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[21]Our predictions are based on a reasonable estimate of $\langle n\rangle$ since it is outside the scope of the present approach to calculate $\langle n\rangle$. For the $\pi^{\pm}p$ case, data on $\langle n\rangle$ is plotted versus $\ln p_{lab}$. For π^+p, we extrapolate and obtain $\langle n\rangle=8$ and 9 for 100 and 200 GeV/c, respectively. For π^-p, we used the value $\langle n\rangle=8.0\pm0.17$ (200 GeV/c) given by F. R. Huson *et al.*, at the International Conference on New Results from Experiments on High Energy Particle Collisions, Vanderbilt University, Nashville, Tennessee, 1973, and interpolate to get the value $\langle n\rangle=7.4$ at 100 GeV/c.

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Pion Exchange in Inclusive Relations*

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Pion-exchange contributions to inclusive reactions at very high energy are calculated and shown to possess scaling behavior. The nearby pion pole is expected to produce clearly identifiable structure, and perhaps even to dominate the cross section in the region $0.9 < x < 1$, $p_\perp^2 < 0.1$ GeV^2. Specific predictions are made for $a + p \rightarrow \Delta +$ anything, where $a = p$, π, or K, and for $\pi + p \rightarrow \rho +$ anything. Measurements of these reactions above 30 GeV could be used to test the Mueller-Regge hypothesis, and if it is valid, to measure the slope of the pion trajectory. A specific proposal in which the Regge hypothesis fails is also discussed. Extrapolation to the pole could be used to measure $\pi\pi$ and πK total cross sections.

Inclusive reactions of the form $a+b\rightarrow c+X$, where all possible states are included in X, have been found theoretically interesting,[1-3] as well as readily accessible to experiment at high energy. In this paper, we contemplate a pion-exchange process which can contribute to some of these reactions—in particular the experimentally attractive ones $a+p\rightarrow\Delta(1236)+X$, where $a=p$, π, or K; and $\pi^{\pm}+p\rightarrow\rho^0+X$ or $K+p\rightarrow K^*(890)+X$. The process is shown in Fig. 1. The pion pole, at $t=m_\pi^2$, is expected to produce structure in the cross section as a function of the squared momentum transfer $t=(p_1-p_3)^2$, for $-t\lesssim m_\pi^2$, as it does in $2\rightarrow2$ reactions.[4] This structure should make it possible to observe the pion contribution experimentally, despite the presence of background processes, which contain no pole. The incident beam energy must be $\gtrsim 30$ GeV, however, in order for $|t|\lesssim m_\pi^2$ to be in the physical region, when the mass of X is beyond the lowest resonances. The resolution in t must be very good in order for the effect of the pole to be visible. The background presumably consists mainly of "quasielastic" scattering in which particle 2 remains intact.

Few people would doubt the existence of the pion pole itself. The motivation for this work is that a measurement of the cross section near the pole would enable one to study the triple-Regge limit,[3] which is an extension of the familiar Regge theory of $2\rightarrow2$ reactions. In addition, the $\pi\pi$ and πK total cross sections could be measured by extrapolating to the pole in $a+p\rightarrow\Delta+X$ for $a=\pi$ and K, provided the extrapolation could be shown to yield the known πp cross section when $a=$proton, as has been noted previously by Salin and Thomas.[5]

If we ignore off-shell corrections for now, Fig. 1 predicts a Lorentz-invariant cross section

$$E_3\frac{d\sigma}{d\vec{p}_3}=\frac{\sigma_\pi}{16\pi^3}\frac{V(t)}{(t-m_\pi^2)^2}\left(\frac{(q\cdot p_2)^2-m_\pi^2m_2^2}{(p_1\cdot p_2)^2-m_1^2m_2^2}\right)^{1/2}, \qquad (1)$$

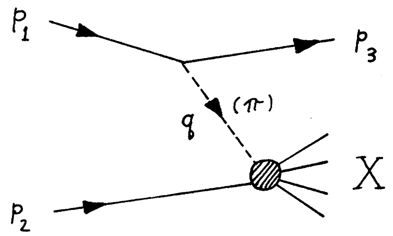

FIG. 1. Pion-exchange graph for the process $1+2 \to 3+X$.

where σ_π is the total cross section for π-2 scattering, and $V(t)$ is the absolute square of the π-1-3 vertex, summed and averaged over helicities.[5] If 1 = proton and 3 = $\Delta^{++}(1236)$,

$$V(t)=\tfrac{1}{2}\sum_{\lambda,\mu}|g\overline{U}_\alpha^{(\mu)}(p_3)(p_1)_\alpha u^{(\lambda)}(p_1)|^2$$
$$=g^2[(m_\Delta+m_p)^2-t]^2[(m_\Delta-m_p)^2-t]/6m_\Delta^{\,2}, \tag{2}$$

where the strength of the coupling is known from the width of the Δ decay into $p\pi^+$:

$$\Gamma=g^2[(m_\Delta+m_p)^2-m_\pi^{\,2}]k^3/(24\pi m_\Delta^{\,2}),$$
$$k=[(m_\Delta+m_p)^2-m_\pi^{\,2}]^{1/2}[(m_\Delta-m_p)^2-m_\pi^{\,2}]^{1/2}/2m_\Lambda. \tag{3}$$

For $\pi^\pm+p\to\rho^0+X$,

$$V(t)=\sum_\lambda|g\epsilon_\alpha^{(\lambda)}(p_3)(p_1+q)_\alpha|^2$$
$$=g^2[(m_\rho+m_\pi)^2-t][(m_\rho-m_\pi)^2-t]/m_\rho^{\,2}, \tag{4}$$

where g^2 is related to the width of the ρ by

$$\Gamma=g^2(m_\rho^{\,2}-4m_\pi^{\,2})^{3/2}/(48\pi m_\rho^{\,2}). \tag{5}$$

It is convenient theoretically to work in the rest frame of particle 2. If we make approximations appropriate to high incident energy, the kinematic variables are described by

$$\begin{aligned}
&p_1=(0,0,P,P+m_1^{\,2}/2P),\\
&p_2=(0,0,0,m_2),\\
&p_3=(p_\perp,0,xP,xP+(p_\perp^{\,2}+m_3^{\,2})/2Px),\\
&s=(p_1+p_2)^2\simeq 2Pm_2,\\
&t=(p_1-p_3)^2\simeq(1-x)(m_1^{\,2}-m_3^{\,2}/x)-p_\perp^{\,2}/x,\\
&M^2=(p_1+p_2-p_3)^2\simeq(1-x)s+m_2^{\,2}.
\end{aligned} \tag{6}$$

The invariant cross section (1) can be written as

$$E_3\frac{d\sigma}{d\vec{p}_3}=\frac{s}{\pi}\frac{d\sigma}{dt\,dM^2}$$
$$=\frac{\sigma_\pi}{16\pi^3}\frac{V(t)}{(t-m_\pi)^2}(1-x). \tag{7}$$

This cross section has a finite limit when $s\to\infty$ for fixed x and $p_\perp$, if we assume σ_π approaches a constant at high energy. The pole term thus exhibits scaling[1] and limiting-fragmentation[2] behavior. This might seem surprising, since in $2\to2$ reactions, pion-exchange cross sections are known to fall like s^{-2}. The s^{-2} result is in fact reproduced by (7) for fixed M^2, but it does not apply to the scaling limit, where M^2 is proportional to s. The scaling behavior of (7) can be understood physically on the basis that the probability for particle 1 to dissociate into $\pi+3$, and the subsequent probability for the π to interact with particle 2, are both constant at high energy. The spin of the pion is irrelevant in this argument, because the pion is not forced to become "wee"[1] in the scaling limit. It is thus perhaps misleading to refer to the pion as "exchanged." In comparing theory with experiment, the dependence of σ_π on M^2 could be taken into account. However, we wish to avoid very small values of M^2, since we assume σ_π consists of a Regge term, which is dual to resonances and approaches zero as $M^2\to\infty$, plus an absorptive term (Pomeranchukon) which is approximately constant.[6] These two terms are likely to behave differently as the pion is taken off the mass shell. Also, M^2 has to be large to obtain the triple-Regge limit.

Let us consider the reaction $pp\to\Delta^{++}(1236)X$. This reaction is convenient experimentally, since a Δ^{++} which is produced with small momentum transfer from the *target* proton has low momentum in the laboratory, and the $p\pi^+$ from its decay can be measured accurately in a bubble chamber. [Still better, the combination $\frac{9}{8}(pp\to\Delta^{++}X)-\frac{3}{8}(pp\to\Delta^0X)$, which isolates isospin 1 in the t channel, could be used.] If we require $|t|\leq 3m_\pi^{\,2}$, then $0.925\leq x$ according to (6). If we require $M\geq 2$ GeV, then $x\leq 1-(3\text{ GeV}^2)/s$. The cross section $d\sigma/dtdx$ has a sharp peak in t in this region, given approximately by the $(t-m_\pi^{\,2})^{-2}$ pion pole factor. Integrating over the peak, using $\Gamma=120$ MeV and a constant $\sigma_\pi=26$ mb, we obtain a total of 0.218 mb in the limit $s\to\infty$, and 0.214 mb at $s=570$ GeV^2. This is a sizable fraction of the 2.2 mb which was observed in the much larger region $|t|<1.0$ GeV^2 in an early National Accelerator Laboratory (NAL) experiment at that energy.[7] In view of the x and $p_\perp^{\,2}$ distributions found in that experiment, we would expect the pion pole term to dominate background in the small region $|t|\lesssim 3m_\pi^{\,2}$, $0.925\lesssim x\lesssim 0.995$. The NAL experiment had too few events (0.2 mb $\simeq$ 10 events) for the pion peak itself to be observed. At a lab momentum of 30 GeV/c, the predicted cross section in the same region $|t|\leq 3m_\pi^{\,2}$ and $M\geq 2$ GeV is only 0.04 mb. This illustrates the advantage of using a very high energy, because of the minimum-momentum-transfer effect.

Experience with $2\to 2$ reactions leads one to expect that the cross section predicted by the simple pole model will be too large in the physical region, when one goes substantially away from the pole. Let us review the results of the Regge analysis.[3,5] The cross section associated with Fig. 1, when summed over the states X, can be represented as in Fig. 2. The absorptive part of π-2 scattering has been expressed as a sum over the Regge poles i. In the limit $s\to\infty$, $M^2\to\infty$, $s/M^2\to\infty$ one obtains

$$E_3\frac{d\sigma}{d\vec{p}_3}=\sum_i R_i(t)\left(\frac{s}{M^2}\right)^{2\alpha_\pi(t)-\alpha_i(0)} s^{\alpha_i(0)-1}\,. \qquad (8)$$

Scaling behavior is produced by a Pomeranchukon term in this sum, with intercept $\alpha_P(0)=1$. The residue functions $R_i(t)$ are unknown, except that they contain the pion pole times factors which presumably vary slowly on the scale of m_π^2. The Regge analysis also predicts additional dependence on x as one moves away from the pole. For if we assume a linear pion trajectory, (8) contains a factor

$$(1-x)^{1+2\alpha_\pi'(0)(|t|+m_\pi^2)}\,. \qquad (9)$$

The rate at which the cross section vanishes as x approaches 1 in the scaling limit therefore accelerates with increasing $|t|$. It would be very interesting to look for this effect experimentally, and thereby measure the slope $\alpha_\pi'(0)$ of the pion trajectory. In the region $0.925\le x\le 0.995$, which corresponds to $|t|\le 3m_\pi^2$ and $M\ge 2$ GeV at $s=570$ GeV^2, the extra Regge factor $(1-x)^{2\alpha_\pi'(0)(|t|+m_\pi^2)}$ changes from $e^{-5.2|t|-0.10}$ to $e^{-10.6|t|-0.11}$ if $\alpha_\pi'(0)=1$, in units where GeV = 1. The integrated cross section of 0.21 mb discussed above is reduced to 0.14 mb. The effect therefore appears to be large enough to be measurable.

In the limit $s\to\infty$ at fixed M^2, $(1-x)\to(m^2-m_2^2-t)/s$ and $t\to -p_\perp^2$. The cross section (7) becomes

$$\frac{d\sigma}{dt\,dM^2}=\frac{\sigma_\pi(M^2)}{16\pi^2}\,\frac{V(t)}{(t-m_\pi^2)^2}\,\frac{M^2-m_2^2-t}{s^2}. \qquad (10)$$

We can argue that in this limit we have essentially

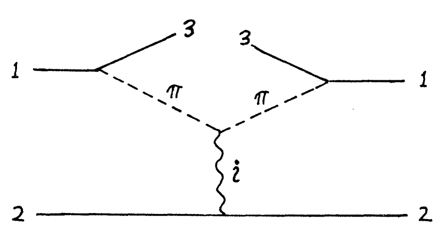

FIG. 2. Triple-Regge representation of the process $1+2\to 3+X$.

a $2\to 2$ reaction, for which we expect the energy dependence $(s/s_0)^{2\alpha(t)-2}$. If the off-shell behavior is to depend on x and t, it must therefore have the form $R(t)(1-x)^{-2\alpha(t)}$. By requiring a smooth continuation from the scaling limit $s\to\infty$ at fixed M^2/s to the two-body Regge limit $s\to\infty$ at fixed M^2, we can therefore obtain the triple-Regge result (9). This result is not inescapable, however. We could argue that Regge behavior in $2\to 2$ reactions is not required when one of the external particles is replaced by the Pomeranchukon, rather than by an ordinary Reggeon, which is dual to particles, and that as a result there is no connection between the scaling limit and the fixed-M^2 Regge limit. As an interesting alternative, we might assume, as is done in Ref. 8, that the off-shell effects depend on the distance from the energy shell in old-fashioned perturbation theory, in a frame such as the rest frame of particle 2 where the dissociation $1\to\pi+3$ takes place at very high momentum, so that the impulse approximation is valid. In that case, the off-shell effects depend on the variable $(t-m_\pi^2)/(1-x)$ rather than the $(t-m_\pi^2)\log(1-x)$ of the Regge theory. If this "off-energy-shell" model is correct, the decrease of the pion-exchange cross section as $x\to 1$ will be very rapid.

The decay distribution of the resonant particle 3 can be used to test for the contribution of pion exchange, or to help in isolating it. The decay distribution of $\Delta^{++}\to p\pi^+$ can be written as

$$\begin{aligned} w(\theta,\phi)=\frac{3}{4\pi}\Big[&\sin^2\theta\,\rho_{3/2,3/2}+(\cos^2\theta+\tfrac{1}{3})\rho_{1/2,1/2}\\ &-\frac{2}{\sqrt{3}}\sin 2\theta\cos\phi\,\mathrm{Re}\rho_{3/2,1/2}\\ &-\frac{2}{\sqrt{3}}\sin^2\theta\cos 2\phi\,\mathrm{Re}\rho_{3/2,-1/2}\Big], \end{aligned} \qquad (11)$$

where θ and ϕ are the spherical angles of the decay proton in the Δ rest frame, and the y axis is perpendicular to the production plane. This distribution can be described conveniently in terms of its spherical moments $\langle Y_{lm}\rangle=\int d\Omega\, w(\theta,\phi)Y_{l,m}(\theta,\phi)$. We have

$$\begin{aligned} \langle Y_{00}\rangle&=(1/4\pi)^{1/2},\\ \langle Y_{10}\rangle&=\langle Y_{11}\rangle=0,\\ \langle Y_{20}\rangle&=(\rho_{1/2,1/2}-\rho_{3/2,3/2})(1/5\pi)^{1/2},\\ \langle Y_{21}\rangle&=\mathrm{Re}\rho_{3/2,1/2}(2/5\pi)^{1/2},\\ \langle Y_{22}\rangle&=-\mathrm{Re}\rho_{3/2,-1/2}(2/5\pi)^{1/2}. \end{aligned} \qquad (12)$$

In the Jackson frame, which is defined by choosing the z axis along the momentum of particle 1, pure π exchange predicts that only $\rho_{1/2,1/2}$ is nonzero, so that $w(\theta,\phi)\propto\cos^2\theta+\frac{1}{3}$. As in $2\to 2$ reactions, the simple pion-exchange prediction might be altered by absorption and Reggeization, as well as by interference with other exchanges. Since s/M^2 is large in the region of interest, absorption effects could presumably be calculated in the same manner as in $2 \to 2$ reactions. The effects would not be expected to be as large as they are in $\pi N \to \rho N$, $np \to pn$, or $NN \to \Delta N$, because the pole terms for those reactions contain factors of $\sqrt{-t}$ for each $NN\pi$ coupling.

We conclude by repeating the numerical calculations for the reaction $\pi^{\pm} + p \to \rho^0 + X$. If we require $|t| \leq 3m_\pi^2$, then $0.906 \leq x$. If we require $M \geq 2$ GeV, then $x \leq 1 - (3\ \text{GeV}^2)/s$. Integrating the cross section given by (7) and (4) over this region, using $\Gamma = 120$ MeV and $\sigma_\pi = 17$ mb, we obtain 0.053 mb in the limit $s \to \infty$ and 0.052 mb at $s = 570\ \text{GeV}^2$. The Regge factor (9) would reduce this to 0.036 mb. The decay distribution of $\rho^0 \to \pi^+\pi^-$ is described by

$$w(\theta, \phi) = \frac{3}{4\pi}[\sin^2\theta\rho_{11} + \cos^2\theta\rho_{00} - \sqrt{2}\sin 2\theta\cos\phi\,\text{Re}\rho_{10} - \rho_{1,-1}\sin^2\theta\cos 2\phi]. \quad (13)$$

The moments are

$$\begin{aligned}
\langle Y_{00}\rangle &= (1/4\pi)^{1/2}, \\
\langle Y_{10}\rangle &= \langle Y_{11}\rangle = 0, \\
\langle Y_{20}\rangle &= (\rho_{00} - \rho_{11})(1/5\pi)^{1/2}, \\
\langle Y_{21}\rangle &= (3/5\pi)^{1/2}\text{Re}\rho_{10}, \\
\langle Y_{22}\rangle &= -(3/10\pi)^{1/2}\rho_{1,-1}.
\end{aligned} \quad (14)$$

Pure pion exchange predicts that only ρ_{00} is nonzero, so $w(\theta, \phi) \propto \cos^2\theta$.

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