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Forward-Peaked ν_e-e Scattering and the Solar-Neutrino Problem*

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The possibility that multiple ν_e-e scattering within the sun can account for the low solar-neutrino counting rates is considered. In the absence of ν_e-e scattering data, bounds are imposed from $\bar{\nu}_e$-e data, introducing a requirement for strong forward peaking. Generalized four-fermion interactions calculated to first order are found to be inadequate. Electromagnetic ν_e-e interactions with a finite neutrino magnetic moment are found to be satisfactory, but require a seemingly unphysical neutrino form factor corresponding to a mean neutrino radius of $\bar{r}_\nu > 7 \times 10^{-10}$ cm.

Considerable interest has been generated by the unexpectedly low counting rates for solar neutrinos.[1] Searches[2-4] for possible explanations have been conducted in many areas of astrophysics, chemistry, and physics. In this paper we report an investigation of the possibility that the anomaly can be explained by the multiple near-forward scattering of neutrinos from solar electrons.

Since the neutrino absorption cross section of ^{37}Cl rises rapidly with increasing neutrino energy[5] above the threshold at 0.814 MeV, the counting rate is very sensitive to the energy spectrum of the neutrinos. Consequently, a significant downward shift in the neutrino energy spectrum resulting from energy loss in multiple scattering would cause a substantial decrease in the counting rate. The preference for considering scattering on electrons rather than nucleons or other more massive particles is based on the familiar enhanced energy loss for lighter targets at corresponding momentum transfers. Although this possibility has been considered by Bahcall[3,6] no quantitative analysis has been reported.

The importance of multiple scattering is usually disregarded on the basis of $V-A$ predictions[7] for the ν_e-e cross section and the apparent success of the model for $\bar{\nu}_e$-e scattering.[8,9] The predicted ν_e-e cross section suggests a solar mean-free path on the order of 10^7 solar radii. Indeed, the expectation of a negligible neutrino interaction within the sun has provided the primary impetus for solar-neutrino experiments, since it has been hoped that this property would make available otherwise unobtainable information about the solar interior.

The chance that forward-peaked ν_e-e scattering might account for the small observed counting rates is left open by the absence of experimental data for the ν_e-e interaction[10] and the possibility of non-$V-A$ neutrino-electron interactions.

If, however, we insist that the ν_e-e cross section must behave at least qualitatively as the $\bar{\nu}_e$-e interaction (e.g., total cross sections of the same order of magnitude), then we should recognize the limited sensitivity of previous $\bar{\nu}_e$-e experiments to near-forward scattering. This experimental fact is a consequence of the minimum observable energy for recoil electrons.[8,9,11] The relationship between the recoil electron-kinetic energy T and the neutrino-scattering angle θ (in the lab. system) is given by

$$T=E-E'=\frac{2(E^2/m)\sin^2(\frac{1}{2}\theta)}{1+2(E/m)\sin^2(\frac{1}{2}\theta)}, \tag{1}$$

where E (E') is the initial (final) neutrino energy and m is the electron mass. The squared momentum transfer t is related to T by $t=-2mT$.

As an example from results of the most recent $\bar{\nu}_e$-e experiment,[9] $T_{\min}=3.6$ MeV, which the authors use for comparison with theory, corresponds to a minimum scattering angle $\theta_{\min}=14°$ for antineutrinos of a maximum energy of 10 MeV. The most-forward data are reported in the 1957 paper of Cowan and Reines[11] for which $T_{\min}=0.1$ MeV and $T_{\max}=0.5$ MeV.

Our approach is to investigate the energy loss for multiple scattering in the sun for particular forms of the ν_e-e interaction which are constrained to satisfy the experimental bound for the $\bar{\nu}_e$-e cross section over the region $T_{\min} \leq T \leq T_{\max}$. The constraint is applied through the expression

$$\Delta = N_e \eta \int_{T_{\min}}^{T_{\max}} S(T)dT \leq \Delta_{\text{expt}}\,, \qquad (2)$$

where $S(T) = \int dE' f(E') d\sigma/dT$. The expression $d\sigma/dT$ is the theoretical differential cross section for a particular form of the ν_e-e interaction. The other quantities are determined by the experimental conditions.[8,9,11] The differential spectrum of reactor antineutrinos is given by $f(E')$, the detector efficiency by η, and the number of electrons in the detector by N_e.

An estimate for the neutrino energy loss to the sun can be obtained from the expected energy loss per collision:

$$\langle T \rangle = \frac{1}{\sigma_{\text{tot}}(E)} \int T \frac{d\sigma}{d\Omega}\, d\Omega\,.$$

The energy loss per cm is then $dE/dr = -\rho_e(r)\sigma_{\text{tot}}(E)\langle T\rangle$, where $\rho_e(r)$ is the solar-electron density. If dE/dr is only weakly dependent on E in the region of interest and if we take

$$\frac{1}{R_\odot}\int_0^{R_\odot} \rho_e(r)dr \equiv \bar{\rho}_e \simeq 1.2\times 10^{25}\ \text{electrons/cm}^3,$$

then the total energy loss for a neutrino which travels from the center to the surface of the sun is given by

$$\Delta E = -R_\odot \bar{\rho}_e \int T \frac{d\sigma}{d\Omega}\, d\Omega\,. \qquad (3)$$

The solar radius $R_\odot$ is taken to be 7×10^{10} cm.

As mentioned previously, the predicted counting rate for the Davis experiment is very sensitive to the incident neutrino spectrum. The relationship between the counting rate Σ in solar neutrino units (SNU) and the energy spectrum of the incident neutrinos $\phi(E)$ is given by $\Sigma = \int dE\, \phi(E)\sigma_{\text{det}}(E)$, where $\sigma_{\text{det}}(E)$ is the neutrino-absorption cross section for the detector.[5] The predicted emission spectrum[4] $\phi_0(E)$ corresponds to a counting rate Σ_0. The shift of the spectrum toward lower energies causes a reduction in the counting rate as a consequence of both the energy dependence of $\sigma_{\text{det}}(E)$ and the decrease in the number of neutrinos with energies above the detection threshold.

An additional simplification occurs at this stage of the analysis if dE/dr is weakly dependent on neutrino energy above the threshold. In this case the energy loss is almost constant and the spectrum is shifted uniformly by the energy loss ΔE to give $\phi(E) = \phi_0(E + \Delta E)$ for E above threshold. Our calculations, using the predicted solar-neutrino spectrum $\phi_0(E)$, indicate that the counting reductions $\Sigma = \frac{1}{5}\Sigma_0$ and $\Sigma = \frac{1}{10}\Sigma_0$ correspond to energy losses of 2.8 and 3.8 MeV, respectively. For each of these shifts, only neutrinos associated with ^{8}B have energies above the detection threshold.

In order for a particular ν_e-e interaction to prove suitable we require that it satisfy Eq. (2) and provide an energy loss ΔE sufficient to substantially reduce the expected solar-neutrino counting rate. Investigation of Eqs. (2) and (3) reveals that a very large contribution must result from scattering in the region $\theta \leq \theta_{\min}$, hence a dramatic forward peak is required. Generalized four-fermion couplings such as those considered by Bardin, Bilenky, and Pontecorvo[12] and Chen[13] provide cross sections to first order which are polynomials in t. Such behavior is easily shown to be too gentle to give the necessary results. The same is true of the Weinberg form[14,15] of the ν_e-e interaction involving massive neutral vector bosons.

Electromagnetic interactions are more suitable since the exchange of a massless photon results in the familiar infinite forward peak. From charge conservation and measurements of the electron, proton, and neutron charge, the neutrino charge is known[16] to be less than $10^{-18}e$ and we will assume that it is neutral. We therefore turn our attention to the ν_e-e interaction with a finite-neutrino magnetic-dipole moment. This interaction was first considered by Bethe[17] in 1935 and has been discussed more recently by several authors.[11,18,12] The contribution of neutrino-antineutrino precession induced by a solar magnetic field on neutrinos has also been considered and discussed by Cisneros.[19] The general matrix element for the ν_e-e interaction to lowest order is given by

$$\mathfrak{M} = -\frac{e^2\kappa}{2m_\nu}\bar{u}(p_\nu')i\sigma^{\mu\phi}q_\phi u(p_\nu)\frac{1}{t}\bar{u}(p_e')\gamma_\mu u(p_e)F(t)\,, \qquad (4)$$

where κ is the neutrino-magnetic moment, m_ν is the neutrino mass, p_ν, p_e (p_ν', p_e') are the initial (final) neutrino and electron four-momenta, and $q = p_\nu - p_\nu'$. $F(t)$ is a possible neutrino form factor, and the spinors and matrices follow the conventions of Bjorken and Drell.[20]

Although a finite magnetic moment requires a neutrino of nonzero rest mass,[21,12] the kinematic dependence of Eq. (1) and the cross section on m_ν is negligible for $m_\nu \ll m$ and $m_\nu \ll E$. The resulting differential cross section has the form

$$\frac{d\sigma}{dT}(T,E) = 4\pi\left(\frac{\alpha\kappa}{2m_\nu}\right)^2 \frac{(E-T)}{ET}\,|F(t)|^2\,, \qquad (5)$$

where α is the fine-structure constant.

The assumption of an electromagnetic ν_e-e coupling also implies the validity of the experimental $\bar{\nu}_e$-e cross section bound (to first order) for the

ν_e-e cross section. This is a consequence of the *CPT* theorem.

Performing the energy-loss calculation for Eq. (5) indicates that a constant form factor $F(t)$ is not sufficient to account for the necessary energy loss. For simplicity we assume a Gaussian form factor $F(t) = e^{t/\Lambda^2}$, where Λ is related to $\bar{r}_\nu$, the mean radius of the magnetic-moment distribution of the neutrino, by $\bar{r}_\nu = 2\hbar c/\Lambda$.

We have calculated the limiting values of the parameters $\kappa/2m_\nu$ and Λ which are necessary to reduce the current best prediction of the counting rate[4] ($\Sigma_0 = 5.5$ SNU) to 1.0, 0.5, and 0.1, respectively, which may be compared with the experimental value[1] $\Sigma \leq 1$ SNU. The most stringent results are obtained for the 1957 ν_e-e data[11] and are presented in Table I. For the appropriate values of the parameters, dE/dr is found to be virtually independent of neutrino energy down to the detection threshold. The validity of the simplified forms for the energy loss and spectrum-shift calculations is thereby justified for this interaction.

The requirement for a diffuse neutrino of exceptionally large dimensions ($\bar{r}_\nu > 7\times 10^{-10}$cm) for this model provides evidence for the dramatic forward peaking necessary to obtain the required energy losses. This seemingly unphysical radius for the neutrino may be considered as evidence for the improbability that ν_e-e scattering can account for the solar-neutrino problem. On the other hand, it is important to note that this suggestion has the virtue that it may be tested directly in future ν_e-e scattering experiments.

TABLE I. Reduced solar-neutrino counting rates and ν_e-e electromagnetic scattering parameters.

Σ [a] (SNU)	ΔE (MeV)	$\left(\frac{\kappa e}{2m_\nu}\right)_{\min}$ (Bohr magnetons)	$\Lambda_{\max}$ (MeV)
1.0	3.0	2.2×10^{-4}	0.067
0.5	3.8	2.5×10^{-4}	0.067
0.1	5.6	3.0×10^{-4}	0.066

[a] Based on predicted[4] solar-neutrino emission spectrum yielding $\Sigma_0 = 5.5$ SNU.

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