# Chiral-Symmetry Breaking and the Electromagnetic Interaction in the $\eta \to 3\pi$ Decay

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We discuss the $\eta \to 3\pi$ decay within the framework of current algebra and soft-meson theorems using both the conventional electromagnetic interaction and isospin-breaking terms ("$U_3$" terms) from the same representation as the strong interaction. The interrelation between the strong Hamiltonian and the SU(2)-violating interactions is discussed. We find that the electromagnetic interaction gives a value of the decay width which is too small by three orders of magnitude. Using isospin-breaking contributions transforming as a member of the same representation of SU(3) × SU(3) as the strong Hamiltonian, the decay width is not adequately described in either the $(3,\bar{3}) + (\bar{3},3)$, $(6,\bar{6}) + (\bar{6},6)$, or (8,8) models. Consistent use of all possible soft-pion theorems demonstrates that it is incorrect to neglect certain $\sigma$ terms that have been discarded in the existing literature.

## I. INTRODUCTION

Recently attempts have been made to explain the $\eta \to 3\pi$ decay by writing the Hamiltonian density in the form[1-9]

$$\mathcal{H} = \mathcal{H}_0 + \epsilon\mathcal{H}_{SB} + e^2\mathcal{H}_{em} + \epsilon_3\mathcal{H}_3 , \qquad (1.1)$$

where $\mathcal{H}_0$ is the SU(3)×SU(3) symmetric part, $\mathcal{H}_{SB}$ is the strong-symmetry–breaking part conserving isospin and hypercharge, $\mathcal{H}_{em}$ is the effective electromagnetic Hamiltonian for this process, i.e.,

$$\mathcal{H}_{em} = \frac{-i}{2\sqrt{3}} \int d^4x\, D^{\mu\nu}(x) \times T\big( V^3_\mu(x) V^8_\nu(0) + V^8_\mu(x) V^3_\nu(0)\big), \qquad (1.2)$$

$V^i_\mu(x)$ being the vector currents whose charges generate SU(3), and $\mathcal{H}_3$ is a "strong" SU(2)-breaking Hamiltonian transforming as the third component of some octet.

$G$-parity considerations require that the $\eta \to 3\pi$ decay proceeds via the SU(2)-violating part of the Hamiltonian, namely

$$\mathcal{H}' = e^2\mathcal{H}_{em} + \epsilon_3\mathcal{H}_3 , \qquad (1.3)$$

but in order to apply current-algebra techniques we need the transformation properties of the axial divergences, and these are determined mainly by $\mathcal{H}_{SB}$.

There are two three-pion decay modes for the $\eta$ meson, $\pi^0\pi^0\pi^0$ and $\pi^+\pi^-\pi^0$, and experiment shows that the matrix element $T_{+-0}$ for the charged mode may be written as

$$T_{+-0} = \alpha + \beta E_0 , \qquad (1.4)$$

where $E_0$ is the energy of the neutral pion.

Sutherland[1] showed that if Eq. (1.4) is valid off-shell and only the conventional electromagnetic interaction $\mathcal{H}_{em}$ [Eq. (1.2)] is used, then the amplitude vanishes.

Subsequently it was suggested[3,4] that this problem could be resolved, still using the conventional $\mathcal{H}_{em}$, by making a more general off-shell expansion, namely

$$T_{+-0} = A + BE_0 + C(q_+{}^2 + q_-{}^2) + Dq_0{}^2 \qquad (1.5)$$

(where $q^\mu_+$, $q^\mu_-$, and $q^\mu_0$ are the four-momenta of the pions), but, as has been more recently shown by Bell and Sutherland,[2] although the amplitude no longer vanishes, it is still far too small to fit the experimental data. It is important to note that in order to obtain a result with the larger expansion in Eq. (1.5) one has to use the transformation properties of $\mathcal{H}_{SB}$ under the chiral group. All these authors[2-4] have effectively used the $(3,\bar{3}) + (\bar{3},3)$ model of Gell-Mann, Oakes, and Renner[10] (GMOR).

Consequently, in order to overcome the problem of the small width caused by $\mathcal{H}_{em}$, the $\mathcal{H}_3$ term was introduced in Eq. (1.1).[5-9] Assuming that $\mathcal{H}_3$ is $u_3$, the third component of the scalar octet in the $(3,\bar{3}) + (\bar{3},3)$ representation, using either Eq. (1.4) or Eq. (1.5) to make off-shell extrapolations of the amplitude, and neglecting certain $\sigma$ terms, a good slope [$\beta/\alpha$ in Eq. (1.4)] is obtained and the decay width is calculated as a function of $\epsilon_3$. If $\epsilon_3$ is deduced from the $K^+$-$K^0$ and $\pi^+$-$\pi^0$ mass differences, the width is too small. But if one uses the value of $\epsilon_3$ obtained by Oakes[9] from a Cabibbo rotation on an SU(2)×SU(2)-symmetric Hamiltonian, a higher value is obtained for the width. A similar conclusion has been drawn if $\mathcal{H}_{SB}$ and $\mathcal{H}_3$ transform as the appropriate components of the (8, 8) (Ref. 11) representation.

In this paper we reinvestigate the $\eta \to 3\pi$ problem and attempt to clarify the interrelation between the strong Hamiltonian ($\mathcal{H}_{SB}$) and the SU(2)-violating interactions ($\mathcal{H}_{em}$ and $\mathcal{H}_3$). In Sec. III we use the conventional electromagnetic Hamiltonian [Eq. (1.2)] but consider other transformation properties for $\mathcal{H}_{SB}$, which we have summarized in Sec. II. We find that in both the (8, 8) (Ref. 12) and $(6,\overline{6})+(\overline{6},6)$ (Ref. 13) models the decay width is far too small, just as in the $(3,\overline{3})+(\overline{3},3)$ model. In Sec. IV possible contributions from an $\mathcal{H}_3$ term are considered. Instead of neglecting $\sigma$ terms arbitrarily, we use all the possible soft-meson theorems and work consistently to lowest orders in the symmetry-breaking parameters. For the case of $(3,\overline{3})+(\overline{3},3)$ we find that (i) the expansion in Eq. (1.4) for the off-shell amplitude gives a vanishing matrix element; (ii) using Eq. (1.5), arbitrarily discarding $\sigma$ terms, also constrains the amplitude to zero; and (iii) the decay width is still far too small. For $\mathcal{H}_{SB}$ and $\mathcal{H}_3$ from the (8, 8) and $(6,\overline{6})+(\overline{6},6)$ models the expansion in Eq. (1.4) is possible, and we also estimate the decay rate in these cases.

We conclude in Sec. V with a discussion of the various results and the assumptions involved.

## II. OFF-SHELL EXTRAPOLATIONS AND CHIRAL-SYMMETRY BREAKING

To first order in the effective electromagnetic Hamiltonian we write

$$\langle \pi\pi\pi | S-1 | \eta\rangle = -i\int d^4x \langle \pi\pi\pi | \mathcal{H}'(x) | \eta\rangle$$
$$= i(2\pi)^4 \delta^4(\textstyle\sum p) T\,. \qquad (2.1)$$

Thus

$$T = -\langle \pi\pi\pi | \mathcal{H}'(0) | \eta\rangle\,, \qquad (2.2)$$

and in order to make an off-shell extrapolation in this amplitude we use the LSZ (Lehmann-Symanzik-Zimmermann) reduction formalism and PCAC (partially conserved axial-vector current). Hence

$$T_{+-0}(q_+^2, E_+; q_-^2, E_-; q_0^2, E_0) = \frac{-(i)^3(m_\pi^2 - q_+^2)}{\sqrt{2} f_\pi m_\pi^2}\frac{(m_\pi^2 - q_-^2)}{\sqrt{2} f_\pi m_\pi^2}\frac{(m_\pi^2 - q_0^2)}{f_\pi m_\pi^2}$$
$$\times \int\int\int d^4x_1\, d^4x_2\, d^4x_3\, e^{i(q_+x_1 + q_-x_2 + q_0x_3)} \langle 0 | T(\partial_\mu A_+^\mu(x_1)\partial_\nu A_-^\nu(x_2)\partial_\lambda A_3^\lambda(x_3)\mathcal{H}'(0)) | \eta\rangle\,, \qquad (2.3)$$

where $f_\pi$ is given by

$$\langle 0 | \partial_\mu A_i^\mu(0) | \pi_j\rangle = \delta_{ij} f_\pi m_\pi^2 \qquad (2.4)$$

and

$$\partial_\mu A_\pm^\mu = \partial_\mu A_1^\mu \pm i\partial_\mu A_2^\mu\,. \qquad (2.5)$$

Now if we take one of the pions, $\pi_0$ for instance, to zero four-momentum, we get

$$T_{+-0}(q_+^2, E_+; q_-^2, E_-; 0, 0) = \frac{i}{f_\pi}\frac{i(m_\pi^2 - q_+^2)}{\sqrt{2} f_\pi m_\pi^2}\frac{i(m_\pi^2 - q_-^2)}{\sqrt{2} f_\pi m_\pi^2}$$
$$\times \int\int d^4x_1\, d^4x_2\, e^{i(q_+x_1 + q_-x_2)}\{\langle 0 | T([Q_3^A, \partial_\mu A_+^\mu(x_1)]\partial_\nu A_-^\nu(x_2)\mathcal{H}'(0)) | \eta\rangle$$
$$+\langle 0 | T(\partial_\mu A_+^\mu(x_1)[Q_3^A, \partial_\nu A_-^\nu(x_2)]\mathcal{H}'(0)) | \eta\rangle$$
$$+\langle 0 | T(\partial_\mu A_+^\mu(x_1)\partial_\nu A_-^\nu(x_2)[Q_3^A, \mathcal{H}'(0)]) | \eta\rangle\}\,, \qquad (2.6)$$

but if we keep a particle on shell its corresponding axial divergence does not appear inside the time-ordered product in Eq. (2.3), so that if, for example, both $\pi^+$ and $\pi^-$ are on shell, one obtains

$$T_{+-0}(m_\pi^2, E_+; m_\pi^2, E_-; 0, 0) = \frac{i}{f_\pi}\langle \pi^+\pi^- | [Q_3^A, \mathcal{H}'(0)] | \eta\rangle\,. \qquad (2.7)$$

Now, if we take two pions to zero four-momentum, we obtain to order $\epsilon e^2$ (or $\epsilon\epsilon_3$)

$$T_{+-0}(0,0;0,0;q_0^2,E_0)=\frac{1}{(\sqrt{2}f_\pi)^2}\frac{i(m_\pi^2-q_0^2)}{f_\pi m_\pi^2}$$
$$\times\int d^4x_3\,e^{iq_0x_3}\langle 0|T(\tfrac{1}{2}\{[Q_+^A,[Q_-^A,\partial_\mu A_3^\mu(x_3)]]\mathcal{H}'(0)+\partial_\mu A_3^\mu(x_3)[Q_+^A,[Q_-^A,\mathcal{H}'(0)]]$$
$$+[Q_+^A,\partial_\mu A_3^\mu(x_3)][Q_-^A,\mathcal{H}'(0)]+[Q_-^A,\partial_\mu A_3^\mu(x_3)][Q_+^A,\mathcal{H}'(0)]$$
$$+(+\longleftrightarrow -)\})|\eta\rangle\,, \tag{2.8}$$

in the case when $\pi^+$ and $\pi^-$ have been taken soft. This involves discarding terms such as $T([Q_+^A,\partial_\mu A_-^\mu]\partial_\nu A_3^\nu\mathcal{H}')$, which is of higher order than the terms retained in Eq. (2.8).

If $\mathcal{H}'$ is the conventional electromagnetic Hamiltonian, $\mathcal{H}_{\text{em}}$, given in Eq. (1.2), then

$$[Q_3^A,\mathcal{H}']=[Q_8^A,\mathcal{H}']=0\,, \tag{2.9}$$

and the amplitude in Eq. (2.7) vanishes. We also observe in Eq. (2.6) that if we are considering off-shell pions we require knowledge of the $\sigma$ terms, $[Q_i^A,\partial_\mu A_j^\mu]$, in order to compare the amplitude at different off-mass-shell points. These $\sigma$ terms are determined by the transformation properties of the strong-symmetry–breaking Hamiltonian, since

$$\partial_\mu A_i^\mu=-i[Q_i^A,\epsilon\mathcal{H}_{\text{SB}}] \tag{2.10}$$

neglecting the SU(2)-breaking contribution. The models we wish to consider have the following transformation properties:

*(a)* $(3,\bar{3})+(\bar{3},3)$ (Ref. 10)

$$\mathcal{H}_{\text{SB}}=u_0+cu_8 \tag{2.11}$$

and

$$[Q_i^V,u_j]=if_{ijk}u_k\,, \tag{2.12}$$

$$[Q_i^V,v_j]=if_{ijk}v_k\,, \tag{2.13}$$

$$[Q_i^A,u_j]=-id_{ijk}v_k\,, \tag{2.14}$$

$$[Q_i^A,v_j]=id_{ijk}u_k\,, \tag{2.15}$$

where $i=1,\ldots,8$ and $j,k=0,1,\ldots,8$. The coefficient $c$ is determined from the pseudoscalar meson masses to be $-1.26$.

*(b)* $(8,8)$ (Ref. 12)

$$\mathcal{H}_{\text{SB}}=\tfrac{1}{8}S_{kk}-\frac{c'}{\sqrt{3}}d_{8lm}S_{lm} \tag{2.16}$$

and

$$[Q_i^V,S_{jk}]=if_{ijp}S_{pk}+if_{ikp}S_{jp}\,, \tag{2.17}$$

$$[Q_i^A,S_{jk}]=if_{ijp}S_{pk}-if_{ikp}S_{jp}\,, \tag{2.18}$$

where $i,j,k,p=1,\ldots,8$. In this model $c'$ is found to be 0.67.

*(c)* $(6,\bar{6})+(\bar{6},6)$ (Ref. 13)

$$\mathcal{H}_{\text{SB}}=\tfrac{1}{8}T_{kk}^+-\frac{\bar{c}}{\sqrt{3}}d_{8lm}T_{lm}^+ \tag{2.19}$$

and

$$[Q_i^V,T_{jk}^\pm]=if_{ijp}T_{pk}^\pm+if_{ikp}T_{jp}^\pm\,, \tag{2.20}$$

$$[Q_i^A,T_{jk}^\pm]=-(d_{ijp}T_{pk}^\mp+d_{ikp}T_{jp}^\mp+\delta_{ij}d_{klm}T_{lm}^\mp+\delta_{ik}d_{jlm}T_{lm}^\mp)\,, \tag{2.21}$$

where

$$T_{ij}^\pm=T_{ji}^\pm\,, \tag{2.22}$$

$i,j,k,p,l$, and $m=1,\ldots,8$, and $T^+$ and $T^-$ have positive and negative parity, respectively. In this case we obtain $\bar{c}=1.59$.

The isospin components of the $\sigma$ terms (i.e., $[Q_i^A,\partial_\mu A_j^\mu]$ for $i,j=1,2,3$) are isospin singlets in the $(3,\bar{3})+(\bar{3},3)$ model, while in $(8,8)$ and $(6,\bar{6})+(\bar{6},6)$ we have $I=0$ and $I=2$ contributions.

We can now use the off-shell extrapolations for the amplitude, developed earlier in this section, together with the transformation properties of $\mathcal{H}'$ and $\mathcal{H}_{\text{SB}}$ to obtain conditions on the coefficients in expansions such as (1.4) and (1.5). We apply these considerations to both the charged and the neutral decay modes. In general the neutral decay amplitude $T_{000}$ is related to $T_{+-0}$ by

$$T_{000}=T_{+-0}+T_{-0+}+T_{0+-}\,. \tag{2.23}$$

This follows from Bose statistics and the assumption that the final state is $I=1$.

## III. CALCULATION WITH $\mathcal{H}_{\text{em}}$

As mentioned in Sec. I, it has been shown[1] that assuming a linear off-shell form for the amplitude, namely Eq. (1.4),

$$T_{+-0}=\alpha+\beta E_0\,,$$

leads to a vanishing amplitude. This conclusion is independent of the form of the strong symmetry breaking, and is deduced as follows. Equations (1.4) and (2.23) give

$$T_{000}=3\alpha+\beta(E_1+E_2+E_3)\,, \tag{3.1}$$

and energy-momentum conservation leads to

$$T_{000}=3\alpha+\beta m_\eta \tag{3.2}$$

in the rest frame of the $\eta$. As pointed out in Sec. II, the right-hand side of Eq. (2.7) vanishes when $\mathcal{H}'$ is $e^2\mathcal{H}_{em}$. Thus Eqs. (1.4) and (3.2) give

$$\alpha = \beta = 0 . \tag{3.3}$$

More generally, however, assuming the larger expansion[3,4] (1.5) for $T_{+-0}$ together with the appropriate expansion for $T_{000}$ [which is given by Eq. (2.23)]

$$T_{000} = 3A + B(E_1 + E_2 + E_3) + (2C + D)(q_1{}^2 + q_2{}^2 + q_3{}^2) \tag{3.4}$$

to obtain a solution is more complicated and in the $(3, \bar{3}) + (\bar{3}, 3)$ model proceeds in three distinct steps:

*Step 1*. In both the amplitudes $T_{+-0}$ and $T_{000}$, let $q_0^\mu \to 0$ and keep the other two pions on the mass shell. Again, Eq. (2.7) shows that both amplitudes vanish, requiring

$$A = -2m_\pi{}^2 C , \tag{3.5}$$

$$B = \frac{2m_\pi{}^2}{m_\eta}(C - D) . \tag{3.6}$$

These two equations are independent of $\mathcal{H}_{SB}$.

*Step 2*. Since in the $(3, \bar{3}) + (\bar{3}, 3)$ model we have

$$[Q_3^A, \partial_\mu A_\pm^\mu] = 0 , \tag{3.7}$$

$T_{+-0}$ (but not $T_{000}$) will vanish if we let $q_0^\mu \to 0$ and allow $\pi^+$ to be off the mass shell, keeping $\pi^-$ on-shell. In this case we have

$$A + C(q_+{}^2 + m_\pi{}^2) = 0 \tag{3.8}$$

for all values of $q_+{}^2$, and hence

$$A = C = 0 . \tag{3.9}$$

At this stage the slope is determined, for the on-shell amplitude may be written, using Eqs. (3.5), (3.6), and (3.9), in the form

$$T_{+-0}(m_\pi{}^2, E_+; m_\pi{}^2, E_-; m_\pi{}^2, E_0) = Dm_\pi{}^2(1 - 2E_0/m_\eta) , \tag{3.10}$$

which agrees very well with the experimental value of the slope.

*Step 3*. For $T_{000}$, taking two of the pions to zero momentum (i.e., $q_1^\mu \to 0$ and $q_2^\mu \to 0$) so that the third goes off shell to $p_\eta^\mu$ (assuming energy-momentum conservation) gives

$$\begin{aligned} T_{000}(0, 0; 0, 0; m_\eta{}^2, m_\eta) &= 3A + Bm_\eta + (2C + D)m_\eta{}^2 \\ &= -\left(\frac{-i}{f_\pi}\right)^2 \frac{i(m_\pi{}^2 - q_3{}^2)}{f_\pi m_\pi{}^2} \int d^4x_3\, e^{iq_3x_3} \langle 0 | T([Q_3^A, [Q_3^A, \partial_\mu A_3^\mu(x_3)]] e^2\mathcal{H}_{em}(0)) | \eta \rangle , \end{aligned} \tag{3.11}$$

where again we have made use of Eq. (2.9). Since in the $(3, \bar{3}) + (\bar{3}, 3)$ model

$$[Q_3^A, [Q_3^A, \partial_\mu A_3^\mu]] = \partial_\mu A_3^\mu , \tag{3.12}$$

we obtain[14]

$$D(m_\eta{}^2 - 2m_\pi{}^2) = \frac{1}{f_\pi{}^2}\langle \pi^0(p_\eta) | e^2\mathcal{H}_{em}(0) | \eta \rangle . \tag{3.13}$$

The right-hand side of Eq. (3.13) can be related to the $K^+$-$K^0$ and $\pi^+$-$\pi^0$ mass differences, and following Bell and Sutherland (and taking into account footnote 14) we get

$$Dm_\pi{}^2 = -3 \times 10^{-2} . \tag{3.14}$$

The decay widths (with the phase-space integrals evaluated relativistically) are given by[15]

$$\Gamma_{+-0} = 489 |x|^2 (1 + 0.02y + 0.02y^2) \text{ eV} , \tag{3.15}$$

$$\Gamma_{000} = 827 |x|^2 \text{ eV} , \tag{3.16}$$

with

$$x = 3\alpha + \beta m_\eta , \tag{3.17}$$

$$y = \frac{m_\eta - 3m_\pi}{m_\eta + 3(\alpha/\beta)} , \tag{3.18}$$

where $\alpha$ and $\beta$ are the on-shell values of the parameters in Eq. (1.4), so that for the larger expansion (1.5)

$$\alpha = A + (2C + D)m_\pi{}^2 , \tag{3.19}$$

$$\beta = B . \tag{3.20}$$

Alternatively, in the literature, frequent use is made of the formula

$$\Gamma_{+-0} = \frac{|\alpha|^2 (m_\eta - 3m_\pi)^2}{3456\sqrt{3}\,\pi^2 m_\eta} , \tag{3.21}$$

when the slope $\beta/\alpha$ is equal to $-2/m_\eta$ and the phase-space integrals are evaluated nonrelativistically. Equations (3.15) and (3.21) are compatible to within 20%. Thus, in this case the decay width can be written

$$\begin{aligned} \Gamma(\eta \to \pi^+\pi^-\pi^0) &= |Dm_\pi{}^2|^2 \frac{(m_\eta - 3m_\pi)^2}{3456\sqrt{3}\,\pi^2 m_\eta} \\ &= 6 \times 10^{-1} \text{ eV}, \end{aligned} \tag{3.22}$$

a value which is three orders of magnitude smaller than the experimental value

$$\Gamma_{exp}(\eta \to \pi^+\pi^-\pi^0) \approx 600 \text{ eV} . \tag{3.23}$$

In models other than $(3,\bar{3})+(\bar{3},3)$ the situation is less straightforward. Step 1 remains unchanged, so we retain Eqs. (3.5) and (3.6). Step 2, however, breaks down if $\mathcal{H}_{SB}$ transforms as the (8, 8) or $(6,\bar{6})+(\bar{6},6)$ representation, since $[Q_3^A, \partial_\mu A_\pm^\mu]\neq 0$ in these models.

But the result of step 2 can be obtained, in general, if one assumes that the expansion in Eq. (3.4) is valid when the $\eta$ meson is taken off the mass shell and that PCAC holds for this particle. In this case we have the following.

*Step 2′.* In $T_{000}$, take all the mesons to zero momentum. Then

$$A \propto \langle 0|[Q_8^A, [Q_3^A, [Q_3^A, [Q_3^A, \mathcal{H}_{em}(0)]]]]|0\rangle = 0\,, \tag{3.24}$$

and we see from Eq. (3.5) that Eq. (3.9) is independent of the transformation properties of $\mathcal{H}_{SB}$; thus Eq. (3.10) still holds, and in all these models we have the correct slope. Note that since step 2′ gives the same answer as step 2 for the $(3,\bar{3})+(\bar{3},3)$ model, the assumptions involved in step 2′ are quite reasonable.

It may appear that in general step 3 also breaks down because in models other than the $(3,\bar{3})+(\bar{3},3)$ model $[Q_3^A,[Q_3^A,\partial_\mu A_3^\mu]]\neq c\partial_\mu A_3^\mu$; for example in the (8, 8) model we have extra pieces transforming as the 10 and $\overline{10}$ representations under SU(3), while in $(6,\bar{6})+(\bar{6},6)$ these are singlet and 27 parts. Thus, at this stage we cannot relate this amplitude to $\langle\pi^0(p_\eta)|\mathcal{H}_{em}(0)|\eta\rangle$. We can overcome this difficulty, however, in the following way.

*Step 3′.* For the amplitude $T_{+-0}$ let the charged pions go soft. Using Eq. (2.8), we obtain

$$\begin{aligned}T_{+-0}(0,0;0,0;m_\eta^2,m_\eta) &= A+Bm_\eta+Dm_\eta^2\\ &= -\left(\frac{-i}{\sqrt{2}f_\pi}\right)^2\frac{i(m_\pi^2-q_0^2)}{f_\pi m_\pi^2}\int d^4x_3\, e^{iq_0x_3}\tfrac{1}{2}\{\langle 0|T([Q_+^A,[Q_-^A,\partial_\mu A_3^\mu(x_3)]]e^2\mathcal{H}_{em}(0))|\eta\rangle\\ &\quad+\langle 0|T(\partial_\mu A_3^\mu(x_3)[Q_+^A,[Q_-^A,e^2\mathcal{H}_{em}(0)]])|\eta\rangle\\ &\quad+(+\longleftrightarrow -)\}+\sigma\text{ terms}\,.\end{aligned} \tag{3.25}$$

Although the $\sigma$ terms appearing here are of the same order in the symmetry-breaking parameters as the terms we are considering, we discard them because we cannot calculate them explicitly. Thus, the calculation which follows can be considered as an order-of-magnitude estimate of the decay width, rather than a strict evaluation.

Remembering the current-current form of $\mathcal{H}_{em}$ [see Eq. (1.2)] and using the current-algebra relations gives us immediately

$$\tfrac{1}{2}[Q_+^A,[Q_-^A,\mathcal{H}_{em}]]+\tfrac{1}{2}[Q_-^A,[Q_+^A,\mathcal{H}_{em}]]=2\mathcal{H}_{em}\,. \tag{3.26}$$

The terms $[Q_+^A,[Q_-^A,\partial_\mu A_3^\mu]]$ are calculated, as is $[Q_3^A,[Q_3^A,\partial_\mu A_3^\mu]]$ in step 3, by using Eqs. (2.10) and (2.16)–(2.22). Then, comparison of steps 3 and 3′ enables us to eliminate the unwanted representations and obtain a linear combination of $T_{+-0}(0,0;0,0;m_\eta^2,m_\eta)$ and $T_{000}(0,0;0,0;m_\eta^2,m_\eta)$ proportional to $\langle\pi_0(p_\eta)|\mathcal{H}_{em}(0)|\eta\rangle$. Thus, in this way we can still calculate the parameter $D$ as was done before. The results are

$$\begin{aligned}D_{(8,8)} &= 2D_{(3,\bar{3})+(\bar{3},3)}\\ &= -6\times10^{-2}m_\pi^{-2}\,,\end{aligned} \tag{3.27}$$

$$\begin{aligned}D_{(6,\bar{6})+(\bar{6},6)} &= D_{(3,\bar{3})+(\bar{3},3)}\\ &= -3\times10^{-2}m_\pi^{-2}\,,\end{aligned} \tag{3.28}$$

where the notation is self-evident.

Step 3′ gives no further information for the case of $(3,\bar{3})+(\bar{3},3)$, as might have been expected, but rather yields the trivial identity $D=D$. Thus, we summarize the results of this section: With $\mathcal{H}'=e^2\mathcal{H}_{em}$, as given in Eq. (1.2), the slope of the amplitude is independent of the form of the strong-symmetry-breaking $\mathcal{H}_{SB}$; the decay width for $\eta\to3\pi$ is far too small for models where $\mathcal{H}_{SB}$ transforms as (8, 8) and $(6,\bar{6})+(\bar{6},6)$, as well as for $(3,\bar{3})+(\bar{3},3)$.

## IV. CALCULATION WITH $\mathcal{H}_3$

We have shown in Sec. III, that the usual techniques of current algebra and PCAC imply that the $\eta\to3\pi$ decay cannot be completely described by the conventional electromagnetic interaction. For this reason, it has been suggested that a further isospin-breaking interaction should be considered, and this has been described by the term $\mathcal{H}_3$ in Eq. (1.1). The simplest choice for $\mathcal{H}_3$, and the one we adopt in the following, is to consider it to be just the third component of the same scalar octet as that which appears in the strong interaction $\mathcal{H}_{SB}$. Thus, in the $(3,\bar{3})+(\bar{3},3)$ model, which we wish to consider first, the full Hamiltonian is just

$$\mathcal{H}(x)=\mathcal{H}_0(x)+\epsilon[u_0(x)+cu_8(x)]+\epsilon_3u_3(x)\,, \tag{4.1}$$

where the $u_i$ were defined in Sec. II.

In the $(3,\bar{3})+(\bar{3},3)$ model one has

$$[Q_\pm^A,u_3]=0\,, \tag{4.2}$$

$$[Q_\pm^A,\partial_\mu A_3^\mu]=0\,, \tag{4.3}$$

as can be seen from Eqs. (2.10)–(2.15).

Using the off-shell extrapolation Eq. (2.3) for the amplitude $T_{+-0}$, and Eqs. (4.2) and (4.3), we find that keeping the negative pion on shell and taking the positive pion to zero four-momentum makes the amplitude vanish for all off-shell values of the four-momentum of the neutral pion. In this case energy-momentum conservation gives

$$2m_\eta E_0 = m_\eta^2 - m_\pi^2 + q_0^2, \tag{4.4}$$

and so we obtain, using Eq. (1.5),

$$\begin{aligned} T_{+-0}(0, 0; m_\pi^2, E_-; q_0^2, E_0) &= A + BE_0 + Cm_\pi^2 + D(2m_\eta E_0 + m_\pi^2 - m_\eta^2) \\ &= 0. \end{aligned} \tag{4.5}$$

Since the above equation is true for all values of $E_0$ this leads to[16,17]

$$A + Cm_\pi^2 + D(m_\pi^2 - m_\eta^2) = 0, \tag{4.6}$$

$$B + 2m_\eta D = 0. \tag{4.7}$$

Using these relations, the on-shell amplitude may be written

$$\begin{aligned} T_{+-0}(m_\pi^2, E_+; m_\pi^2, E_-; m_\pi^2, E_0) &= (A + 2Cm_\pi^2 + Dm_\pi^2) \\ &\quad \times \left(1 - \frac{2(A + Cm_\pi^2 + Dm_\pi^2)}{m_\eta(A + 2Cm_\pi^2 + Dm_\pi^2)} E_0\right). \end{aligned} \tag{4.8}$$

At this stage we would like to point out that the off-shell expansion of the amplitude given in Eq. (1.4) gives a vanishing amplitude, for we see from Eqs. (4.6) and (4.7) that $C = D = 0$ also implies $A = B = 0$.

Next, consider the charged amplitude when both charged pions are soft ($q_+^\mu \to 0$; $q_-^\mu \to 0$). It is immediately clear from Eqs. (2.8), (4.2), and (4.3) that the amplitude vanishes. Thus,

$$\begin{aligned} T_{+-0}(0, 0; 0, 0; m_\eta^2, m_\eta) &= A + Bm_\eta + Dm_\eta^2 \\ &= 0. \end{aligned} \tag{4.9}$$

Solving Eqs. (4.6), (4.7), and (4.9) gives

$$A = -\tfrac{1}{2} Bm_\eta = -Cm_\eta^2 = Dm_\eta^2. \tag{4.10}$$

Substituting these values in Eq. (4.8) gives

$$\begin{aligned} T_{+-0}(m_\pi^2, E_+; m_\pi^2, E_-; m_\pi^2, E_0) = A\left(1 - \frac{m_\pi^2}{m_\eta^2}\right)\left[1 - \left(1 - \frac{m_\pi^2}{m_\eta^2}\right)^{-1} \frac{2E_0}{m_\eta}\right], \end{aligned} \tag{4.11}$$

yielding the correct value for the slope. To determine the value of $A$ and obtain the decay width, we proceed as in Sec. III, step 2′, and take all the mesons to zero momentum, which gives in this case[18]

$$\begin{aligned} A &= \frac{\sqrt{2}\,\epsilon_3}{9 f_\pi^4}\langle 0|u_0|0\rangle \\ &= \frac{(m_{K^+}^2 - m_{K^0}^2)_{u_3}}{3\sqrt{3} f_\pi^2}, \end{aligned} \tag{4.12}$$

where the quantity $(m_{K^+}^2 - m_{K^0}^2)_{u_3}$ is the contribution of $\epsilon_3 u_3$ to the $K^+ - K^0$ mass difference (which can be obtained by use of $f_\pi^2 m_i^2 \delta_{ij} = \langle 0|[Q_i^A, [Q_j^A, \mathcal{H}]]|0\rangle$).

To evaluate the right-hand side of Eq. (4.12) we can use the Dashen sum rule[19]

$$(m_{K^+}^2 - m_{K^0}^2)_{\text{em}} = (m_{\pi^+}^2 - m_{\pi^0}^2)_{\text{em}} \tag{4.13}$$

(where em denotes the contribution from the usual electromagnetic interaction) together with

$$(m_{\pi^+}^2 - m_{\pi^0}^2)_{u_3} = 0 \tag{4.14}$$

(to first order in $\epsilon_3$) and the experimental masses to get

$$(m_{K^+}^2 - m_{K^0}^2)_{u_3} = -0.0053 \text{ GeV}^2. \tag{4.15}$$

Alternatively, if we follow Oakes[9] and assume that the $u_3$ term arises from a Cabibbo rotation of an SU(2)×SU(2)-symmetric Hamiltonian, we obtain

$$\begin{aligned} (m_{K^+}^2 - m_{K^0}^2)_{u_3} &= -m_\pi^2 \\ &= -0.019 \text{ GeV}^2. \end{aligned} \tag{4.16}$$

This result is independent of the strong-symmetry–breaking transformation properties.[20] Using the expression for the decay width given in Eq. (3.15), together with Eqs. (4.11), (4.12), and (4.15), gives

$$\Gamma_{+-0} = 4.8 \text{ eV}, \tag{4.17}$$

whereas using Eq. (4.16) instead of Eq. (4.15) yields

$$\Gamma_{+-0} = 62 \text{ eV}; \tag{4.18}$$

which is again far too small ($\Gamma_{+-0}^{\exp} \approx 600$ eV).

Before discussing the (8, 8) and $(6, \overline{6}) + (\overline{6}, 6)$ models, we would like to exhibit explicitly why it is not allowed to neglect $\sigma$ terms arbitrarily. Consider the charged amplitude when $\pi^+$ and $\pi^0$ are soft (so that $q_-^2 = m_\eta^2$). One obtains

$$\begin{aligned} T_{+-0}(0, 0; m_\eta^2, m_\eta; 0, 0) &= \frac{\epsilon_3}{f_\pi^2}\langle \pi^0(p)|u_3|\eta(p)\rangle \\ &\quad + \sigma \text{ terms} \\ &= 0, \end{aligned} \tag{4.19}$$

where the $\sigma$ terms are of the same order of magnitude in the symmetry-breaking parameters as the term $(\epsilon_3/f_\pi^2)\langle\pi^0|u_3|\eta\rangle$, which is proportional to $A$ (since $A \sim (\epsilon_3/f_\pi^4)\langle 0|u_0|0\rangle \sim (\epsilon_3/f_\pi^2\langle\pi^0|u_3|\eta\rangle)$. Therefore neglect of $\sigma$ terms in Eq. (4.19) would cause $A$ to vanish, and consequently the whole amplitude would also vanish.

Now we would like to consider a "strong" SU(2)-breaking Hamiltonian transforming as the (8, 8) or $(6,\bar{6})+(\bar{6},6)$ representations of SU(3)×SU(3). In these cases, we choose $\mathcal{H}_3$ to be the third component of the same octet as $\mathcal{H}_{SB}$, so that we have for the (8, 8) model

$$\mathcal{H}_3 = d_{3ij} S_{ij} , \qquad (4.20)$$

and

$$\mathcal{H}_3 = d_{3ij} T^+_{ij} \qquad (4.21)$$

for the case of $(6,\bar{6})+(\bar{6},6)$, where $S_{ij}$ and $T^+_{ij}$ were defined in Sec. II. Equations (4.2) and (4.3) are not true for these models, and so we do not expect to obtain Eqs. (4.6) and (4.7) in general. However, in the (8, 8) model, taking $\pi^+$ soft in $T_{+-0}$ and keeping the other particles on the mass shell, we obtain

$$T_{+-0}(0, 0; m_\pi{}^2, E_-; m_\pi{}^2, E_0) \propto \epsilon_3 \langle \pi^- \pi^0 | [Q^A_+, d_{3ij} S_{ij}] | \eta \rangle . \qquad (4.22)$$

Since the final $\pi^-\pi^0$ state is in the $S$ wave,[21] Bose symmetry forces this state to be $I=2$. However, $[Q^A_+, d_{3ij}S_{ij}]$ has only 8, 10, and $\overline{10}$ SU(3) components which do not contain $I=2$ parts. Therefore the amplitude in Eq. (4.22) vanishes. This is the only current-algebra zero which we can obtain without arbitrary neglect of $\sigma$ terms. Thus, we may use the expression given in Eq. (1.4) for the off-shell expansion of the amplitude. Then the relation $T_{+-0}(0, 0; m_\pi{}^2, E_-; m_\pi{}^2, E_0) = 0$ leads to

$$\alpha + \tfrac{1}{2}\beta m_\eta = 0 , \qquad (4.23)$$

where we have used energy-momentum conservation to obtain $E_0 = \frac{1}{2} m_\eta$. Therefore, in the (8, 8) model we also get the correct slope. However, in the $(6,\bar{6})+(\bar{6},6)$ model, the amplitude in Eq. (4.22) does not vanish from symmetry considerations, since $[Q^A_+, d_{3ij} T^+_{ij}]$ has a 27 SU(3) component with an $I=2$ part. Thus, in the $(6,\bar{6})+(\bar{6},6)$ model, we are unable to predict the slope. To estimate the value of the parameter $\alpha$, we again take all particles soft; this gives

$$\alpha = \frac{(m_{K^+}{}^2 - m_{K^0}{}^2)_{u_3}}{3\sqrt{3} f_\pi{}^2} \qquad (4.24)$$

for the (8, 8) case, while

$$\alpha = \frac{31(m_{K^+}{}^2 - m_{K^0}{}^2)_{u_3}}{21\sqrt{3} f_\pi{}^2} \qquad (4.25)$$

in the $(6,\bar{6})+(\bar{6},6)$ model. Hence, we are able to calculate the decay width in the (8, 8) model, but for the $(6,\bar{6})+(\bar{6},6)$ case we have to assume the correct slope. Using Eq. (3.15), the decay widths are found to be

$$\Gamma_{+-0} = 4.8 \text{ or } 62 \text{ eV} \qquad (4.26)$$

in (8, 8) [which is the same result as in $(3,\bar{3})+(\bar{3},3)$], and

$$\Gamma_{+-0} = 94 \text{ or } 1200 \text{ eV} \qquad (4.27)$$

in $(6,\bar{6})+(\bar{6},6)$, where, as before, the two values quoted come from using Eqs. (4.15) and (4.16), respectively. The $(6,\bar{6})+(\bar{6},6)$ result seems to be better than the results of the two other models, but is still not in good agreement with the experimental value of 600 eV.

## V. DISCUSSION

We have tried to explain the $\eta \to 3\pi$ decay, using both a conventional electromagnetic interaction and "strong" isospin-breaking terms within the framework of standard current-algebra techniques, such as PCAC and soft-meson theorems. Since the strong interaction plays an important role in defining the divergences of the axial-vector currents, we have considered three different possible representations for the Hamiltonian, namely $(3,\bar{3})+(\bar{3},3)$, $(6,\bar{6})+(\bar{6},6)$, and (8, 8).

In order to proceed, it is necessary to make further assumptions. These vary from model to model, and in each case we assume a minimum set in order to obtain predictions. In particular, we always have to choose an expansion for the off-mass-shell amplitude. Experiment shows that the decay is well described by an amplitude which depends linearly on the energy of the neutral pion, Eq.(1.4). In each case we attempt to use this expression for the off-mass-shell extrapolation. When this leads to a vanishing amplitude, we use the next simplest assumption which includes linear dependence on the squares of the four momenta of the pions, Eq.(1.5). With energy-momentum conservation, and the $\eta$-meson on the mass shell, this is the most general amplitude to this order. In taking the $\eta$ meson soft, it is, in principle, possible to include an extra $\eta$-momentum-dependent ($p^\mu_\eta$-dependent) term. Generally, we are unable to predict the decay width by assuming $p_\eta{}^2$ dependence; however, in the $(3,\bar{3})+(\bar{3},3)$ model with the conventional electromagnetic interaction, it can be seen that the $\eta$ dependence does not appear. Thus, we do not consider this term.

Our results can be summarized in Table I. When using $\mathcal{H}_{em}$, we are forced to use the enlarged expansion (1.5) in order to obtain a nonzero decay rate. For a strong Hamiltonian transforming in the $(6,\bar{6})+(\bar{6}+6)$ and (8, 8) representations, our results for the decay widths are only an order-of-magnitude estimation, since we have arbitrarily neglected $\sigma$ terms. The decay width is too small

TABLE I. Calculated and observed decay width for $\eta \to 3\pi$ decay.

| | $\Gamma(\eta \to \pi^+\pi^-\pi^0)$ (eV) | | |
|---|---|---|---|
| | | "$U_3$" term | |
| $\mathcal{H}_{SB}$ | $\mathcal{H}_{em}$ | a | b |
| $(3,\bar{3})+(\bar{3},3)$ | 0.6 | 4.8 | 62 |
| $(6,\bar{6})+(\bar{6},6)$ | 0.6 | 94 | 1200 |
| (8, 8) | 2.4 | 4.8 | 62 |
| Expt. decay width | | ~600 | |

[a] $(m_{K^+}{}^2 - m_{K^0}{}^2)_{u_3} = -0.0053$ GeV$^2$, determined by fitting pseudoscalar meson masses.
[b] $(m_{K^+}{}^2 - m_{K^0}{}^2)_{u_3} = -0.019$ GeV$^2$, determined by Oakes's rotation.

by three orders of magnitude, and thus the *conventional* electromagnetic interaction cannot describe this decay with any of the accepted modes of strong symmetry breaking.

Considering "strong" isospin breaking in the $(3,\bar{3})+(\bar{3},3)$ model, we find that it is still necessary to use an enlarged off-shell amplitude. Using all possible soft-meson theorems, it is shown that arbitrarily neglecting $\sigma$ terms leads to a vanishing amplitude. When the isospin-breaking term transforms as a member of either the $(6,\bar{6})+(\bar{6},6)$ or the (8, 8) representation, the off-shell extrapolation may be described by the smaller expansion, Eq.(1.4). All these models give small decay widths, although the $(6,\bar{6})+(\bar{6},6)$ model is somewhat larger than the others. The hypothesis due to Oakes that the strong interaction is in some sense given by a Cabibbo rotation of a Hamiltonian describing zero-mass pions allows one to obtain decay widths larger by a factor of 13.

In all the models, apart from that with "strong" isospin breaking transforming as $(6,\bar{6})+(\bar{6},6)$, the slope is predicted to be $-2/m_\eta$, in excellent agreement with experiment. In our calculation with the $(6,\bar{6})+(\bar{6},6)$ model, we have insufficient information to calculate the slope, and so in this case, to predict the width, the slope must be taken as an input.

These results tend to suggest that within the usual framework of current algebra the "strong" isospin-breaking term is a more acceptable candidate to describe $\eta\to3\pi$ decay process than is the usual current-current electromagnetic interaction, but the results are still far from satisfactory. However, the values of the decay width could clearly be improved for the "$U_3$" term by slight variations in some of our assumptions, such as PCAC for the $\eta$ meson,[15] or a larger expansion for the amplitude, but then, although the experimental value could be fitted, no definite prediction could be made.

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[18]Here we must be careful to write the neutral amplitude as $3A + B(E_1 + E_2 + E_3) + (2C + D)(q_1{}^2 + q_2{}^2 + q_3{}^2)$, so that in the soft limit the amplitude is $3A$ and not $3A + Bm_\eta = A$ [using Eq. (4.10)], as can be checked by considering the appropriate soft limit for the charged amplitude. Thus, we get a value of $A$ which is three times smaller than that obtained by other authors (Refs. 5, 9, 16).

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# Forward-Peaked $\nu_e$-$e$ Scattering and the Solar-Neutrino Problem*

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The possibility that multiple $\nu_e$-$e$ scattering within the sun can account for the low solar-neutrino counting rates is considered. In the absence of $\nu_e$-$e$ scattering data, bounds are imposed from $\bar{\nu}_e$-$e$ data, introducing a requirement for strong forward peaking. Generalized four-fermion interactions calculated to first order are found to be inadequate. Electromagnetic $\nu_e$-$e$ interactions with a finite neutrino magnetic moment are found to be satisfactory, but require a seemingly unphysical neutrino form factor corresponding to a mean neutrino radius of $\bar{r}_\nu > 7\times 10^{-10}$ cm.

Considerable interest has been generated by the unexpectedly low counting rates for solar neutrinos.[1] Searches[2-4] for possible explanations have been conducted in many areas of astrophysics, chemistry, and physics. In this paper we report an investigation of the possibility that the anomaly can be explained by the multiple near-forward scattering of neutrinos from solar electrons.

Since the neutrino absorption cross section of $^{37}$Cl rises rapidly with increasing neutrino energy[5] above the threshold at 0.814 MeV, the counting rate is very sensitive to the energy spectrum of the neutrinos. Consequently, a significant downward shift in the neutrino energy spectrum resulting from energy loss in multiple scattering would cause a substantial decrease in the counting rate. The preference for considering scattering on electrons rather than nucleons or other more massive particles is based on the familiar enhanced energy loss for lighter targets at corresponding momentum transfers. Although this possibility has been considered by Bahcall[3,6] no quantitative analysis has been reported.

The importance of multiple scattering is usually disregarded on the basis of $V-A$ predictions[7] for the $\nu_e$-$e$ cross section and the apparent success of the model for $\bar{\nu}_e$-$e$ scattering.[8,9] The predicted $\nu_e$-$e$ cross section suggests a solar mean-free path on the order of $10^7$ solar radii. Indeed, the expectation of a negligible neutrino interaction within the sun has provided the primary impetus for solar-neutrino experiments, since it has been hoped that this property would make available otherwise unobtainable information about the solar interior.

The chance that forward-peaked $\nu_e$-$e$ scattering might account for the small observed counting rates is left open by the absence of experimental data for the $\nu_e$-$e$ interaction[10] and the possibility of non-$V-A$ neutrino-electron interactions.

If, however, we insist that the $\nu_e$-$e$ cross section must behave at least qualitatively as the $\bar{\nu}_e$-$e$ interaction (e.g., total cross sections of the same order of magnitude), then we should recognize the limited sensitivity of previous $\bar{\nu}_e$-$e$ experiments to near-forward scattering. This experimental fact is a consequence of the minimum observable energy for recoil electrons.[8,9,11] The relationship between the recoil electron-kinetic energy $T$ and the neutrino-scattering angle $\theta$ (in the lab. system) is given by

$$T=E-E'=\frac{2(E^2/m)\sin^2(\frac{1}{2}\theta)}{1+2(E/m)\sin^2(\frac{1}{2}\theta)}, \tag{1}$$

where $E$ ($E'$) is the initial (final) neutrino energy and $m$ is the electron mass. The squared momentum transfer $t$ is related to $T$ by $t=-2mT$.

As an example from results of the most recent $\bar{\nu}_e$-$e$ experiment,[9] $T_{\min}=3.6$ MeV, which the authors use for comparison with theory, corresponds to a minimum scattering angle $\theta_{\min}=14°$ for antineutrinos of a maximum energy of 10 MeV. The most-forward data are reported in the 1957 paper of Cowan and Reines[11] for which $T_{\min}=0.1$ MeV and $T_{\max}=0.5$ MeV.

Our approach is to investigate the energy loss for multiple scattering in the sun for particular forms of the $\nu_e$-$e$ interaction which are constrained to satisfy the experimental bound for the $\bar{\nu}_e$-$e$ cross section over the region $T_{\min}\leqslant T\leqslant T_{\max}$. The