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Sum Rules for Mass Differences Within the Baryon Decuplet and the Meson Nonets

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A group-theoretic approach, previously used to derive a precise hybrid formula relating electromagnetic mass differences of pseudoscalar mesons and baryons, is extended here to the barycn decuplet and the vector- and tensor-meson nonets. We derive three sum rules relating mass differences within isotopic-spin multiplets of the decuplet, as well as a hybrid formula relating baryon-octet and -decuplet electromagnetic mass differences. Using the present experimental average for the Ξ^{*-}-Ξ^{*0} mass difference, the hybrid formula can be used to predict $M_{Y^-} - M_{Y^+} = 3.2 \pm 0.5$ MeV, $M_{\Delta^0} - M_{\Delta^+} = -0.24 \pm 0.16$ MeV, and $M_{\Delta^{++}} - M_{\Delta^-} = 0.72 \pm 0.48$ MeV. No assumptions concerning U-spin symmetry are required in obtaining these results. With the 27-dimensional part of the SU(2)-breaking interaction taken to be a U-spin singlet, two additional predictions follow: $M_{Y^-} - M_{Y^0} = 3.4 \pm 0.6$ MeV and $M_{\Delta^{++}} - M_{\Delta^+} = 3.9 \pm 0.9$ MeV. Hybrid mass formulas are also derived for meson nonets. Using the present experimental average for the K^{*0}-K^{*+} mass difference, we predict the neutral ρ meson to be heavier than its charged counterparts by approximately 5 ± 2 MeV.

I. INTRODUCTION

In a recent paper[1] we have used a group-theoretical approach to derive a new hybrid mass formula, relating electromagnetic mass differences of baryons and pseudoscalar mesons. The experimental accuracy of (2 ±2)% of the mass formula derived

$$\frac{5(m_{K^0}{}^2 - m_{K^+}{}^2) + 2(m_{\pi^+}{}^2 - m_{\pi^0}{}^2)}{2m_K{}^2 + m_\eta{}^2 - 3m_\pi{}^2} = \frac{3}{2}\;\frac{M_{\Sigma^-} - M_{\Sigma^+}}{M_\Xi - M_N} \tag{1}$$

is essentially within the accuracy of the measured electromagnetic mass differences. Among the mass formulas which have been derived over the last twelve years for "electromagnetic mass differences," only the well-known formula of Coleman and Glashow,[2] relating mass differences within the baryon octet, is of a comparable degree of accuracy. In fact, the derivation of Ref. 1 emphasizes the similarity of the physical content of the two formulas.

In obtaining Eq. (1) one assumes that the symmetry-breaking Hamiltonian can be treated to first order in perturbation theory, after being separated into SU(3)- and SU(2)-breaking parts. The symmetry-breaking parts are allowed to belong to 8- and 27-dimensional representations of SU(3). Assuming that the $\underline{27}$ SU(2)-breaking part behaves like a singlet under U-spin rotations, the hybrid formula (1) is obtained by equating the ratio of the octet matrix elements of the SU(3)- and SU(2)-breaking parts within the pseudoscalar octet, to the corresponding ratio of the antisymmetric octet matrix elements within the baryon octet.

Accepting the theoretical basis which is suggested in Ref. 1 for the formula derived there, one is led to expect that the same approach can be extended to other multiplets, and under similar conditions it would lead to hybrid mass formulas of comparable accuracy. Specifically, if the success of (1) is due to a fundamental operator identity of $H^{(8)}$ and $\overline{H}^{(8)}$, as suggested in Ref. 1, accurate formulas should hold also when one considers multiplets other than the pseudoscalar octet, for which there is also only one (symmetry-wise) octet matrix element.

From the other multiplets, we select the better established decuplet of baryons and the nonets of vector and tensor mesons, as the suitable objects for our considerations. Although mass differences within isotopic-spin multiplets are much more difficult to establish experimentally for the wide resonances,[3] and there are also some theoretical ambiguities related to their relatively large decay widths, their theoretical calculation is of obvious interest and numerous authors have considered the question of mass sum rules since the advent of higher symmetries. In particular, a large number of theoretical approaches have been advanced to analyze the electromagnetic mass differences of the decuplet. Reference 4 has a fairly complete list of these approaches and of their various predictions. It is interesting to remark on the wide variety of predictions for the electromagnetic mass differences of models, which otherwise give identical results in the realm of medium-strong symmetry breaking. Thus, the electromagnetic mass formulas acquire a special role in judging the validity of these models.

Although we conform with the practice of using the wording "electromagnetic mass differences," there is no commitment in our approach to a pure electromagnetic origin for the mass differences within the isotopic spin multiplets. Our framework solely requires an SU(2)-breaking interaction which is appreciably weaker than the SU(3)-breaking one, and has the symmetry properties specified in (2). Part of our results will be obtained without any assumption about $\mathrm{SU}(2)_U$ invariance (obeyed by electromagnetic interactions), and those results obtained by using some assumption concerning $\mathrm{SU}(2)_U$ properties are carefully isolated.

II. MASS FORMULAS FOR THE DECUPLET

Following the approach outlined in Ref. 1, we express the symmetry-breaking (SB) Hamiltonian to first order in SU(3)-breaking couplings (g_i) and the SU(2)-breaking couplings $(\alpha_i, \beta_i, \gamma_i)$ with the conventional notation $H^{(N)}_{(T_3TY)}$

$$H_{\mathrm{SB}} = g_8 H^{(8)}_{(000)} + g_{27} H^{(27)}_{(000)} + \alpha_8 \overline{H}^{(8)}_{(000)} + \alpha_{27} \overline{H}^{(27)}_{(000)} + \beta_8 \overline{H}^{(8)}_{(010)} + \beta_{27} \overline{H}^{(27)}_{(010)} + \gamma_{27} \overline{H}^{(27)}_{(020)} . \tag{2}$$

Using (2), the mass differences within the isotopic-spin multiplets of the decuplet are given by

$$M_{\Delta^{++}} - M_{\Delta^+} = 2(\tfrac{1}{30})^{1/2}\beta_8 \overline{\Delta}_8 + 2(\tfrac{1}{70})^{1/2}\beta_{27}\overline{\Delta}_{27} + 2(\tfrac{3}{14})^{1/2}\gamma_{27}\overline{\Delta}_{27} , \tag{3a}$$

$$M_{\Delta^+} - M_{\Delta^0} = 2(\tfrac{1}{30})^{1/2}\beta_8 \overline{\Delta}_8 + 2(\tfrac{1}{70})^{1/2}\beta_{27}\overline{\Delta}_{27} , \tag{3b}$$

$$M_{\Delta^0} - M_{\Delta^-} = 2(\tfrac{1}{30})^{1/2}\beta_8 \overline{\Delta}_8 + 2(\tfrac{1}{70})^{1/2}\beta_{27}\overline{\Delta}_{27} - 2(\tfrac{3}{14})^{1/2}\gamma_{27}\overline{\Delta}_{27} , \tag{3c}$$

$$M_{Y^+} - M_{Y^0} = 2(\tfrac{1}{30})^{1/2}\beta_8 \overline{\Delta}_8 - 3(\tfrac{1}{70})^{1/2}\beta_{27}\overline{\Delta}_{27} + 3(\tfrac{1}{42})^{1/2}\gamma_{27}\overline{\Delta}_{27} , \tag{3d}$$

$$M_{Y^0} - M_{Y^-} = 2(\tfrac{1}{30})^{1/2}\beta_8 \overline{\Delta}_8 - 3(\tfrac{1}{70})^{1/2}\beta_{27}\overline{\Delta}_{27} - 3(\tfrac{1}{42})^{1/2}\gamma_{27}\overline{\Delta}_{27} , \tag{3e}$$

$$M_{\Xi^{*0}} - M_{\Xi^{*-}} = 2(\tfrac{1}{30})^{1/2}\beta_8 \overline{\Delta}_8 - 8(\tfrac{1}{70})^{1/2}\beta_{27}\overline{\Delta}_{27} . \tag{3f}$$

Within our framework, a possible $\underline{64}$ contribution to the mass differences is neglected.[5] $\overline{\Delta}_i$ are the normalized reduced matrix elements of $\overline{H}^{(i)}$ with the convention of de Swart.[6] The six equations for the three unknown matrix elements allows the derivation of three sum rules:

$$M_{\Delta^{++}} - M_{\Delta^-} = 3(M_{\Delta^+} - M_{\Delta^0}), \tag{4a}$$

$$M_{Y^+} - 2M_{Y^0} + M_{Y^-} = M_{\Delta^{++}} - 2M_{\Delta^+} + M_{\Delta^0} , \tag{4b}$$

$$(M_{\Delta^+} - M_{\Delta^0}) + (M_{\Xi^{*0}} - M_{\Xi^{*-}}) = M_{Y^+} - M_{Y^-} . \tag{4c}$$

We stress that these formulas hold to first order of SU(2)-symmetry breaking [i.e., to order e^2 and to the same order of possible additional SU(2)-breaking interactions, as expressed in (2)] and are obtained without any assumption about the properties of the SU(2)-breaking interaction under U-spin transformations. Equation (4a), which has been noted by various authors,[7-9] is essentially a consequence of charge independence within the $T=\frac{3}{2}$ multiplet and is therefore independent of SU(3) considerations. Equation (4b) expresses the equality of the $\Delta T=2$ contributions to the mass differences within the two multiplets, while (4c) is an equality of a linear combination of $\Delta T=1$ contributions from both $\underline{8}$ and $\underline{27}$. Sum rules (4b) and (4c) have not been obtained before from symmetry considerations, to the best of our knowledge. Nevertheless, either one or both have been shown to be valid in various variants of the quark model.[10-12]

The physical content of Eq. (4c) is very similar to that of the well-known Coleman-Glashow equation[2] for mass differences within the baryon octet. The latter can also be derived[1] without any assumption about $\mathrm{SU}(2)_U$ invariance, and is an expression of the magnitude of the antisymmetric octet reduced matrix element. Thus, we consider (4c) to be the analog of the Coleman-Glashow relation within the realm of the baryon decuplet.

For the decuplet, there is no need for any additional assumption in order to isolate $\beta_8\overline{\Delta}_8$, whose explicit expression in terms of mass differences is needed for obtaining a hybrid formula. Thus, we do not have to proceed as in Ref. 1 and to make the $\mathrm{SU}(2)_U$ assumption relating γ_{27} to β_{27}, at this stage. From (3b) and (3f) we find

$$\beta_8\overline{\Delta}_8 = \frac{\sqrt{3}}{\sqrt{10}}\left[4(M_{\Delta^+} - M_{\Delta^0}) + (M_{\Xi^{*0}} - M_{\Xi^{*-}})\right]. \tag{5}$$

Neglecting the SU(2)-breaking interaction in (2), one can easily derive

$$g_{27}\Delta_{27} = (\tfrac{21}{10})^{1/2}(M_Y - 2M_{\Xi^*} + M_\Omega) = \sqrt{2.1}\ (-3 \pm 4)\ \mathrm{MeV} \simeq 0, \tag{6a}$$

$$g_8\Delta_8 = (\tfrac{2}{5})^{1/2}(M_\Delta - 4M_{\Xi^*} + 3M_Y) = 459 \pm 12\ \mathrm{MeV}, \tag{6b}$$

where Δ_i are the normalized reduced matrix elements of $H^{(i)}$. In order to obtain a hybrid formula, we follow the argumentation of Ref. 1 in making the identification

$$\Delta_8 = \overline{\Delta}_8 . \tag{7}$$

Using also the equality of the appropriate antisymmetric octet matrix element within the baryon octet, i.e., $D_A = \overline{D}_A$ as well as their explicit expressions derived in Ref. 1 from the Hamiltonian (2), we find by equating $\beta_8 \overline{\Delta}_8 / g_8 \Delta_8$ to $\beta_8 \overline{D}_A^8 / g_8 D_A^8$

$$\frac{4(M_{\Delta^+} - M_{\Delta^0}) + M_{\Xi^{*0}} - M_{\Xi^{*-}}}{M_\Delta + 3M_Y - 4M_{\Xi^*}} = \frac{M_{\Sigma^-} - M_{\Sigma^+}}{M_\Xi - M_N}. \quad (8)$$

Using the equal-spacing rule resulting from the vanishing $\underline{27}$ contribution of (6a), we can rewrite (8) as

$$\frac{(M_{\Xi^{*-}} - M_{\Xi^{*0}}) + 4(M_{\Delta^0} - M_{\Delta^+})}{M_{\Xi^*} - M_\Delta} = \frac{2}{5} \frac{M_{\Sigma^-} - M_{\Sigma^+}}{M_\Xi - M_N}. \quad (8')$$

Formula (8′) is the decuplet hybrid mass formula analogous to our previously derived[1] Eq. (1), except that *no assumption concerning invariance under $SU(2)_U$ transformations* has been made in obtaining (8) or (8′). In this respect, its similarity to the Coleman-Glashow formula[2,1] goes even further than that of Eq. (1).

By using (4c) we can obtain a different form for (8′), which is also more transparent for comparing to other previously derived[13,14] hybrid formulas, as follows

$$\frac{4(M_{Y^-} - M_{Y^+}) - 3(M_{\Xi^{*-}} - M_{\Xi^{*0}})}{M_{\Xi^*} - M_\Delta} = \frac{2}{5} \frac{M_{\Sigma^-} - M_{\Sigma^+}}{M_\Xi - M_N}. \quad (9)$$

The formulas (8′) and (9) are different from their predecessors. In the Coleman and Glashow tadpole model,[13] the $\underline{27}$ contribution is evidently neglected in deriving their formula, while Eq. (13) of Radicati *et al.*[14] is obtained from assumptions different from ours, leading to hybrid formulas having numerators and denominators with different SU(3) transformation properties.

The assumption of $SU(2)_U$ invariance for the $\underline{27}$ part of $\overline{H}$ implies

$$\gamma_{27} = (\tfrac{5}{3})^{1/2} \beta_{27}, \quad (10)$$

and then several additional relations hold, namely

$$M_{\Delta^-} - M_{\Delta^0} = M_{\Xi^{*-}} - M_{\Xi^{*0}} = M_{Y^-} - M_{Y^0}, \quad (11a)$$

$$M_{Y^+} - M_{Y^-} = M_{\Delta^+} - M_{\Delta^-}. \quad (11b)$$

These are the relations obtained by many authors in the past (e.g., Refs. 7–9) for the electromagnetic mass differences within the decuplet, by using the conventional approach of $SU(2)_U$ invariance. Combining (11a) and (11b) one can recover our sum rule (4c), which we have shown to hold independently of the $SU(2)_U$-invariance hypothesis.

The experimental situation concerning decuplet mass differences is such that it is not possible at present to check the relations derived here with reasonable degree of confidence. A search of the literature reveals the following situation:

(A) *Δ Mass Differences.* We found two relevant references, giving

$$M_{\Delta^0} - M_{\Delta^{++}} = 2.9 \pm 0.85 \text{ MeV} \quad (\text{Carter } et\ al.^{15}),$$

$$M_{\Delta^-} - M_{\Delta^{++}} = 7.9 \pm 6.8 \text{ MeV} \quad (\text{Gidal } et\ al.^{16}).$$

(B) *Y Mass Differences.* The "Review of Particle Properties" lists[3] nine experiments, which give for $(M_{Y^-} - M_{Y^+})$ values between 2 to 17 MeV.

(C) *Ξ^* Mass Difference.* The world average of five experiments which are reasonably close in their findings is given[3,4] as

$$M_{\Xi^{*-}} - M_{\Xi^{*0}} = 3.4 \pm 0.6 \text{ MeV}. \quad (12)$$

At this stage, we can make some predictions if we use the world average (12) for what seems to be the better established mass difference. Numerically, our formulas (8′) and (9) read

$$\begin{aligned}(M_{\Xi^{*-}} - M_{\Xi^{*0}}) + 4(M_{\Delta^0} - M_{\Delta^+}) &= 4(M_{Y^-} - M_{Y^+}) - 3(M_{\Xi^{*-}} - M_{\Xi^{*0}}) \\ &= 2.45 \pm 0.05 \text{ MeV}. \end{aligned} \quad (13)$$

Using the value (12), we predict from (13)

$$M_{Y^-} - M_{Y^+} = 3.2 \pm 0.5 \text{ MeV}, \quad (14)$$

$$M_{\Delta^0} - M_{\Delta^+} = -0.24 \pm 0.16 \text{ MeV}, \quad (15)$$

and using (4a) one further has

$$M_{\Delta^{++}} - M_{\Delta^-} = 0.72 \pm 0.48 \text{ MeV} \quad (16)$$

We emphasize that the predictions (14)–(16) are obtained by using the present world average for $M_{\Xi^{*-}} - M_{\Xi^{*0}}$, and a conclusive test of our approach requires the accurate determination of the quantities appearing in (13). For comparison, the hybrid mass formula of Radicati *et al.*[14] predicts $M_{Y^-} - M_{Y^+} = 6.1 \pm 0.3$ MeV (the uncertainty of their prediction is smaller since their formula reads

$$(M_{Y^-} - M_{Y^+}) = (M_{\Sigma^-} - M_{\Sigma^+})(M_{\Xi^*} - M_\Delta)/(M_\Xi - M_N)).$$

The tadpole model[13] has all the six mass differences (3) equal among themselves, and from their hybrid formula equal to -3.1 ± 0.3 MeV.

If we use the relations (11a) and (11b) derived by the additional assumption (10) of $SU(2)_U$ invariance for the $\underline{27}$ part of $\overline{H}$, in combination with our hybrid formula (8′) or (9), one can predict all the other five independent decuplet mass differences from the experimental input (12). One thus obtains

$$\begin{aligned}
&M_{Y^-}-M_{Y^0}=3.4\pm0.6\text{ MeV},\\
&M_{\Delta^-}-M_{\Delta^0}=3.4\pm0.6\text{ MeV},\\
&M_{Y^0}-M_{Y^+}=-0.2\pm1.1\text{ MeV},\\
&M_{\Delta^0}-M_{\Delta^+}=-0.24\pm0.16\text{ MeV},\\
&M_{\Delta^+}-M_{\Delta^{++}}=-3.9\pm0.9\text{ MeV}.
\end{aligned}\tag{17}$$

III. HYBRID MASS FORMULA FOR MESON NONETS

Let us consider first the nonet of vector mesons. It can be treated along the same lines as the derivation for pseudoscalar mesons.[1] In this way a hybrid formula analogous to (1) is obtained by replacing $m_K\to m_{K^*}$, $m_\pi\to m_\rho$, and $m_\eta\to m_{V_8}$. Making the assumption that the 27 part of the SU(3)-breaking Hamiltonian is negligible compared to the octet part, one can use $2m_{K^*}^2+m_{V_8}^2-3m_\rho^2=(10/3)(m_{K^*}^2-m_\rho^2)$ to obtain

$$\frac{5(m_{K^{*0}}^2-m_{K^{*+}}^2)+2(m_{\rho^+}^2-m_{\rho^0}^2)}{\frac{10}{3}(m_{K^*}^2-m_\rho^2)}=\frac{3}{2}\,\frac{M_{\Sigma^-}-M_{\Sigma^+}}{M_\Xi-M_N}\,. \tag{18}$$

In obtaining (18) from (2), one has to assume $\mathrm{SU}(2)_U$ invariance for the 27 part of $\overline{H}$ in dealing with the meson mass differences.

There is nevertheless the delicate question of the correct procedure of formulating mass sum rules for vector mesons. The arguments of Coleman and Schnitzer[17] indicate that the mass sum rules obtained as a result of symmetry breaking should hold for the inverse masses. If we rewrite the left-hand side of (18) with this approach, the hybrid formula can be recast in the following form:

$$\frac{5(m_\rho^2/m_{K^*}^2)(m_{K^{*0}}^2-m_{K^{*+}}^2)+2(m_{K^*}^2/m_\rho^2)(m_{\rho^+}^2-m_{\rho^0}^2)}{\frac{10}{3}(m_{K^*}^2-m_\rho^2)}=\frac{3}{2}\,\frac{M_{\Sigma^-}-M_{\Sigma^+}}{M_\Xi-M_N}\,. \tag{19}$$

There are no good data for the ρ^0-ρ^+ mass difference at present. On the other hand, an average[3] of several experiments indicates a K^* mass difference of $m_{K^{*0}}-m_{K^{*+}}=6.1\pm1.5$ MeV. If we accept this figure, we can use (18) or (19) to predict the Δm_ρ mass difference. One thus obtains

$$m_{\rho^0}-m_{\rho^+}=10.7\pm3.5\text{ MeV} \tag{18$'$}$$

or

$$m_{\rho^0}-m_{\rho^+}=4.5\pm1.5\text{ MeV} \tag{19$'$}$$

depending on whether one uses the mass-squared or the inverse mass-squared approach for obtaining the sum rule.

In view of the known difficulties involved in the calculation of electromagnetic mass differences of vector mesons, very little has been done on this subject. From a quark model, Gal and Scheck[12] obtain a mass difference of opposite sign and approximately the same magnitude as in (18′) and (19′). Using finite-energy sum rules, Bucella *et al.*[18] obtain $m_{\rho^+}-m_{\rho^0}\simeq 1$ MeV and a similar result is obtained in a quark model by Barton and Dare.[19] Recently, Brown and Mich[20] have used a renormalizable Lagrangian with spontaneous symmetry breaking to obtain a finite contribution to the charged-ρ-meson selfmass. Their calculation gives 0.52 MeV for the charged-ρ mass shift. On the experimental side, an analysis by Pišút and Roos[21] of data on ρ production reveals a heavier neutral ρ, giving $m_{\rho^0}-m_{\rho^-}=2.4\pm2.1$ MeV.

It is clear, therefore, that at present our prediction cannot be tested yet. It should be stressed that we anticipate the neutral ρ meson to be heavier than the charged one, if the present data on $m_{K^{*0}}-m_{K^{*+}}$ are correct. In view of the remarkable accuracy of Eq. (1), it is an intriguing question to find out whether the same approach holds experimentally with a similar accuracy when applied to the vector nonet.

Our derivation for the vector mesons can be also used for the tensor nonet—and a hybrid formula similar to (18) would then result with $m_{K^*}\to m_{K_N(1420)}$ and $m_\rho\to m_{A_2}$. At present, there is no way to analyze such a formula in any detail.

IV. SUMMARY

By using the Hamiltonian (2) with decuplet states, we have derived three relations for mass differences within isotopic-spin multiplets, of which (4a) does not require any SU(3) considerations for its derivation. The other two relations, (4b) and (4c), do not require the usually made assumption of U-spin invariance for deriving mass differences induced by $\mathrm{SU}(2)_T$ breaking.

Following the approach of Ref. 1, we then assume the equality of the reduced octet matrix elements of H and $\overline{H}$, thus deriving the hybrid mass formulas (8′) and (9). These relations are also derived without any assumption about U-spin invariance, thus underlying their connection to the nonhybrid Coleman-Glashow relation.[2,1]

The presently available experimental information is not sufficient to allow a check of these formulas. However, if we use the experimental world average for the $\Xi^{*-} - \Xi^{*0}$ mass difference, our hybrid formulas can be used to predict

$$M_{\Delta^0} - M_{\Delta^+} = -0.24 \pm 0.16 \text{ MeV},$$

$$M_{Y^-} - M_{Y^+} = 3.2 \pm 0.5 \text{ MeV},$$

and

$$M_{\Delta^{++}} - M_{\Delta^-} = 0.72 \pm 0.48 \text{ MeV}.$$

If we use $SU(2)_U$ invariance for $\overline{H}^{(27)}$, the hybrid formulas (8′) and (9) combined with the presently available experimental input on $M_{\Xi^{*-}} - M_{\Xi^{*0}}$ [Eq. (12)], permit the calculation of all the mass differences within the decuplet, the prediction following from this approach being listed in Eq. (17).

The application of our approach to the vector nonet results in the hybrid mass formula (18) or (19). The latter holds when the symmetry breaking is originally introduced through the kinetic energy term of the Hamiltonian. Using the present world average for the $K^{*0} - K^{*+}$ mass difference, we predict a neutral ρ appreciably heavier than the charged one [by ~5 or ~10 MeV, from Eqs. (19′) or (18′), respectively].

An experimental effort towards the measurement of the mass differences discussed here would be very timely and helpful at this stage of understanding of the "electromagnetic" mass differences.

Note Added in Proof

(1) In lieu of accurate "direct" measurements for $M_{Y^-} - M_{Y^+}$ one can use the average masses of M_{Y^-} and M_{Y^+} for comparing with the calculated mass difference of Eq. (14), $(M_{Y^-} - M_{Y^+})_{\text{th}} = 3.2 \pm 0.5$. An average over a large number of experiments gives[3] for these masses $M_{Y^-} = 1385.9 \pm 1.5$ and $M_{Y^+} = 1382.81 \pm 0.68$, which results in $(M_{Y^-} - M_{Y^+})_{\text{exp}} = 3.1 \pm 2.2$, in remarkable agreement with our calculated value. We are indebted to Professor L. Wolfenstein for this remark.

(2) In connection with our prediction for the $\rho^0 - \rho^+$ mass difference [Eq. (19′)], it is of interest to mention that calculations of the charge-dependent corrections to the two-body nuclear potential require a neutral ρ meson heavier than the charged ones by several MeV, in order to account for the observed proton-proton and neutron-proton scattering lengths [see C. Yalçin and A. N. Akbay, Nuovo Cimento 13A, 181(1973), where references to previous works can also be found].

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[1]S. Eliezer and P. Singer, Phys. Rev. D 8, 672 (1973).
[2]S. Coleman and S. L. Glashow, Phys. Rev. Lett. **6,** 423 (1961).
[3]Particle Data Group, Rev. Mod. Phys. 45, 332 (1973).
[4]L. Kirsch *et al.*, Nucl. Phys. **B40,** 349 (1972).
[5]See also H. Harari, Phys. Rev. **139,** B1323 (1965).
[6]J. J. de Swart, Rev. Mod. Phys. **35,** 916 (1963).
[7]S. P. Rosen, Phys. Rev. Lett. **11,** 100 (1963).
[8]A. J. Macfarlane and E. C. G. Sudarshan, Nuovo Cimento **31,** 1176 (1964).
[9]S. Okubo, J. Phys. Soc. Jap. **19,** 1507 (1964).
[10]S. Ishida, K. Konno, and H. Shimodaira, Nuovo Cimento **46A,** 194 (1966).
[11]D. B. Lichtenberg, Nuovo Cimento **49A,** 435 (1967).
[12]A. Gal and F. Scheck, Nucl. Phys. **B2,** 110 (1967).
[13]S. Coleman and S. L. Glashow, Phys. Rev. **134B,** 671 (1964).
[14]L. A. Radicati, L. E. Picasso, D. P. Zanello, and J. J. Sakurai, Phys. Rev. Lett. **14,** 160 (1965).
[15]A. A. Carter *et al.*, Nucl. Phys. **B26,** 445 (1971).
[16]G. Gidal *et al.*, Phys. Rev. **141,** 1261 (1966).
[17]S. Coleman and H. J. Schnitzer, Phys. Rev. **134,** B863 (1964); see also S. Gasiorowicz and D. A. Geffen, Rev. Mod. Phys. **41,** 531 (1969).
[18]F. Bucella *et al.*, Nuovo Cimento **64A,** 927 (1969).
[19]G. Barton and D. Dare, Nuovo Cimento **46A,** 433 (1966).
[20]L. M. Brown and J. D. Mich (unpublished).
[21]J. Pišút and M. Roos, Nucl. Phys. **B6,** 325 (1968).