the resonance effects are generally visible only as interference with the background. An absolute bump of magnitude 8 μb would be difficult to extract experimentally.

The author wishes to thank Professor J. Button-Shafer and Professor R. R. Kofler for stimulating discussions.

†This work is supported in part through funds provided by the Atomic Energy Commission under Contract No. AT11-1-3069.

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[1]R. Bizzari, P. Guidoni, F. Marzano, G. C. Moneti, P. Rossi, D. Zanello, E. Castelli, and M. Sessa, Nuovo Cimento Lett. 1, 749 (1969).

[2]B. Lorstad, Ch. d'Andlau, A. Astier, J. Cohen-Ganouna, M. Della Negra, M. Aguilar-Benitez, J. Barlow, L. D. Jacobs, P. Malecki, and L. Montanet, in *Proceedings of the Fifth International Conference on Elementary Particles, Lund, 1969,* edited by G. von Dardel (Berlingska Boktryckeriet, Lund, Sweden, 1970).

[3]H. Nicholson, B. C. Barish, J. Pine, A. V. Tollestrup, J. K. Yoh, C. Delorme, F. Lobkowicz, A. C. Melissinos, Y. Nagashima, A. S. Carroll, and R. H. Phillips, Phys. Rev. Lett. 23, 603 (1969).

[4]A. Benvenuti, D. Cline, R. Rutz, D. D. Reeder, and V. R. Scherer, Phys. Rev. Lett. 27, 283 (1971).

[5]R. G. Carson, J. Button-Shafer, S. S. Hertzbach, R. R. Kofler, and S. S. Yamamoto, in *Proceedings of the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973).

[6]D. C. Peaslee, Particles and Nucl. 4, 73 (1972).

[7]H. J. Lipkin, Phys. Rev. 176, 1709 (1968).

[8]D. Cline and R. Rutz, Phys. Rev. D 5, 778 (1972).

[9]D. C. Peaslee and D. M. Rosalky, report (unpublished).

PHYSICAL REVIEW D VOLUME 8, NUMBER 1 1 JULY 1973

Amplitudes and Model for πN Backward Scattering at 6 GeV/c

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(Received 24 October 1972)

Using reasonable assumptions concerning the s-channel helicity-nonflip amplitude $M^{3/2}_{++}$ of isospin $\frac{3}{2}$ in the u channel and the helicity-flip amplitude $M^{1/2}_{+-}$, we extract from the data at 6 GeV/c on πN backward scattering the remaining amplitudes. We find that the isospin-$\frac{3}{2}$ amplitudes can be described in a Regge-pole model with absorption. A description of $M^{1/2}$ amplitudes is given, using degenerate N_α and N_γ trajectories. The N_γ couplings are found to be fairly important, especially in the $M^{1/2}_{++}$ amplitude which then has to be strongly absorbed.

I. INTRODUCTION

Whereas a description of forward meson-baryon scattering in terms of Regge-pole exchanges with absorption is sufficiently well established so that refinements to it in terms of Regge-Regge cuts can be examined, backward scattering is still far from being understood. The question of which poles exactly contribute to πN backward scattering is not even resolved, despite the fact that $\pi N \to N\pi$ has been rather extensively studied, both theoretically and experimentally. Pion-nucleon backward scattering is described by isospin-$\frac{1}{2}$ and -$\frac{3}{2}$ exchange in the u channel. The isospin $\frac{1}{2}$ is very probably dominated by N_α exchange. What mechanism—wrong-signature nonsense zero or pole-cut interference—is responsible for the dip observed in the $\pi^+ p \to p\pi^+$ differential cross section is a problem which is not settled. Unanswered too is the question of the presence or absence of the N_γ trajectory in the isospin-$\frac{1}{2}$ amplitude. A very strong N_γ, besides the N_α, seems needed in pion backward photoproduction and $pp \to d\pi^+$ where no dip corresponding to the one in $\pi N \to N\pi$ is observed. What part the N_γ plays in πN backward scattering however is difficult to resolve as long as the dip mechanism is associated with the N_α trajectory. What is clear is that the isospin-$\frac{3}{2}$ amplitudes, which are the only ones contributing to $\pi^- p \to p\pi^-$, are dominated by Δ_δ exchange. However, there is discussion about whether the Δ_δ residue should change sign or not when it varies from the backward direction to the N_{33} pole. This is related to the question of whether the residue should contain a factor $\alpha_\Delta - \frac{1}{2}$ (where α_Δ is the Δ_δ trajectory) as is demanded by SU(3) symmetry, the strong exchange degeneracy of Σ_γ and Σ_β in $K^+p \to pK^+$, and the absence of a $\frac{1}{2}^+$ particle on the Σ_β trajectory.

This paper attempts to clarify some of these points. No complete amplitude analysis of course is possible, since at 6 GeV/c there exists no complete set of measurements, but only results on the three differential cross sections, and the polarizations in $\pi^\pm p \to p\pi^\pm$. Using some model-independent results obtained by Barger and Olsson,[1] we try to derive some general features of the amplitudes by assuming a definite form for some of them, and deducing the others from the data. We then propose a model for the amplitudes obtained this way. In Sec. II we report the model-independent results and discuss the isospin amplitudes and their interferences. Section III contains our results and the description of our model. We conclude in Sec. IV with a general discussion of our results.

II. GENERAL DISCUSSION OF THE $I_u = \frac{1}{2}, \frac{3}{2}$ AMPLITUDES

A. Model-Independent Results

Barger and Olsson[1] have obtained the following model-independent results from the 6-GeV/c data[2]:

(a) The modulus squared of the isospin-$\frac{1}{2}$ amplitude σ_N has approximately a double zero at $u \simeq -0.15$ (GeV/c)^2. We present σ_N in Fig. 1 (see Refs. 3 and 4) at 5.9 GeV/c and moreover at 9.85 GeV/c to show its energy dependence.

(b) The isospin-$\frac{1}{2}$ and -$\frac{3}{2}$ interference term $\text{Re}I_1I_3^*$ has an approximately double zero at $u \simeq -0.15$ (GeV/c)^2. We present $\text{Re}I_1I_3^*$ in Fig. 2 (see Refs. 3 and 4), conveniently normalized, at 5.9 GeV/c and moreover at 9.85 GeV/c.

(c) The polarization in $\pi^+p \to p\pi^+$ outside of the dip region is essentially given by the $I_u = \frac{1}{2}$ amplitude interfering with itself.

If one looks for amplitudes the form of which is close to that of Regge poles, the natural way to interpret the result of point (a) is that each isospin-$\frac{1}{2}$ amplitude is very small around $u = -0.15$ (GeV/c)^2. Barger and Olsson[1] moreover conclude from point (b) that $\text{Im}M_{+-}^{3/2}$ (the s-channel helicity-nonflip amplitude of isospin $\frac{3}{2}$ in the u channel) has a simple zero at $u \simeq -0.15$ (GeV/c)^2. Their conclusion is however based on the following additional assumptions: At $u = -0.15$ (GeV/c)^2 the helicity-nonflip amplitudes dominate the helicity-flip amplitudes and $\text{Re}M_{+-}^{1/2}$ has a double zero at that momentum transfer.

B. Qualitative Discussion of the Amplitudes and Assumptions

We now proceed to describe our procedure. As a first guess, we consider that $M_{+-}^{1/2}$ is essentially given by a simple Regge pole, the N_α trajectory. Therefore the additional assumption involved relative to our interpretation of point (a) in Sec. IIA is that $\text{Re}M_{+-}^{1/2}$ has a double zero at $u \simeq -0.15$ (GeV/c)^2. This zero structure is evidently attributed to the wrong-signature nonsense (WSN) zero at $\alpha_{N_\alpha}(u) = -\frac{1}{2}$, where $\alpha_{N_\alpha}(u)$ is the nucleon trajectory. Our choice for $M_{+-}^{1/2}$ seems the most

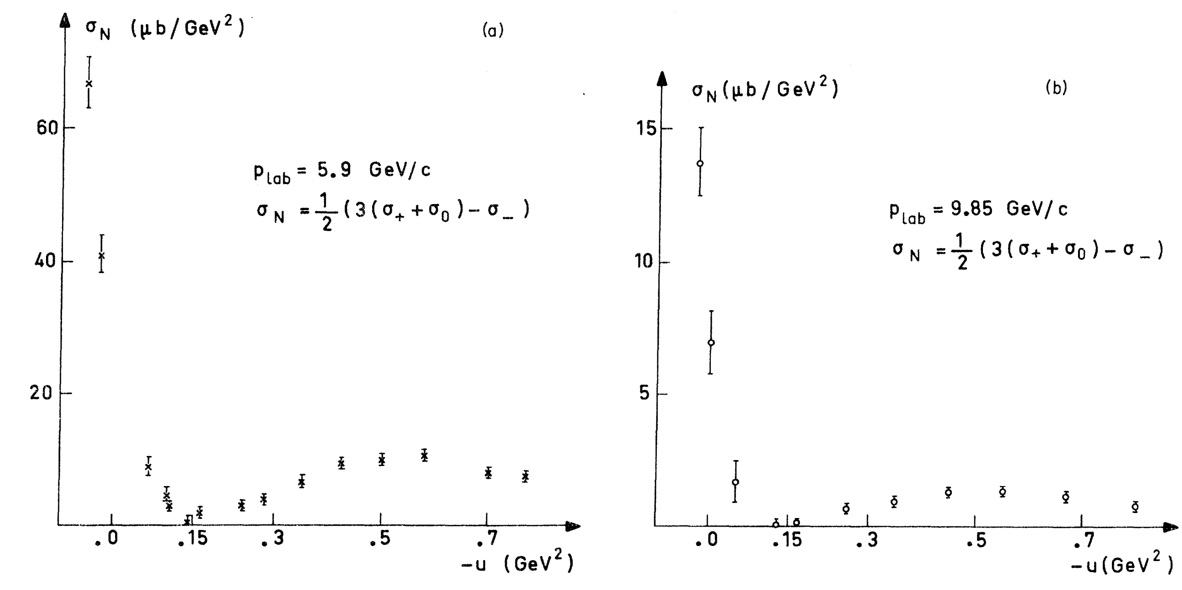

FIG. 1. The modulus squared of the isospin-$\frac{1}{2}$ amplitude σ_N at (a) 5.9 GeV/c; (b) 9.85 GeV/c. Data from Refs. 3 and 4.

natural starting point, considering that at[5] $u = u_0$ the energy dependence of the $\pi^+ p \to p\pi^+$ differential cross section is compatible with that given by the N_α trajectory, and that the normalization at that same momentum transfer is in good agreement with the extrapolation of $M^{1/2}_{+-}$ to the nucleon pole. There clearly exists a strong correlation between $M^{1/2}_{+-}$ at $u = u_0$ and the nucleon. From this choice for $M^{1/2}_{+-}$ follows a certain number of consequences for the other amplitudes. In particular, let us consider the question of the $\frac{1}{2}$, $\frac{3}{2}$ interference term at small transfer. Before the dip, the pole extrapolation yields the nucleon Regge-pole amplitude in the third quadrant of the phase diagram, as shown in Fig. 3. The value of $\mathrm{Re} I_1 I_3^*$ near $u = 0$ (cf. Fig. 2) implies that the angle between $M^{3/2}_{+-}$ and $M^{1/2}_{+-}$ has to be of the order of 45°. So, if the phase of $M^{3/2}_{+-}$ is related to the Δ_δ trajectory α_Δ, and its coupling to the Δ_δ width, the only solution consists in choosing $M^{3/2}_{+-}$ in the fourth quadrant (Fig. 3), which means in other terms that the coupling does not change sign between $u = m_\Delta{}^2$ and $u = 0$ (no $\alpha_\Delta - \frac{1}{2}$ factor in the residue).

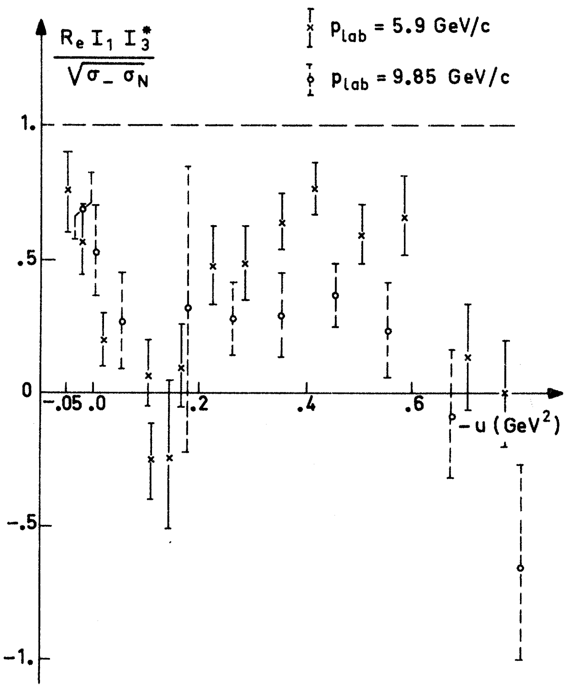

FIG. 2. Ratio of Re $I_1 I_3^*$ to $(\sigma_N \sigma_-)^{1/2}$, with Re $I_1 I_3^* = \frac{3}{4}(\sigma_+ + \frac{1}{3}\sigma_- - 2\sigma_0)$ at 5.9 GeV/c and 9.85 GeV/c. Data from Refs. 3 and 4.

III. STRUCTURE OF THE AMPLITUDES AND MODEL

A. The $M^{3/2}$ Amplitudes

For the extraction of amplitudes, we deal first with the $I_u = \frac{3}{2}$ amplitude, as constrained by σ_- and P_-. The differential cross section does not have any particular structure, and the polarization suggests an interpretation in terms of a Regge-pole model with absorption. In such a model, the absorption will be stronger in the nonflip amplitude than in the flip amplitude, and stronger in the real part than in the imaginary part, due to the vanishing of the latter at the right-signature nonsense point $\alpha_\Delta = -\frac{1}{2}$. At small $|u|$, the two Regge amplitudes have a relative phase equal to zero or π, depending on whether $M^{3/2}_{++}$ has a zero or not, respectively, at $\alpha_\Delta = \frac{1}{2}$. Only in the former case, a conventional absorption model will yield a positive polarization, and therefore we conclude that $M^{3/2}_{++}$ is in the fourth quadrant at small $|u|$, and consequently that its residue does contain an $\alpha_\Delta = \frac{1}{2}$ factor.[6] An actual extraction can be made as follows. We take $M^{3/2}_{++}$ as a Regge-pole amplitude Δ_{++} with its coupling related to the Δ_δ width through pole extrapolation [see Eq. (1)] and extract $M^{3/2}_{+-}$ from the data at 6 GeV/c, σ_-, and P_-. In doing that, we fix the trajectory α_Δ around the value given by the Chew-Frautschi plot, and vary the scale parameter s_0 in $M^{3/2}_{++}$ [Eq. (1)]. As expected, the resulting $M^{3/2}_{+-}$ amplitude has the general shape of an absorbed Regge Δ_δ term: Whereas the imaginary part looks like that of a Regge amplitude, the real part is smaller in magnitude and more peaked. Since we are to present a definite model for Δ_δ exchange, we shall not give further details and numerical results for the extracted amplitude.

B. Model for $M^{3/2}$ Amplitudes

Our model for $M^{3/2}$ amplitudes is as follows:

$$M^{3/2}_{+\pm} = \Delta_{+\pm} + C_{+\pm}\,,$$

where $\Delta_{+\pm}$ is the pole term, and $C_{+\pm}$ the absorptive correction, calculated in the usual way from a pole-Pomeranchukon convolution. The convolution formulas which have been used are shown in the Appendix. Here, since a weak cut model is clearly sufficient for accounting for the P_- polarization, we have fixed the λ cut enhancement factor to 1. We find that

$$\Delta_{++} = 2\pi i e^{-i\pi(\alpha_\Delta - 1/2)/2} \frac{\Delta_0}{m_\Delta} \Gamma\left(\frac{\frac{3}{2} - \alpha_\Delta}{2}\right)\left(\frac{s}{s_0}\right)^{\alpha_\Delta} \times (\alpha_\Delta - \tfrac{1}{2})(-u + u_0)^{1/2}\,, \quad (1)$$

$$\Delta_{+-} = -2\pi i e^{-i\pi(\alpha_\Delta - 1/2)/2} \Delta_1 \Gamma\left(\frac{\frac{3}{2} - \alpha_\Delta}{2}\right)\left(\frac{s}{s_1}\right)^{\alpha_\Delta} \times \left[1 + \delta_1\left(1 - \frac{u}{m_\Delta{}^2}\right)(1 + \delta_2 u)\right]. \quad (2)$$

$\alpha_\Delta = \alpha_{0\Delta} + \alpha'_\Delta u$, with $\alpha'_\Delta = (\frac{3}{2} - \alpha_{0\Delta})/m_\Delta{}^2$ and m_Δ

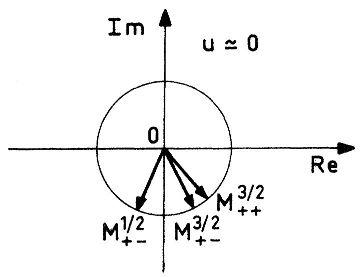

FIG. 3. Phase diagram at $u \simeq 0$ for $M^{1/2}_{+-}$, $M^{3/2}_{+-}$, and $M^{3/2}_{++}$.

the Δ_δ mass. s_0 and s_1 are the two scale parameters. δ_1 and δ_2 are two other parameters which will be explained below, and

$$\Delta_i = \tfrac{3}{2}\alpha'_\Delta(s_i)^{3/2}\frac{m_\Delta{}^2\Gamma_\Delta}{(E_\Delta+m)q_\Delta{}^3},$$

with $\Gamma_\Delta = 0.12$ GeV being the Δ_δ width, m the nucleon mass, and q_Δ the πN c.m. momentum at an energy equal to the Δ_δ mass. The index i takes the values 0 and 1 for Δ_{++} and Δ_{+-}, respectively.

Some comments are in order. First, one can verify that these parametrizations ensure a correct pole extrapolation. The pole and zero structure is clearly exhibited by the argument of the Γ function, and the $\alpha_\Delta - \frac{1}{2}$ factor is made explicit in the Δ_{++} term. Apart from the scale parameters s_0 and s_1, we allow for a supplementary variation of the coupling in Δ_{+-}, through a parabolic residue determined by δ_1 and δ_2. In fact we find that if $\delta_1 = 0$ (constant residue), the value of s_1 which fits to the σ_- cross section at $u = u_0$ is too small for providing a correct slope at higher momentum transfers. These two additional free parameters are the price we have to pay for realizing a smooth extrapolation to the Δ_δ pole. We believe it is not too much, owing to the fact that the Δ_δ pole is far away from the physical region.

For calculating the absorption amplitudes, we take a Pomeranchukon term of the form

$$P = is\sigma_T \exp[\alpha'_P t(\ln s - \tfrac{1}{2}i\pi)]e^{at}, \tag{3}$$

where σ_T has been fixed at 22 mb. a and α'_P have been fitted.

The results of our fit are shown in Figs. 4 (see Ref. 3) and 5 (see Ref. 7) for the differential cross section and the polarization, respectively. The values of the parameters are found to be

$$s_0 = 0.574 \text{ GeV}^2, \quad s_1 = 0.680 \text{ GeV}^2,$$
$$\delta_1 = -0.637, \quad \delta_2 = 2.09 \text{ GeV}^{-2}, \quad \alpha_{0\Delta} = -0.0648, \tag{4}$$

$$\alpha'_P = 0.3 \text{ GeV}^{-2}, \quad a_P = 1.955 \text{ GeV}^{-2}. \tag{5}$$

We shall discuss these values later on (see Sec. IV). Let us just say for the moment that none of these parameters is sharply fixed by the σ_- and P_- data alone. For example, comparable fits can be obtained keeping s_0 and s_1 equal, or fixing *a priori* α'_P to any reasonable value. But the Δ_δ cannot be completely studied without referring to the other data, since it turns out that the polarization P_+ in the dip region, and the cross section σ_0, are fairly sensitive to the detailed structure of $M^{3/2}_{+\pm}$, in particular to the ratio of the helicity-flip to the helicity-nonflip $\frac{3}{2}$ amplitude. Note that since in our final result the helicity-flip contribution $|M^{3/2}_{++}|^2$ to σ^- amounts to about $\frac{1}{3}$ of the helicity-nonflip contribution, the argument of Ref. 1 as to the occurrence of a zero at $u \simeq -0.15$ in $M^{3/2}_{+-}$ (see Sec. II A) does not hold in our model.

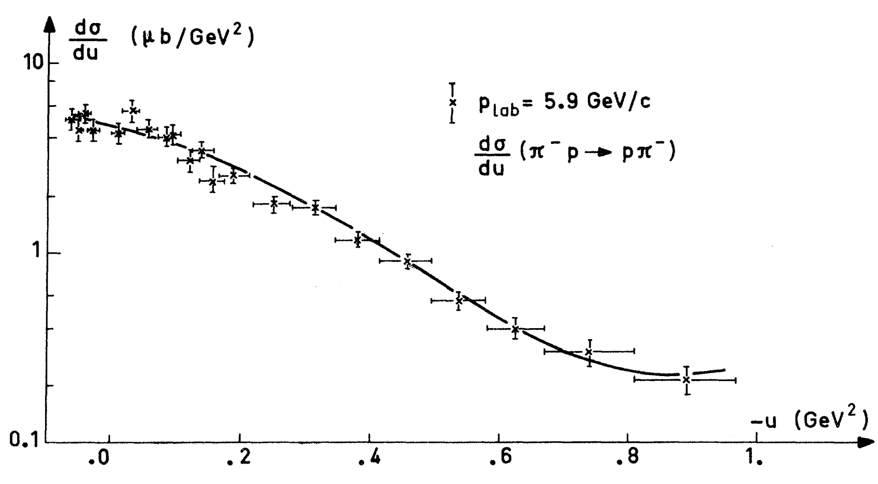

FIG. 4. $d\sigma/du$ for $\pi^-p \to p\pi^-$ at 5.9 GeV/c from our model. Data from Ref. 3.

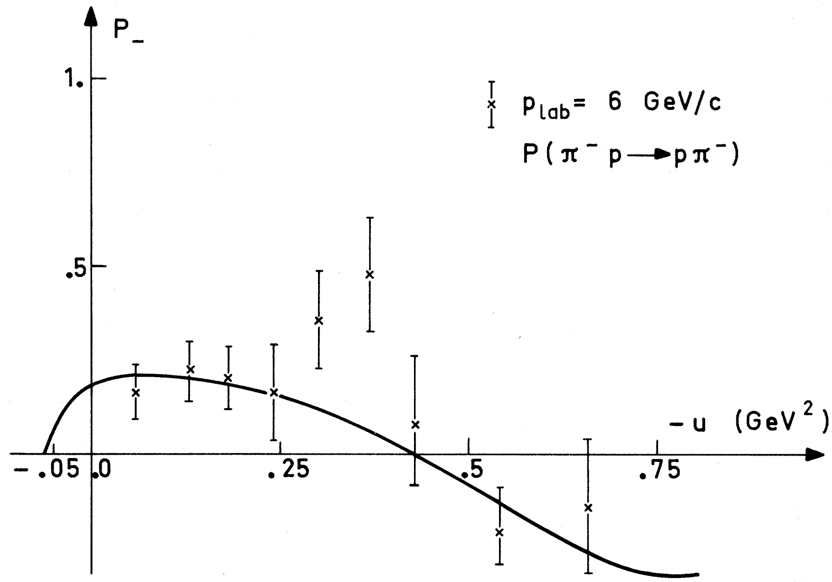

FIG. 5. Polarization in $\pi^- p \to p\pi^-$ at 6 GeV/c from our model. Data from Ref. 7.

C. The $M^{1/2}$ Amplitudes

Once we are given a model for the $M^{3/2}$ amplitudes, we can proceed to the extraction of the $M^{1/2}_{++}$ amplitude from σ_+, σ_0, and P_+, for a given $M^{1/2}_{+-}$ amplitude, first chosen to be of the Regge-pole type with WSN zero at $\alpha_N = -\frac{1}{2}$, as explained previously. The striking feature of the result is that, for any reasonable scale parameter and N_α trajectory, $\mathrm{Re}M^{1/2}_{++}$ happens to be large and positive beyond the dip, whereas $\mathrm{Im}M^{1/2}_{++}$ remains small.[8] In Fig. 6, for $u \simeq -0.5$, we show the positions of our amplitudes on a qualitative phase diagram. The position of $M^{1/2}_{+-}$ comes out from the WSN-zero mechanism which has been assumed explicitly, the position of $M^{3/2}_{+\pm}$ from the fact that their imaginary parts both vanish near $\alpha_\Delta = -\frac{1}{2}$, accounting for the vanishing of the P_- polarization. Finally, the position of $M^{1/2}_{++}$ in the first quadrant for $|u| \gtrsim 0.5$ can be qualitatively understood as follows: First, since the P_+ polarization is large, there has to be a sizable phase difference between $M^{1/2}_{++}$ and $M^{1/2}_{+-}$, and moreover, since it is negative, $M^{1/2}_{++}$ must be to the right of $M^{1/2}_{+-}$. This position is furthermore confirmed by the large positive value of $\mathrm{Re}I_1 I_3^*$ (Fig. 2), which is easily obtained by a dominant positive interference between $M^{1/2}_{++}$ and $M^{3/2}_{++}$.

D. Model for $M^{1/2}$ Amplitudes

The main problem we have to deal with is of course that of an interpretation of the large positive value of $\mathrm{Re}M^{1/2}_{++}$ after the dip [remember that the WSN-zero mechanism would yield a negative value, of the same order of magnitude as the (positive) imaginary part]. The fact that this feature survives at higher energies, as shown by the existence of a still substantially positive value of $\mathrm{Re}I_1 I_3^*$ at 9.85 GeV/c (Fig. 2), seems to be in disagreement with a possible interference effect with "nonasymptotic" contributions (resonances, or maybe t-channel Regge poles). On the contrary, an almost purely real $M^{1/2}_{++}$ with a simple zero can be easily obtained on a wide range of energies by assuming exchange degeneracy between N_α and N_γ trajectories, degeneracy of the couplings, and strong absorption in $M^{1/2}_{++}$. We then proceed to construct a Regge model with absorption in which N_α and N_γ trajectories are degenerate, and the ratios $\gamma_\pm$ of the N_γ to N_α couplings left free. γ_+ is expected to be substantially different from zero and we allow for an important cut enhancement factor λ_+ on $M^{1/2}_{++}$, whereas as for the two $M^{3/2}$ amplitudes, this factor is taken to be 1 for $M^{1/2}_{+-}$. The amplitudes read as follows:

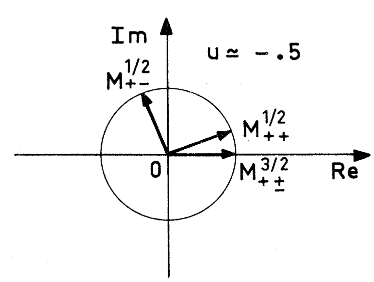

FIG. 6. Phase diagram at $u \simeq -0.5$ GeV^2 for $M^{1/2}_{+\pm}$ and $M^{3/2}_{+\pm}$ amplitudes.

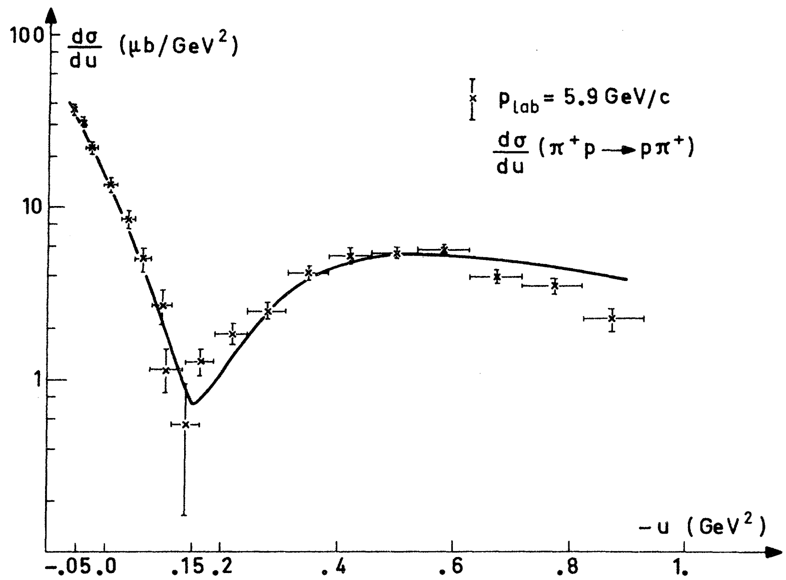

FIG. 7. $d\sigma/du$ for $\pi^+p\to p\pi^+$ at 5.9 GeV/c from our model. Data from Ref. 3.

$$M^{1/2}_{++} = N^{\alpha\gamma}_{++} + \lambda_+ D_{++}\,,$$
$$M^{1/2}_{+-} = N^{\alpha\gamma}_{+-} + D_{+-}\,, \qquad (6)$$

where $D_{+\pm}$ are again the absorptive corrections, calculated with the Pomeranchuk term of Eq. (3) with parameters of Eq. (5), and

$$N^{\alpha\gamma}_{++} = \frac{2\pi}{m} N_0[1+\gamma_+ + (1-\gamma_+)e^{-i\pi(\alpha_N-1/2)}] \times \Gamma(\tfrac{1}{2}-\alpha_N)\left(\frac{s}{s_0}\right)^{\alpha_N}\frac{(-u+u_0)^{1/2}}{m}\,, \qquad (7)$$

$$N^{\alpha\gamma}_{+-} = 2\pi N_1[1+\gamma_- + (1-\gamma_-)e^{-i\pi(\alpha_N-1/2)}] \times \Gamma(\tfrac{1}{2}-\alpha_N)\left(\frac{s}{s_1}\right)^{\alpha_N}\,. \qquad (8)$$

α_N is the N_α, N_γ trajectory with intercept α_{0N} and slope $\alpha_N' = (0.5-\alpha_{0N})/m^2$. s_0 and s_1 are scale parameters, and the N_i are obtained by pole extrapolation:

$$N_0 = -3m\sqrt{s_0}\;\alpha_N'\, g^2/4\pi\,,$$
$$N_1 = -3m\sqrt{s_1}\;\alpha_N'\, g^2/4\pi\,, \qquad (9)$$

with a πN coupling constant $g^2/4\pi = 14.8$.

A fit to the σ_+, σ_0, and P_+ data is then performed in two steps. First, for fixed Δ_δ and Pomeranchuk parameters, as previously determined by σ_- and P_-, we adjust $\gamma_\pm$, s_0, s_1, α_{0N}, and λ_+. Then an over-all fit to all data is done, which leads to a slight readjustment of the Δ_δ and Pomeranchuk parameters. As already stated, this is essentially due to the sensitivity of the $M^{1/2}$, $M^{3/2}$ interference to the helicity-flip to helicity-nonflip ratio of $M^{3/2}$ amplitudes, a ratio which is badly determined by σ_- and P_- data alone. Our final result for σ_+, P_+, and σ_0 is shown in Figs. 7 (see Ref. 3), 8 (see Ref. 4), and 9 (see Ref. 7). The values of our parameters are

$$s_0 = 0.609\ \text{GeV}^2,\quad s_1 = 0.262\ \text{GeV}^2,\quad \alpha_{0N} = -0.315,$$
$$\gamma_+ = 0.508,\quad \gamma_- = 0.183,\quad \lambda_+ = 2.78\,. \qquad (10)$$

The rather high value of the λ_+ enhancement factor is to be attributed to the necessity of getting a zero in $\text{Re}M^{1/2}_{++}$ near the dip both for having a small amplitude in this region and a large positive real part beyond. We think this parameter is the only one which has a rather unusual value.

IV. CONCLUSION AND DISCUSSION OF OUR RESULTS

Let us first sum up the general lines of our reasoning. Our main assumptions concerning extraction and interpretation of the πN backward amplitudes are the following ones:

(i) The helicity-nonflip amplitude $M^{1/2}_{+-}$ has something to do with N_α exchange. The arguments in favor of this assumption are essentially the energy dependence of the backward peak in $\pi^+p\to p\pi^+$ and the position of the dip around $\alpha_N = -\frac{1}{2}$.

(ii) Both $I_u = \frac{3}{2}$ amplitudes are dominated by Δ_δ exchange (energy dependence of the σ_- cross section), and they can be actually described in the framework of a Regge-pole model with absorption (shape of the P_- polarization).

On the basis of these two assumptions, we then infer from the data

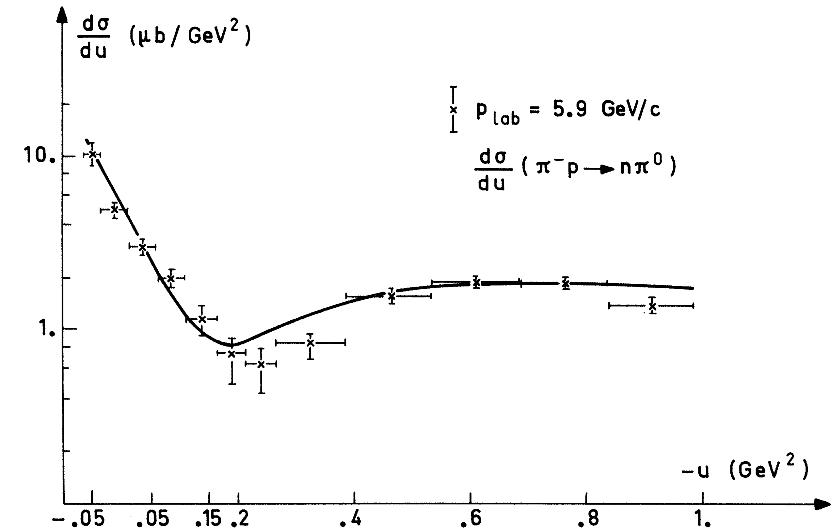

FIG. 8. $d\sigma/du$ for $\pi^- p \to n\pi^0$ at 5.9 GeV/c from our model. Data from Ref. 4.

(a) that $\mathrm{Im}M^{3/2}_{+-}$ does not vanish at the WS point $\alpha_\Delta = \frac{1}{2}$, whereas $\mathrm{Im}M^{3/2}_{++}$ does, and

(b) that after the dip in the σ_+ cross section, $\mathrm{Re}M^{1/2}_{++}$ is large and positive, in contrast with what is obtained in an N_α exchange model with WSN zero at $\alpha_N = -\frac{1}{2}$.

Finally we present a model for $I_u = \frac{3}{2}$ and $\frac{1}{2}$ amplitudes, using Δ_δ, and $N_\alpha N_\gamma$ exchange, respectively, taking care of the extrapolations to the Δ_{33} and nucleon poles. Absorption is quite normal $(\lambda = 1)$ for both $I_u = \frac{3}{2}$ amplitudes and for $M^{1/2}_{+-}$. The only point which would require some interpretation is the rather high value of the cut enhancement factor λ_+ in $M^{1/2}_{++}$. The model, with its fitted parameters at 6 GeV/c, is in very good agreement with all known data at that energy.

In Figs. 10(a) (see Ref. 3), 10(b) (see Ref. 3), and 10(c) (see Ref. 4), we compare the predictions of our model, with its parameters as determined at 6 GeV/c, with the data at 10 GeV/c on differential cross sections. The agreement is quite reasonable. The discrepancy around the dip

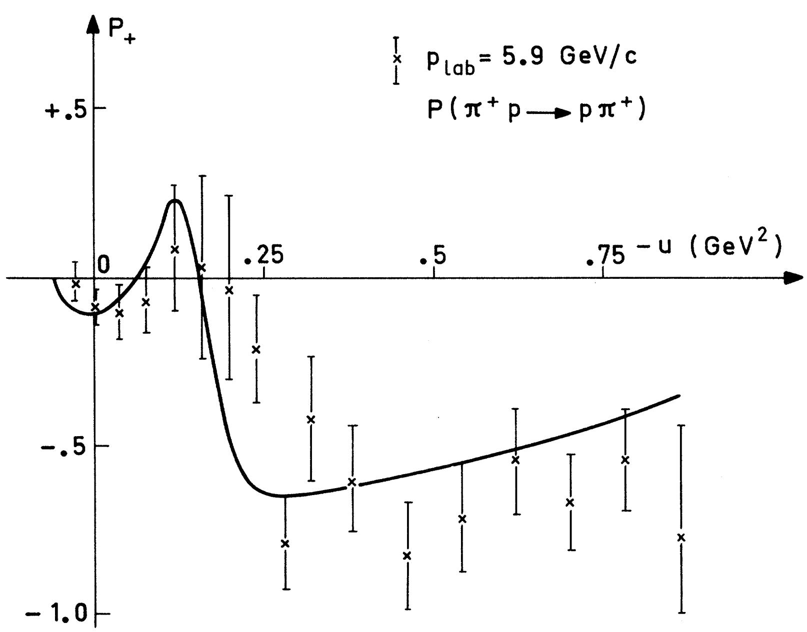

FIG. 9. Polarization for $\pi^+ p \to p\pi^+$ at 5.9 GeV/c from our model. Data from Ref. 7.

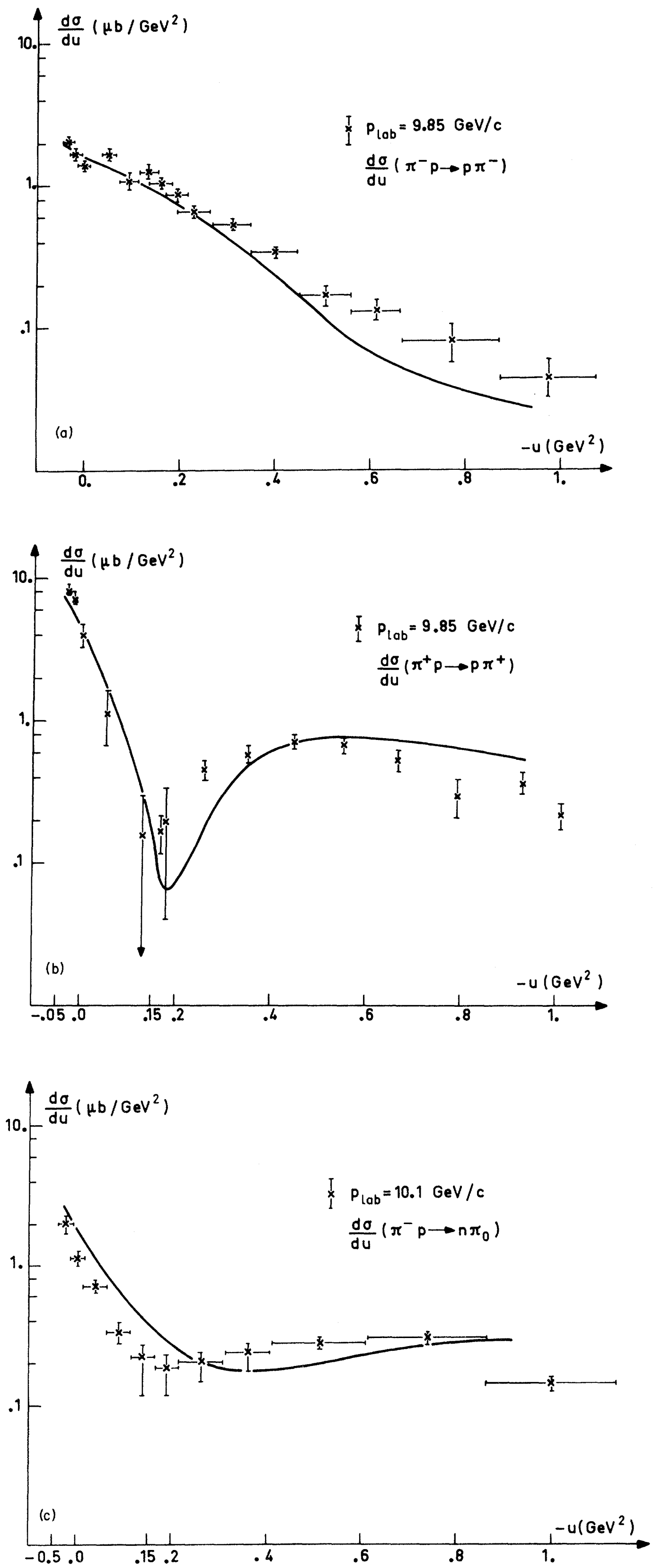

FIG. 10 Comparison of the prediction of our model with the data on differential cross sections (a) $\pi^- p \to p\pi^-$ at 9.85 GeV/c—data from Ref. 3; (b) $\pi^+ p \to p\pi^+$ at 9.85 GeV/c—data from Ref. 3; (c) $\pi^- p \to n\pi^0$ at 10.1 GeV/c—data from Ref. 4.

region in $\pi^- p \to n\pi^0$ is related to the fact that our model does not reproduce well the energy dependence of the quantity $\mathrm{Re}(I_1 I_3^*)/(\sigma_- \sigma_N)^{1/2}$. We point out that there is some evidence that around 6 GeV/c one cannot consider that the backward πN amplitudes have completely reached their asymptotic behavior: Comparing the 5.9-GeV/c data of Ref. 3 with the 5.2- and 7-GeV/c data of Ref. 9 on $\pi^+ p \to p\pi^+$, one observes that, before the dip, σ_+ at 5.9 GeV/c practically coincides with σ_+ at 5.2 GeV/c, whereas after the dip it is close to σ_+ at 7 GeV/c. [See also the 180° σ_+ compilation in Ref. 10 which indicates that the 5.9-GeV/c point could lie in a still oscillating region.] Therefore it is possible that the parameters at 6 GeV/c still reflect some nonasymptotic behavior. Unfortunately, 6 GeV/c is the only momentum at which polarization data exist. At any rate, we believe that the predictions and inclusions we have drawn from the 6-GeV/c data remain generally valid, despite the discrepancies at 10 GeV/c which could be reduced by small variations of the parameters.

We want to emphasize that the present model predicts that the general behavior of the πN backward amplitudes stays stable on a wide range of energies. In particular, we predict the $\pi^+ p$ backward polarization P_+ remains significantly negative at higher energies, at least outside the dip region. As far as the dip region is concerned, we cannot conclude in a definite way. If both $M^{1/2}$ amplitudes were strictly zero at $\alpha_N = -\frac{1}{2}$, of course one would obtain $P_+ = P_-$ at all energies. If they are not, P_+ is very sensitive to the N, Δ interference term, and then to their respective helicity-flip to helicity-nonflip ratios. For example, a substantially positive spike in P_+ cannot be completely excluded, which could result from a dominant $\mathrm{Im}(\Delta_{++} N_{+-}^*)$ contribution. This could occur in particular if Δ_δ exchange was predominantly a helicity-flip amplitude, as obtained in a strong cut model where Δ_{+-} would vanish at $u \simeq -0.2$ to 0.3. Note that in such a case, $\mathrm{Re} M_{+-}^{3/2}$ vanishes at a smaller $|u|$ than $\mathrm{Im} M_{+-}^{3/2}$. Then, one can distinguish between weak and strong cut models for $M_{+-}^{3/2}$ by an R measurement in the transfer region where P_- vanishes ($u \simeq -0.5$), since accordingly $\mathrm{Re} M_{++}^{3/2} M_{+-}^{3/2*}$ will be, respectively, positive or negative. Our point of view is that the 6-GeV/c data for P_+ and σ_- are not in favor of strong absorption in $M_{+-}^{3/2}$.

As a conclusion, we have presented a model which is in fair agreement with high-energy πN backward data, which incorporates significant N_γ contributions to $M^{1/2}$ amplitudes, thus allowing for the absence of dips in other reactions dominated by $I_u = \frac{1}{2}$ baryon exchange, and which predicts polarizations in $\pi^+ p$ and $\pi^- p$ to be slowly varying functions of energy, but may be, for P_+, in the dip region.[11]

APPENDIX

1. Kinematics

At large c.m. energy squared, the two helicity amplitudes M_{++} and M_{+-} are related to the invariant functions $A(s,u)$ and $B(s,u)$ by

$$M_{++} = (-u + u_0)^{1/2} B\sqrt{s}\ ,$$

$$M_{+-} = (A + mB)\sqrt{s}\ ,$$

where m is the nucleon mass. Our normalizations are such that the differential cross section is given by

$$\frac{d\sigma}{du} = \frac{1}{64\pi s q^2}\left(|M_{++}|^2 + |M_{+-}|^2\right),$$

and the polarization by

$$P = \frac{2\mathrm{Im}(M_{++} M_{+-}^*)}{|M_{++}|^2 + |M_{+-}|^2}\ .$$

The u-channel isospin decomposition is

$$F(\pi^- p \to p\pi^-) = M^{3/2}\ ,$$

$$F(\pi^+ p \to p\pi^+) = \tfrac{2}{3} M^{1/2} + \tfrac{1}{3} M^{3/2}\ ,$$

$$F(\pi^- p \to n\pi^0) = \tfrac{1}{3}\sqrt{2}\,(M^{1/2} - M^{3/2})\ .$$

2. Formulas for Absorption Calculations

In calculating the Pomeranchuk–Regge-pole–cut contributions to $M_{+\pm}$, we have used the following approximations to the usual convolution integrals:

$$I_{00} \equiv e^{a(u-u_0)} \circledast e^{bt} = \frac{i}{8\pi s(a+b)} \exp\left[\frac{ab}{a+b}(u-u_0)\right],$$

$$J_{00} \equiv [(-u+u_0)^{1/2} e^{a(u-u_0)}] \circledast e^{bt} = \frac{b}{a+b} I_{00}(-u+u_0)^{1/2}\ ,$$

$$I_{n0} \equiv [(u-u_0)^n e^{a(u-u_0)}] \circledast e^{bt} = \frac{d}{da^n} I_{00}\ ,$$

$$J_{n0} \equiv [(u-u_0)^n(-u+u_0)^{1/2} e^{a(u-u_0)}] \circledast e^{bt} = \frac{d}{da^n} J_{00}\ .$$

In the cut calculations, we have neglected the variation in u of the Γ functions present in the Regge-pole parametrization, so that all terms appear as products of exponentials and polynomials in u.

[1]V. Barger and M. G. Olsson, University of Wisconsin report, 1971 (unpublished). Similar conclusions were reached by J. K. Storrow and G. A. Winbow, in *Proceedings of the Fourth International Conference on High Energy Collisions, Oxford, 1972,* edited by J. R. Smith (Rutherford High Energy Laboratory, Chilton, Didcot, Berkshire, England, 1972).

[2]We denote by the indices +, −, 0, differential cross sections $\sigma_{+,-,0}$ and polarizations $P_{+,-,0}$ of the reactions $\pi^+ p \to p\pi^+$, $\pi^- p \to p\pi^-$, and $\pi^- p \to n\pi^0$, respectively.

[3]D. P. Owen, F. C. Peterson, J. Orear, A. L. Read, D. G. Ryan, D. H. White, A. Ashmore, C. J. S. Damerell, W. R. Frisken, and R. Rubinstein, Phys. Rev. 181, 1794 (1969).

[4]J. P. Boright, D. R. Bowen, D. E. Groom, J. Orear, D. P. Owen, A. J. Pawlicki, and D. H. White, Phys. Lett. 33B, 615 (1970).

[5]$u = u_0$ corresponds to $\cos\theta_{\text{c.m.}} = -1$.

[6]In terms of the invariant amplitudes A and B, the fact that $M^{3/2}_{++}$ has a factor $\alpha_\Delta - \frac{1}{2}$ whereas $M^{3/2}_{+-}$ has not means that only B contains such a factor (see in the Appendix the relations between M_{++} and A, B).

[7]H. Aoi, N. Booth, C. Caverzasio, L. Dick, A. Gonidec, A. Janout, K. Kuroda, A. Michalowicz, M. Poulet, D. Sillou, C. Spencer, and W. Williams, Phys. Lett. 35B, 90 (1971); report (unpublished).

[8]If, at small $|u|$, $\text{Re}M^{1/2}_{++}$ had the sign given by a Regge N_α pole, our result implies that $\text{Re}M^{1/2}_{++}$ has a simple zero in the dip region, thus ruling out WSN zero.

[9]W. F. Baker, K. Berkelman, P. J. Carlson, G. P. Fisher, P. Fleury, D. Hartill, R. Kalbach, A. Lundby, S. Muklin, R. Nierhaus, K. P. Pretzl, and J. Woulds, Nucl. Phys. B25, 385 (1971).

[10]See, for example, M. Derrick, in CERN report No. 68-7, 1968 (unpublished), Vol. I.

[11]While finishing this work, we received a report [CERN-TH 1490, 1972 (unpublished)] from C. Ferro Fontan on πN backward scattering at 6 GeV/c. The author arrives at the same conclusion as we do concerning $\text{Re}M^{1/2}_{+-}$. He also considers $M^{1/2}_{+-}$ and $M^{3/2}_{++}$ as Regge-pole amplitudes, without specifying, however, the parametrization used, and in particular, mentions only briefly the question of the presence of a $\alpha_\Delta - \frac{1}{2}$ factor in the Δ_δ residue. We disagree with his $\text{Im}M^{1/2}_{++}$ having a zero at $u = -0.6$ rather than at $\alpha_N = -\frac{1}{2}$, which is in conflict with our interpretation, and that of Ref. 1, of point (a) in Sec. II A. We also arrive at a somewhat different conclusion for $M^{3/2}_{+-}$. Moreover Ferro Fontan does not present any model for the $M^{1/2}_{++}$ and $M^{3/2}_{+-}$ amplitudes he extracts from the data.

PHYSICAL REVIEW D VOLUME 8, NUMBER 1 1 JULY 1973

Multichannel-Dispersion-Theory Calculation of Photopion Production

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We have performed a multichannel-dispersion-relation calculation of low-energy ($E_\gamma \lesssim 450$ MeV) pion photoproduction. We are able to fit the present γp data along with the less-well-known $\gamma n \to \pi^- p$ data without the introduction of an $I = 2$ electromagnetic current. Predictions for the reaction $\gamma n \to \pi^0 n$ are made. Parameters are introduced to describe the photoproduction Born terms for the inelastic hadronic channels. We find that through the rescattering integrals, the inelastic effects strongly influence the $\gamma N \to \pi N$ amplitudes even at low energy (in particular, in the E_{0+} and M_{1-} multipoles).

I. INTRODUCTION

Considerable interest in low-energy pion-photoproduction experiments and phenomenology has been recently stimulated by (a) the suggestion that an $I = 2$ electromagnetic current might exist[1] and (b) the possibility that time-reversal invariance might be violated in electromagnetic interactions.[2] If one can obtain as accurate measurements of reactions

$$\gamma + n \to \pi^0 + n\,, \tag{1a}$$

$$\gamma + n \to \pi^- + p \tag{1b}$$

as have been done for the reactions

$$\gamma + p \to \pi^0 + p\,, \tag{2a}$$

$$\gamma + p \to \pi^+ + n\,, \tag{2b}$$