PHYSICAL REVIEW D VOLUME 8, NUMBER 7 1 OCTOBER 1973

Radiative Corrections to $p + p \to l^+ + l^- +$ "Anything" and Application to Muon-Electron Symmetry*

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(Received 20 February 1973)

Radiative corrections, to order α, are carried out to the lepton lines in the process $p + p \to l^+ + l^- +$ "anything." Expressions are derived which relate the radiative corrections, due to the emission of soft photons, and of hard photons from the lepton lines, to the observed cross section for production of electron or muon pairs. These expressions are used to predict the difference, based purely on electromagnetic interactions, between the cross sections for production of electron and muon pairs. As a specific example, computations are done using the BNL data for production of lepton pairs in hadron-hadron collisions.

RADIATIVE CORRECTIONS TO THE DECAY OF A VECTOR MESON[1,2]

To perform the radiative corrections to the lepton lines, in the one-photon-exchange production of lepton pairs (Fig. 1), in the reaction[3]

$$p+p\to l^+ + l^- + \text{"anything"}, \tag{1}$$

we first carry out, for simplicity, the radiative corrections to the decay of a massive vector meson of mass Q into a lepton pair.

To the lowest order in α, the decay of the vector meson takes place, as shown in Fig. 2, with a transition probability per unit time, W_0, given by

$$W_0 \to Q\alpha/3, \tag{2}$$

where α is the fine-structure constant, and terms of order m_l^2/Q^2 have been neglected (m_l is the lepton mass). The radiative correction, to the lowest order in α, arises from the square of the diagrams 3(a) plus 3(b) and the interference between diagrams 2 and 3(c) to 3(e). As usual, the contribution from real-photon emission, Figs. 3(a) and 3(b) is divided into two regions, with photon energy $\omega < \omega_m$ (soft-photon region) and $\omega > \omega_m$ (hard-photon region). The contribution from the soft real photons is added to the virtual-photon contribution arising from diagrams 3(c), 3(d), and 3(e), and the resultant correction to W_0 is given by (again letting $m_l^2/Q^2 \to 0$)

$$d_s(l) = -\frac{\alpha}{\pi}\left[4\left(\ln\frac{Q}{m_l}\right)\ln\frac{Q}{2\omega_m} - 3\ln\frac{Q}{m_l} - 2\ln\frac{Q}{2\omega_m} + 2 - \frac{\pi^2}{3}\right]. \tag{3}$$

The contribution due to the real hard photons ($\omega > \omega_m$) is given by

$$\frac{dW_h}{dM^2} \to \frac{W_0}{Q^2}\frac{\alpha}{\pi}\frac{1+z^2}{1-z}\left(\ln\frac{M^2}{m_l^2} - 1\right), \tag{4}$$

where M is the dilepton mass, and $z = M^2/Q^2$.

APPLICATIONS TO $p+p \to l^+ + l^- +$"ANYTHING"

We now apply the expressions given in the previous section to reaction (1). If $d\sigma_0/dM^2$ be the cross section for the process shown in Fig. 1, then the correction due to the soft photons changes it to $d\sigma_s(l)/dM^2$ given by

$$\frac{d\sigma_s(l)}{dM^2} = \frac{d\sigma_0}{dM^2}[1 + d_s(l)], \tag{5}$$

where $d_s(l)$ is given by (3) with $Q \approx M$ for $\omega_m \ll M$. If ω_m is the experimental energy resolution, then $d\sigma_s(l)/dM^2$ represents the cross section (to order α^3) for production of all lepton pairs which are not accompanied by any detectable photons emitted by the leptons.

The hard-photon emission corrects $d\sigma_0/dM^2$ to $d\sigma_h(l)/dM^2$ which, from (4), is given by

$$\frac{d\sigma_h(l)}{dM^2} = \frac{\alpha}{\pi}\int_{(M+\omega_m)^2}^{Q^2\max}\frac{d\sigma_0}{dQ^2}\frac{dQ^2}{Q^2}\frac{1+z^2}{1-z}\left(\ln\frac{M^2}{m_l^2} - 1\right). \tag{6}$$

The total cross section, to order α^3, is then given by

$$\frac{d\sigma_{\text{obs}}(l)}{dM^2} = \frac{d\sigma_s(l)}{dM^2} + \frac{d\sigma_h(l)}{dM^2},$$

and it is free of ω_m since the dependence on ω_m can be shown to cancel between the two contributions. We can invert this relation and obtain an expression for $d\sigma_0/dM^2$ in terms of $d\sigma_{\text{obs}}(l)/dM^2$, that is

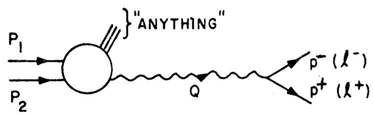

FIG. 1. The reaction $p+p \to l^+ + l^- +$ "anything" in lowest order in α. p_1, p_2 are incident proton momenta, p^+, p^- are momenta of the emerging leptons l^+ and l^-, and Q is the momentum of the massive photon exchanged.

$$\frac{d\sigma_0}{dM^2} = [1 - d_s'(l)] \frac{d\sigma_{\text{obs}}(l)}{dM^2} - \frac{\alpha}{\pi}\left(\ln\frac{M^2}{m_l^2} - 1\right) \int_{M^2}^{Q_{\max}^2} \frac{dQ^2}{Q^2 - M^2} \times \left[(1+z^2)\frac{d\sigma_{\text{obs}}(l)}{dM^2} - \frac{2d\sigma_{\text{obs}}(l)}{dM^2}\right], \tag{7}$$

where

$$d_s'(l) = -\frac{\alpha}{\pi}\left(-3\ln\frac{M}{m_l} + 2 - \frac{\pi^2}{3}\right) + \frac{2\alpha}{\pi}\left(\ln\frac{M^2}{m_l^2} - 1\right)\ln\frac{Q_{\max}^2 - M^2}{M^2}. \tag{8}$$

The total radiative corrections $d_t(l)$, defined as

$$d_t(l) = \left(\frac{d\sigma_{\text{obs}}(l)}{dM^2} - \frac{d\sigma_0}{dM^2}\right)\left(\frac{d\sigma_0}{dM^2}\right)^{-1}, \tag{9}$$

is now simply obtained from (7) in terms of $d\sigma_{\text{obs}}(l)/dM^2$. Finally, we can use (7) to get an expression for $d\sigma_{\text{obs}}(l')/dM^2$ and hence for $d_t(l')$ in terms of $d\sigma_{\text{obs}}(l)/dM^2$. In Fig. 4 we show the total radiative corrections for electron and muon pairs. To do so the BNL data[3] for muon pair production [reaction (1)] was used.

TEST OF MUON - ELECTRON SYMMETRY

The relations derived above yield simple expressions for testing muon-electron symmetry in hadronic collisions. For this purpose, we introduce asymmetry parameters ϵ_s and ϵ, as a measure of the difference, based purely on electromagnetic interactions, in the electron and muon pair production cross sections. We define ϵ_s as

$$\epsilon_s = \left(\frac{d\sigma_s(e)}{dM} - \frac{d\sigma_s(\mu)}{dM}\right)\left(\frac{d\sigma_s(e)}{dM} + \frac{d\sigma_s(\mu)}{dM}\right)^{-1}. \tag{10}$$

Using (5) and (3), we get

$$\epsilon_s \approx \frac{3\alpha}{2\pi}\ln\frac{m_\mu}{m_e}\left(1 - \tfrac{4}{3}\ln\frac{M}{2\omega_m}\right) \tag{11}$$

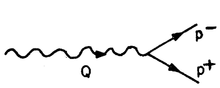

FIG. 2. Decay of a massive vector meson into a lepton pair in lowest order in α.

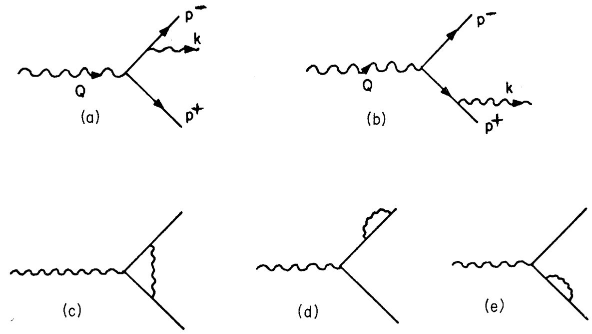

FIG. 3. (a)–(e) Radiative corrections, to order α, to the lepton lines in Figs. 1 and 2. k is the momentum of the real photon emitted by the leptons.

which is about −10% for $\omega_m/M = \frac{1}{2}\%$ and goes down to −4% for $\omega_m/M = 5\%$.

Similarly, we introduce another asymmetry parameter, ϵ, defined as

$$\epsilon = \left(\frac{d\sigma_{\text{obs}}(e)}{dM} - \frac{d\sigma_{\text{obs}}(\mu)}{dM}\right)\left(\frac{d\sigma_{\text{obs}}(e)}{dM} + \frac{d\sigma_{\text{obs}}(\mu)}{dM}\right)^{-1}. \tag{12}$$

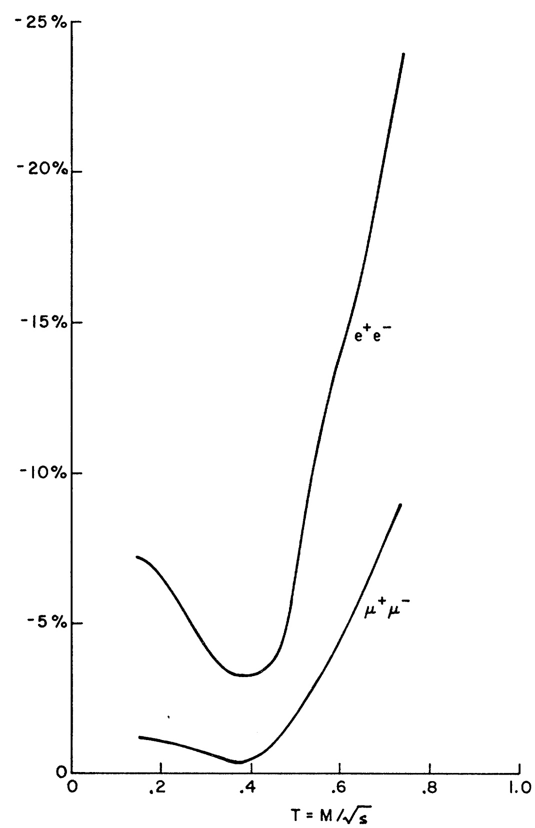

FIG. 4. Total radiative corrections, defined by Eq. (9), extracted from the BNL data of Ref. 3, are shown for production of electron and muon pairs.

On the substitution into (12) of the $O(\alpha^3)$ expressions for cross sections derived in the previous section, we get the lowest-order-in-α contribution to ϵ. This[4] contribution we term as ϵ_1, and compute it, using again the BNL data on reaction (1). Figure 5 shows a graph of ϵ_1. It is seen to vary considerably over the dilepton mass spectrum.

FURTHER REMARKS

1. Since the C-asymmetric process for production of lepton pairs, via an interference between one-photon exchange (C-odd lepton pair) and those two-photon-exchange diagrams which produce a C-even lepton pair, does not contribute to $d\sigma_{obs}/dM$, ϵ_1 is the complete contribution, to order α^3 in the cross section, to ϵ.

2. If the cross section for production of lepton pairs was assumed to scale,[5,6] that is, $d\sigma_0/dT = F(T)/s$, then from the expressions obtained in the previous sections

$$\epsilon_1 \simeq \frac{d_t(e) - d_t(\mu)}{2} = \ln \frac{m_\mu}{m_e} f(T), \qquad (13)$$

where $T = M/\sqrt{s}$, so that ϵ_1 also scales and the plot of ϵ_1 in Fig. 5 then holds for all values of $s \gg m_{\text{nucleon}}^2$.

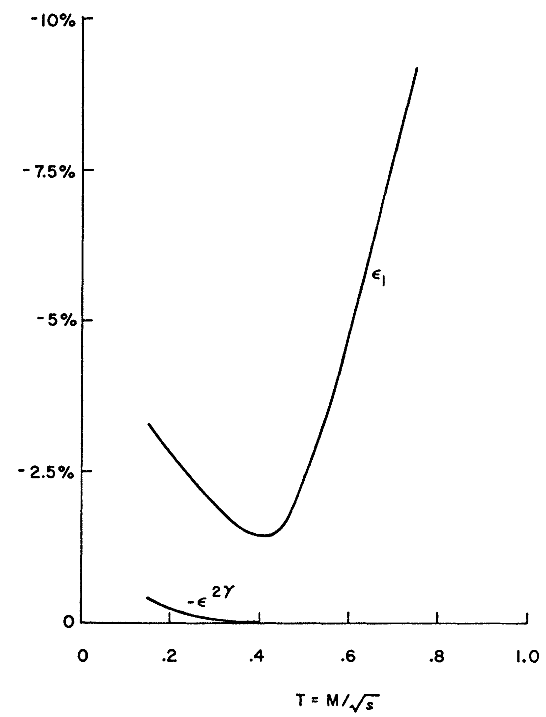

FIG. 5. ϵ_1, the contribution from $O(\alpha^3)$ cross section to the asymmetry ϵ [defined by Eq. (12)], is shown for BNL experiment of Ref. 3 taking $s = 60$ GeV^2. However, if the cross section is assumed to scale, then this curve should hold for all values of s [see Eq. (13)]. For comparison, we also show $\epsilon^{2\gamma}$ for $s = 60$ GeV^2 computed in the last paper of Ref. 7. This is the contribution to the μ-e asymmetry arising from Fig. 4 and calculated in the leading-logs approximation as $s \to \infty$ holding M^2/s fixed.

3. A notable contribution to ϵ comes from the two-photon[7] exchange diagram of Fig. 6. The cross section, as $s \to \infty$, of this $O(\alpha^4)$ process has a logarithmic dependence on the lepton mass and for $M^2/s \ll 1$ it contributes up to a few percent to ϵ. This contribution is termed[8] $\epsilon^{2\gamma}$ and is shown in Fig. 5 for $s = 60$ GeV^2 and is seen to be very small compared to ϵ_1. Unless s is very very large, the two-photon-exchange contribution to the asymmetry is never too large. For example, for $s = 3200$ GeV^2, $\epsilon^{2\gamma}$ is $\sim +3\%$ for $T \simeq 0.04$ ($M = 2$ GeV) and drops to $+\frac{1}{2}\%$ for $T \simeq 0.18$ ($M = 10$ GeV). It is interesting to note that $\epsilon^{2\gamma}$ and ϵ_1 are of opposite signs, that is, the order α^3 radiative cross section for production of muon pairs in reaction (1) is greater than that for production of electrons, but in order α^4, the two-photon-process contribution (of Fig. 6) is just the other way round. $\epsilon^{2\gamma}$ also drops sharply with increase in M or T unlike ϵ_1.

4. As stated earlier we are dealing here with the radiative corrections to the process (1) due to the emission of photons from the leptons only. Radiative corrections can, of course, also arise by emission of photons from hadrons which are not considered in this work. There are, however, situations when a knowledge of the leptonic contributions to the radiative corrections could prove useful. One such example is the problem of muon-electron symmetry where the hadronic contributions are not expected to play any role.

5. The radiative corrections evaluated here assume a uniform detecting efficiency of the leptons over a complete 4π solid angle. This, in general, is not true and one would in specific experiments have to incorporate the experimental situation in greater detail. The angular distribution of the

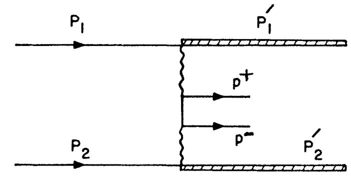

FIG. 6. Two-photon-exchange production of a C-even lepton pair in the reaction $p + p \to l^+ + l^- +$ "anything." The thick lines in the figure represent the unobserved final hadronic states of momenta p_1' and p_2'.

emerging leptons produced via an exchange of a single photon (based on the parton model) is given in the last paper [Eq. (35)] of Ref. 7 and may be of some use.

ACKNOWLEDGMENT

I am sincerely grateful to Professor Norman Christ for his sponsorship of this problem.

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*This research was supported by the U. S. Atomic Energy Commission.

[1]F. Ehlotzky and H. Mitter, Nuovo Cimento 55A, 181 (1968), and references therein.

[2]This is to acknowledge the help of Professor L. M. Lederman in carrying out a similar calculation on radiative corrections to the decay of an indefinite-metric photon (unpublished).

[3]J. Christenson *et al.*, Phys. Rev. Lett. 25, 1523 (1970).

[4]That is, $\epsilon = \epsilon_1 + \cdots$ contributions from higher-order radiative corrections to the lepton-pair production cross section.

[5]J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969).

[6]S. D. Drell and T.-M. Yan, Phys. Rev. Lett. 25, 316 (1970).

[7]K. Fujikawa, Nuovo Cimento 12A, 83 (1972), 12A, 117 (1972); V. M. Budnev *et al.*, Phys. Lett. 39B, 526 (1972); A. Soni, Phys. Rev. D 8, 880 (1973).

[8]$\epsilon^{2\gamma}$ is computed in the last paper of Ref. 7.

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Some Consequences of a Modified Kuti-Weisskopf Quark-Parton Model*

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We attempt to modify the Kuti-Weisskopf quark-parton model so as to obtain agreement with recent experimental and theoretical results. We find that, at the cost of sacrificing some simplicity, reasonable phenomenological fits can be obtained.

I. INTRODUCTION

The parton model has been very useful in helping us to guess at regularities in deep-inelastic lepton-nucleon scattering. The interesting quantities which we study are the deep-inelastic structure functions which cannot be calculated without making strong dynamical assumptions about the parton distribution functions. The SLAC data[1] require that most of the charged partons have spin $\frac{1}{2}$ so a rather natural assumption is that the charged partons are quarks.

Once this choice is made, one can derive, on the basis of a few assumptions, the quark-parton momentum distributions which, in turn, allow us to predict all that can be measured about deep-inelastic lepton processes.

Kuti and Weisskopf[2] proposed such a model which, at the time, fit the available experimental data. Recent data, however, have shown their explicit model to be incorrect, since it fails to correctly predict the observed behavior on the F^{en}-F^{ep} ratio.[3] If we alter one of the quark-parton probability distributions, according to a suggestion of Friedman, and discussed briefly by Kuti and Weisskopf, agreement with some of the data is improved. We describe in this note the results of our attempts to reconcile this simple quark-parton model with all of the recent experimental and theoretical results.

II. THE QUARK-PARTON MODEL

In a simple quark-parton model, the physical nucleon is seen to be composed of three valence quarks (qqq) which contribute all of the nucleon's quantum numbers, plus a "sea" of $\bar{q}q$ pairs and neutral gluons. The natural interpretation of such an arrangement is to suppose that the valence quarks contribute only toward peripheral scattering off the nucleon, while the core contributes only to the diffractive scattering. In Regge language, this is equivalent to assuming that the valence contributions correspond to normal Regge exchanges (P', A_2, etc.) while the "sea" of $\bar{q}q$'s correspond to Pomeron exchange.

In such a model, the deep-inelastic structure functions are described by six independent func-