.) while the "sea" of $\bar{q}q$'s correspond to Pomeron exchange.

In such a model, the deep-inelastic structure functions are described by six independent functions, $u(x)$, $\bar{u}(x)$, $d(x)$, $\bar{d}(x)$, $s(x)$, and $\bar{s}(x)$, which represent, respectively, the numbers of $\mathcal{P}, \bar{\mathcal{P}}, \mathcal{N}$, $\bar{\mathcal{N}}$, λ, and $\bar{\lambda}$ quarks with momentum between xP and $P(x+dx)$ in a *proton* of momentum P.

If we separate the valence and core contributions, according to

$$\begin{aligned} u(x) &= u_v(x) + u_c(x) \\ &= u_v(x) + c(x), \\ \bar{u}(x) &= \bar{u}_c(x) = c(x), \\ d(x) &= d_v(x) + c(x), \\ \bar{d}(x) &= c(x), \\ s(x) &= \bar{s}(x) = c(x), \end{aligned}$$

where the subscript v (c) refers to the valence (core) contribution, then besides positivity, the conditions which these distribution functions must satisfy are:

$$\begin{aligned} &\int_0^1 [u(x) - \bar{u}(x)]\, dx = \int_0^1 u_v(x) dx = 2, \\ &\int_0^1 [d(x) - \bar{d}(x)]\, dx = \int_0^1 d_v(x) dx = 1, \\ &\int_0^1 [s(x) - \bar{s}(x)]\, dx = 0. \end{aligned} \tag{1}$$

In terms of these distributions, the deep-inelastic structure functions have the following simple forms:

$$\begin{aligned} W_2^{ep}(x) &= F^{ep}(x) \\ &= x[\tfrac{4}{9} u_v(x) + \tfrac{1}{9} d_v(x) + \tfrac{4}{3} c(x)], \\ W_2^{en}(x) &= F^{en}(x) \\ &= x[\tfrac{1}{9} u_v(x) + \tfrac{4}{9} d_v(x) + \tfrac{4}{3} c(x)]. \end{aligned} \tag{2}$$

Present data suggest[3] that, as $x \to 1$, the ratio F^{en}/F^{ep} approaches $\frac{1}{4}$. Saturation of this limit would imply, as seen from Eq. (2),[4] that the $\mathcal{N}$-type quark distribution function, $d_v(x)$, vanish as $x \to 1$; i.e.,

$$\lim_{x \to 1} \frac{d_v(x)}{u_v(x)} = 0. \tag{3}$$

We know of no detailed model study which explicitly satisfies this condition. Kuti and Weisskopf did, however, propose a specific quark-parton model, wherein they derived detailed distribution functions for partons (quarks) within a proton, which can easily be modified to meet this requirement. The authors themselves became aware that a modification of the quark-parton distribution functions would be necessary in order for their model to possess the correct qualitative behavior displayed in the newer data.

III. THE KUTI-WEISSKOPF MODEL

The Kuti-Weisskopf model was based upon a relatively few reasonable assumptions. For the probability distribution of the constituent partons they assumed the following: The distribution for core quarks and gluons is according to phase space

$$dP_c(x) \sim g \frac{dx}{(x^2 + \mu^2/P^2)^{1/2}}, \tag{4}$$

where μ is an "average" parton mass (assumed to be small compared to P, the proton momentum), while the probability distribution of valence quarks is determined by phase space *and* Regge asymptotics for small x:

$$dP_v(x) \sim \frac{x^{1-\alpha(0)} dx}{(x^2 + \mu^2/P^2)^{1/2}}, \tag{5}$$

where $\alpha(0)$ is the intercept of the leading (non-Pomeron) trajectory. Using these analytical forms, and certain simple assumptions, one can derive the various valence and core quark momentum distributions, $u_v(x)$, $d_v(x)$, and $c(x)$.[5] Since Kuti and Weisskopf explicitly assumed the equality of the probability distributions of both the $\mathcal{P}$- and $\mathcal{N}$-type valence quarks, the resulting deep-inelastic structure functions disagree with recent experimental results. If we modify the $\mathcal{N}$-type valence probability distribution, however, according to

$$dP_v(x) \sim \frac{(1-x) x^{1-\alpha(0)} dx}{(x^2 + \mu^2/P^2)^{1/2}} \tag{6}$$

with no change in the corresponding $\mathcal{P}$-type valence distribution, then we recover the behavior suggested by experiment, and obtain what we shall refer to as the "modified" Kuti-Weisskopf model. In the next section we shall discuss, in some detail, the results implied by this replacement.

IV. THE MODIFIED KUTI-WEISSKOPF MODEL

If we use the modified valence probability distributions as the basis of our calculations, we obtain the following quark number distributions[6]:

$$\begin{aligned} u_v(x) &= 2Z\, x^{-\alpha(0)} (1-x)^{2+2[1-\alpha(0)]} \\ &\quad \times \left(1 - \frac{1-\alpha(0)}{3+2[1-\alpha(0)]}(1-x)\right), \\ d_v(x) &\; Z\, x^{-\alpha(0)} (1-x)^{3+2[1-\alpha(0)]}, \\ c(x) &= \tfrac{1}{6} g\, x^{-1} (1-x)^{2+3[1-\alpha(0)]}, \end{aligned} \tag{7}$$

where $Z^{-1} = B(1-\alpha(0), 4+2[1-\alpha(0)])$. $B(a,b)$ is the beta function and g is an adjustable constant[7] which is determined by fitting to the data. The leading non-Pomeron trajectory is assumed to correspond to the P' or A_2, which both have $\alpha(0) \cong \frac{1}{2}$.

In the original version of the model, the choice $g=1$ gave the best agreement with the available data.[8] The newer data, however, definitely favors a smaller value of g and, hence, a smaller core contribution than previously allowed. If we choose $g=\frac{1}{2}$, we are able to obtain a reasonable fit to the data on the F^{en}-F^{ep} ratio (see Fig. 1). Figure 2 shows that, although the data on $F^{ep}(x)$ is not described extremely well in detail, the "modified" Kuti-Weisskopf model does reproduce the qualitative behavior of the data.[9]

We prefer to adopt a "realistic" point of view with regard to the quality of the fit to the data on $F^{ep}(x)$. With such a one-parameter fit, and considering that our ignorance of this subject still must be regarded as profound, one could not expect perfect, or even very close, agreement with the data. This is not to say that good agreement cannot be obtained, since many interesting possibilities are still available to the phenomenologist who is intent upon fitting all of the available data. In fact, in Sec. V we shall show the results of a phenomenological fit based on the Kuti-Weisskopf model, which led to excellent agreement with the data. Such efforts, however, we regard as merely refinements of a basic theoretical result, since they rarely lead to any deep understanding beyond the original.

After the constant g has been adjusted by fitting to the data, the "modified" Kuti-Weisskopf model makes unique predictions, some of which demonstrate surprising agreement with the data.

The recent CERN data[10] on total cross sections suggest the following sum rule.

$$\frac{1}{2}\int_0^1 [F_2^{\nu p}(x)+F_2^{\nu n}(x)]\,dx = 0.47\pm0.07\,.$$

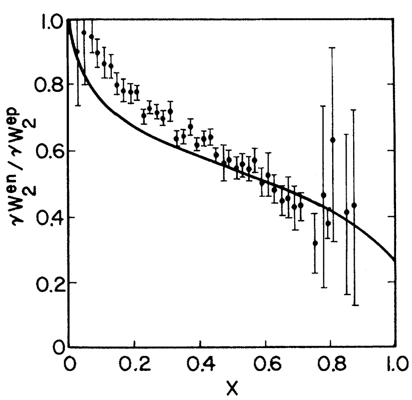

FIG. 1. Description of the deep-inelastic electron scattering structure-function ratio $\nu W_2^{en}/\nu W_2^{ep}$ by the "modified" Kuti-Weisskopf model. The curve shown is for $g=\frac{1}{2}$.

Explicitly integrating the structure functions formed from the quark number distributions [Eq. (7)], we find

$$\frac{1}{2}\int_0^1 [F_2^{\nu n}(x)+F_2^{\nu p}(x)]\,dx = 0.5005$$

demonstrating remarkable agreement with experiment.

An interesting consequence of the "modified" Kuti-Weisskopf model concerns the ratio of total cross sections, $\sigma^{\nu n}/\sigma^{\nu p}$. As a function of g, this model requires that

$$\frac{\sigma^{\nu n}}{\sigma^{\nu p}}=\frac{2.25+0.593g}{0.909+0.593g}\,,$$

where g $(0\le g\le 3)$ determines the relative core contribution to the structure functions. This gives us the following inequalities for the cross section ratio:

$$1.545\le\frac{\sigma^{\nu n}}{\sigma^{\nu p}}\le 2.48\,.$$

Our best fit, with $g=\frac{1}{2}$, requires that $\sigma^{\nu n}/\sigma^{\nu p}=2.10$. It is interesting to note that Landshoff and Polkinghorne[11] find 1.8 for this quantity in their model.

There is one quark-charge sum rule which can be derived from the expression for the structure functions, $F^{ep}(x)$ and $F^{en}(x)$:

$$I^{n,p}=\int_0^1 F_2^{n,p}(x)\frac{dx}{x}\,.$$

The sum rule takes the form:

$$I^p - I^n = \frac{1}{3}\,.$$

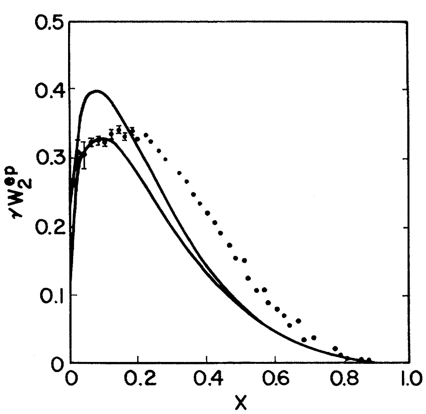

FIG. 2. Description of the deep-inelastic electron scattering structure function νW_2^{ep} by the "modified" Kuti-Weisskopf model, as a function of g. The upper curve has $g=1$, while the lower curve corresponds to $g=\frac{1}{2}$.

This value does not depend upon the choice of constants in the model but only upon the quark charges. It is difficult to extract the correct value from experiment, since the behavior of $F(x)$ at small x is important. Sakurai *et al.*,[12] extrapolated the difference $(F^{ep}-F^{en})$ toward $x=0$ (i.e., for $x \leq \frac{1}{10}$) with Regge behavior assumed and obtained an estimate of $I^p - I^n = 0.19$. This is quite far below the expected value. Quite recently, however, there has been a revision of this estimate using currently available empirical knowledge of ω $(=1/x)$ in the range 10 to 20 for *ep-en.* This places the value closer to 0.28,[13] which is more consistent with the quark-charge sum-rule result of $\frac{1}{3}$.

In view of these results, which can only be regarded as encouraging, it is important that the solution which we obtain also satisfy recent theoretical constraints as well as the experimental; specifically, the convex positivity domains of Nachtmann.[14] These constraints on the $F^{\nu n, \nu p}$ structure functions exploit the greatest information obtainable from the electroproduction data in terms of both $F^{ep}(x)$ *and* the ratio $y = F^{en}(x)/F^{ep}(x)$. The positivity conditions lead to domains in the (η, ξ) plane where

$$\eta = \frac{F_2^{\nu p}}{F_2^{ep}} \quad \text{and} \quad \xi = \frac{F_2^{\nu n}}{F_2^{ep}}$$

as a function of $y(x)$. These constraints are reflected by the following inequalities[14]:

(I) $\eta \leq \frac{18}{5}(1+y) - \xi, \quad \frac{1}{4} \leq y \leq \frac{2}{3}$

(II) $6(1-y) \leq \xi \leq \frac{18}{5}(1+y), \quad \frac{1}{4} \leq y \leq \frac{2}{3}$

(III) $\eta \leq \frac{48}{5}(y - \frac{1}{4}), \quad \frac{1}{4} \leq y \leq \frac{2}{3}$

(IV) $\eta \leq 2\xi, \quad \frac{2}{3} \leq y \leq 1$.

These inequalities are interesting in the sense that *all* of them are easily satisfied by any reasonable quark-parton model in which the quark distribution functions $[u(x),\ d(x),\ \text{etc.}]$ are explicitly defined and subject to the normal constraints [cf. Eq. (1)]. The first three inequalities are automatically satisfied due to the positivity of the valence and core quark distributions, while the fourth inequality requires that

$$d_v(x) \leq 2u_v(x).$$

Since present data suggest that $d_v(x) < \frac{1}{2}u_v(x)$, and the normalization conditions [Eq. (1)] must be satisfied, any model not satisfying this requirement could hardly expect to agree with experiment.

For the ratio $F_2^{\nu p}/F_2^{\nu n} = \eta/\xi$, we obtain a much less restrictive positivity domain since information on F_2^{ep} alone is not used. The latter domain leads to the Paschos inequalities.[15]

$$\begin{aligned} 0 \leq R(x) &= \frac{F_2^{\nu p}}{F_2^{\nu n}} \\ &\leq \frac{18}{5}\left[\frac{y(x) - \frac{1}{4}}{1 - y(x)}\right], \quad \frac{1}{4} \leq y(x) \leq \frac{2}{3} \\ &\leq 2, \quad \frac{2}{3} \leq y(x) \leq 1\,. \end{aligned}$$

Again, and not surprisingly, these inequalities are automatically satisfied due to the positivity of the valence and core quark distributions. As we expect, the quark-parton positivity conditions place no constraints upon a model unless the parton distribution functions themselves are not defined. For example, one of us[14] did a purely phenomenological analysis of the structure functions, and found the Nachtmann quark-parton positivity conditions to lead to very strict constraints for the parametrization. This analysis led to a larger ratio of cross sections than found here; namely, $\sigma^{\nu n}/\sigma^{\nu p} \gtrsim 3$.

It has been suggested by Sakurai and collaborators[12,14] that the rate of convergence of the Adler sum rule, based on the longitudinal coherence length argument, should be fairly rapid. Convergence of the Adler sum rule is most conveniently characterized by the value of ω for 90% saturation; i.e., the value ω_0 defined by

$$\frac{1}{2}\int_1^{\omega_0} [F_2^{\nu n}(x) - F_2^{\nu p}(x)]\,\frac{dx}{x} = 0.9 \quad \left(x_0 = \frac{1}{\omega_0}\right).$$

In the original version of the Kuti-Weisskopf model, 90% saturation occurred at $\omega_0 \cong 476$. This is to be contrasted to the suggested value of ω_0 for 90% saturation $\lesssim 50$ proposed by Sakurai *et al.*[12] Modifying the model improves the convergence only slightly; the ω value now required is of the order $\omega_0 \cong 416$. The arguments against slow convergence of the Adler sum rule[12] has led us to consider alternatives to Eq. (7).

V. DAUGHTER TRAJECTORIES

A simple alternative, and one with some theoretical justification, is to add a low-lying "daughter" trajectory contribution $[\alpha(0) = -\frac{1}{2}]$ in linear combination with the basic "modified" Kuti-Weisskopf functions [Eq. (7)], according to the following phenomenological prescription:

$$\begin{aligned} u_v(x) &= Z_1 x^{-1/2}(1-x)^3(1+ax)\,, \\ d_v(x) &= Z_2 x^{-1/2}(1-x)^{3+\beta}(1+bx)\,, \\ c(x) &= \tfrac{1}{6} g x^{-1}(1-x)^{7/2}\,, \end{aligned} \qquad (8)$$

where a, b, β, and g are determined by a phenomenological fit to the data.[16] Our best fit using the above four free parameters occurred with the following parameter values:

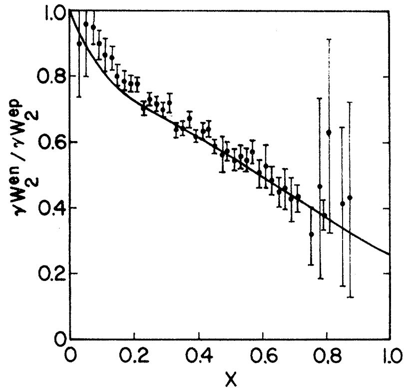

FIG. 3. Best-fit curve of the deep-inelastic structure-function ratio $\nu W_2^{en}/\nu W_2^{ep}$ by a model which includes daughter trajectories.

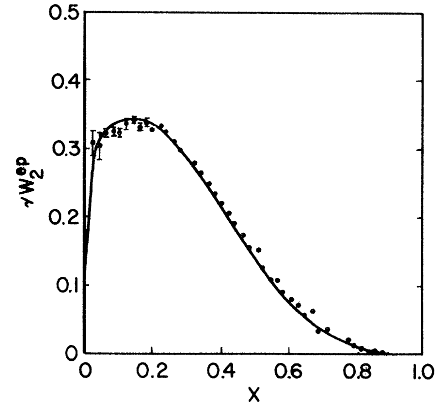

FIG. 4. Best-fit curve of the deep-inelastic structure function νW_2^{ep} by a model which includes daughter trajectories.

$$a=2.3\,,\quad b=0\,,\quad \beta=0.1\,,\quad g=0.6\,. \tag{9}$$

Figures 3 and 4 demonstrate the nature of the fit, which is quite good; a result not unexpected with so many free parameters. The solution again favors a relatively small core contribution.

The convergence of the Adler sum rule was improved only slightly; 90% convergence now occurred at $\omega_0 = 265$. While we do not regard it as being especially significant, we *were* able to obtain solutions with various values of a and b which allowed for 90% convergence of the sum rule at an ω value as low as 25, with a resulting fit better than the one-parameter fit of the modified Kuti-Weisskopf model, but, of course, not comparable to our fits as given by Eqs. (8) and (9). Our only conclusion of this fact is that rapid convergence of the Adler sum rule need not be incompatible with a reasonable form of the quark-parton model.

VI. FURTHER CONSIDERATIONS

There are an interesting set of structure function inequalities which can be derived from the quark model:

$$-xF_3^{(\nu n+\nu p)} \leqslant F_2^{(\nu n+\nu p)} \leqslant \tfrac{18}{5} F_2^{en+ep}\,. \tag{10}$$

Recent CERN data suggest that both of these inequalities are very close to saturation.[17] Saturation of these inequalities has very special consequences for the quark-parton model. The left-hand inequality is saturated when there is an absence of $\bar{\mathcal{P}}$ and $\bar{\mathcal{N}}$ quarks; saturation of the right-hand inequality requires the absence of λ and $\bar{\lambda}$ quarks. In terms of a model in which the valence and core contributions are separated, saturation of Eq. (10) would require that the entire core contribution vanish, leaving only the three valence quarks to determine the structure functions. Our result, which favors a relatively small g and thus a small-core contribution, is perfectly consistent with near saturation of these inequalities. It is interesting to note, however, that complete saturation of these inequalities would create severe problems for this sort of quark-parton model, since we must isolate the Pomeron contribution in the core [it is not possible to include Pomeron behavior in the valence contribution, due to the normalization conditions Eq. (1)].

Although the data seem to require the condition Eq. (3) to be satisfied, there is, as yet, no theoretical understanding of this point. We know, for example, that near $x=1$ all the quarks except one must have a small x, but it is not understood why it is that the $\mathcal{P}$-type quark is the last quark in the proton and not the $\mathcal{N}$-type quark.[18]

We are greatly indebted to Professor R. P. Feynman and Professor V. F. Weisskopf for helpful discussions and communications.

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[1]E. Bloom *et al.*, MIT-SLAC Report No. SLAC-PUB-796, 1970 (unpublished), presented at the Fifteenth International Conference on High Energy Physics, Kiev, U.S.S.R., 1970.

[2]J. Kuti and V. Weisskopf, Phys. Rev. D 4, 3418 (1971).

[3]A. Bodek *et al.*, Phys. Rev. Lett. 30, 1087 (1973);

G. Miller *et al.*, Phys. Rev. D 5, 528 (1972); H. Kendall *et al.*, in *Proceedings of the 1971 International Symposium on Electron and Photon Interactions at High Energies*, edited by N. B. Mistry (Laboratory of Nuclear Studies, Cornell University, Ithaca, N. Y., 1972).

[4]The core contribution, being dual to Pomeron exchange, vanishes quite rapidly as $x \to 1$.

[5]These rather complex calculations are explained in detail in Ref. 2, Appendix B.

[6]We have assumed that the $(1-x)$ behavior for $u_v(x)$ be given by $(1-x)^3$ for $\alpha(0)=\frac{1}{2}$, which was the assumption used in Ref. 2.

[7]There is only the single constraint upon g: $0 \le g \le 3$.

[8]Not only did $g=1$ seem to fit the data, but there were certain theoretical arguments which favored this value of g. (See Ref. 2.)

[9]If we express the data in terms of x', the Bloom-Gilman scaling variable, as was done in Ref. 2, the agreement with the model is improved to within 10%.

[10]D. H. Perkins, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 4, p. 189; see also A. Benvenuti *et al.*, Phys. Rev. Lett. 30, 1084 (1973).

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[12]J. J. Sakurai, H. B. Thacker, and S. F. Tuan, Nucl. Phys. B48, 353 (1972); see also the related discussion in J. D. Bjorken and S. F. Tuan, Comments Nucl. Part. Phys. 5, 71 (1972).

[13]Private communications from J. D. Bjorken and V. F. Weisskopf.

[14]See, for instance, S. F. Tuan, Phys. Rev. D 7, 2092 (1973); O. Nachtmann, *ibid.* 7, 3340 (1973); Nucl. Phys. B38, 397 (1972).

[15]E. Paschos, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 2, p. 166; see also NAL Report No. NAL-THY-87, 1972 (unpublished).

[16]The requirement of Eq. (3) can be satisfied with the general form $dP_{\mathfrak{N}}(x)=\frac{1}{2}(1-x)^{\beta}dP_{\mathcal{P}}(x)$, as long as $\beta>0$. For simplicity we have chosen $\beta=1$ for the "modified" Kuti-Weisskopf model. Simply introducing β as an additional parameter to the "modified" Kuti-Weisskopf model (i.e., $a=b=0$) was found not to significantly improve the agreement with the data.

[17]C. Callan, Comments Nucl. Part. Phys. 5, 125 (1972).

[18]Feynman points out that if the two wee valence quarks were in the $I=0$ state, this result would naturally follow. See, R. P. Feynman, in *Neutrino '72,* proceedings of the Euro-Physics Conference, Balatenfüred, Hungary, 1972, edited by A. Frenkel and G. Marx (OMKDK-TECHNOIN FORM, Budapest, 1972).

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Rapidity Charge Densities and the Leading-Particle Effect*

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The rapidity charge density in inelastic proton-proton collisions is discussed. Within a short-range-order picture, the experimental data are useful in defining the central plateau region of production processes where the properties of the initial particles are unimportant.

One simple thing that can be done with single-particle inclusive distributions is to combine the inclusive spectra $ab\to c_1$, $ab\to c_2$, etc. to form a charge density,

$$\delta Q^{ab}(s,\vec{p})=\sigma_{ab}{}^{-1}\sum_i Q_i\left[E_i\,\frac{d^3\sigma^{ab\to c_i}}{d^3p_i}(s,\vec{p})\right]. \quad (1)$$

Because charge is conserved, the integral over invariant phase space of the charge density must, of course, be given by the charge in the initial state[1]:

$$\int\frac{d^3p}{E}\,\delta Q^{ab}(s,\vec{p})=\sum_i Q_i\langle n_i(s)\rangle = Q_a+Q_b. \quad (2)$$

In Figs. 1(a)–1(d) the charge density in pp inelastic collisions for four different energies[2–4] is integrated over transverse momentum and plotted as a function of rapidity. Let

$$y=\sinh^{-1}[p_L/(m^2+p_T{}^2)^{1/2}]$$

be the target-frame rapidity and write

$$\delta Q^{ab}(y,y-Y)=\int d\vec{p}_T\delta Q(s,\vec{p}). \quad (3)$$

In Eq. (3), $Y=\cosh^{-1}(s^{1/2}/m_p)$ is the rapidity of the beam in the target rest frame.

Because the right-hand side of (2) only contains the charge of the initial particles, the charge density provides a convenient tool for the study of what has come to be called the leading-particle