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"Naive" Quark-Pair-Creation Model of Strong-Interaction Vertices

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We explicitly formulate the vacuum quark-pair-creation model (QPC) of strong-interaction vertices (Micu, Carlitz and Kislinger) in terms of the harmonic-oscillator spatial-SU(6) wave functions and an explicit vacuum quark-antiquark–pair transition matrix (both displaying the quark internal momenta). The coupling constants are expressed as functions of masses and the oscillator radius. The structure of the formulas is in agreement with the expressions coming from VMD (vector-meson dominance) and PCAC (partial conservation of axial-vector current), and the quark-model calculation of leptonic decays. We carefully investigate the relation of this QPC model with the additive quark model with elementary meson emission, which is known to explain most of the hadronic decays. We show that we recover this model in a given limit. It is shown that in this limit, a term $\vec{\sigma}(i)\cdot(\vec{k}_\pi-\vec{k}_i)$ (depending on the internal quark momentum) appears in place of the usual $\vec{\sigma}(i)\cdot\vec{k}_\pi$ term; the additional contribution is similar to the well-known "recoil" term of Mitra and Ross. The main limits of the model lie (i) in the presence of a phenomenological pair-creation constant and (ii) in the nonrelativistic character of the treatment. A critical test of our model is provided by prediction of the decay-products polarization. We find a striking agreement with experiment for the crucial A_1 and B decays. We make a comparison with the parameter-dependent model of Colglazier and Rosner.

I. INTRODUCTION

In the simple additive quark model of strong vertices, one is able to relate the hadron coupling constants to the coupling constant of elementary meson emission by quarks. Thus, one is left with some quark-quark–meson coupling constants, e.g., $g_{qq\pi}$ and $f_{qq\rho}$ which remain theoretically undetermined. Nevertheless, π and ρ are themselves composed of quarks. Then, this simple additivity leads to an asymmetric treatment of the two outgoing mesons in three-mesons vertices, although we expect both to play the same role. This is a well-known problem of this additivity scheme. Moreover, since mesons are composite systems of a $q\bar{q}$ pair, one should be able to relate the coupling constants $g_{qq\pi}$ and $f_{qq\rho}$ to the $q\bar{q}$ structure of π and ρ, namely, to their quark wave function.

We present two approaches to this problem. In Sec. II, using VMD (vector-meson dominance) and PCAC (partially conserved axial-vector current), we relate the strong-interaction vertices to the leptonic decay of mesons, which is interpreted as quark-antiquark annihilation, and thus reveals in a very straightforward way the quark structure of mesons. We finally express the strong coupling constants in terms of spatial quark wave functions of mesons.

In Sec. III, instead of taking information from leptonic interactions, we consider a quark model of strong-interaction vertices, namely, the vacuum-pair–creation models of Micu and of Carlitz and Kislinger[1] [QPC (quark-pair-creation) model]. However, in these models, the internal motion of quarks is either omitted (by Carlitz and Kislinger who are just reproducing $SU(6)_W$ predictions) or treated through fitting of spatial matrix elements (Micu). Here, we explicitly express the matrix elements in terms of the harmonic-oscillator wave functions and an explicit quark-pair-creation T-matrix element, depending on internal quark momenta. With this QPC model, we obtain formulas rather similar to those of Sec. II, namely, for the dimensionless coupling constants $G\sim m^{3/2}R^{3/2}$, where m is some hadron mass and R is some wave-function radius.

We feel that there is a deep relation between the two approaches, although we are not able to formulate it at present owing to the nonrelativistic character of our quark model and the lack of crossing.

In Sec. IV, we investigate the limiting procedure in which the usual additivity calculation is obtained from the QPC model. We show that simple additivity is recovered when the emitted meson becomes pointlike. However we do not recover, e.g., for π emission, the simple $\vec{\sigma}(i)\cdot\vec{k}_\pi$ interaction, but rather $\vec{\sigma}(i)\cdot(\vec{k}_\pi-\vec{k}_i)$, where $\vec{k}_i$ is the initial momentum of the interacting quark, much like the phenomenological interaction of Mitra and Ross.[2]

We then explain the ρ and ω polarization in $A_1\to\rho\pi$ and $B\to\omega\pi$ without any free parameters [$SU(6)_W$ predictions are wrong]; the results depend only on the oscillator radius, which has been fixed once and for all by the Regge slope and other phenomena.[3] The theoretical interest of these decays, which badly contradict $SU(6)_W$, has been emphasized by Colglazier and Rosner.[4] In their relativistic pair-creation model there is no unique treatment of all the hadron vertices, and the explanation of 1^+-mesons decay polarization, e.g., $A_1 \to \rho\pi$ and $B \to \omega\pi$, needs a specific parameter. This polarization test is, in our opinion, a very critical test of the QPC model.

II. RELATION BETWEEN STRONG-COUPLING CONSTANTS AND QUARK-MODEL WAVE FUNCTIONS COMING FROM VMD AND PCAC

VMD and PCAC provide relations between the weak or electromagnetic leptonic decays of pseudoscalar or vector mesons and their strong coupling to hadrons.[5] On the other hand, Van Royen and Weisskopf derive the amplitudes for these weak and electromagnetic leptonic decays by interpreting them as quark-antiquark annihilation.[6] The transition amplitude is proportional to the wave function at the origin, that is, to the amplitude for the quark and antiquark being at the same point. It is then possible to relate in a straightforward way the strong-coupling constants to the quark model nonrelativistic wave functions. This has been suggested by Feynman, Kislinger, and Ravndal.[7]

A. Decay Constants and Quark-Model Wave Functions

Here we only quote well-known calculations of the amplitudes for mesons decaying into leptons:

$$\begin{aligned} &\rho^0 \to e^+e^- , \\ &\omega \to e^+e^- , \\ &\pi^- \to \mu^-\bar{\nu} , \\ &K^- \to \mu^-\bar{\nu} . \end{aligned} \tag{2.1}$$

These transitions are described by the following phenomenological Lagrangians:

$$\mathcal{L} = -\frac{e}{2f_\rho}\,\rho_{\mu\nu}F_{\mu\nu} \tag{2.2}$$

(for ω, we replace f_ρ by f_ω and $\rho_{\mu\nu}$ by $\omega_{\mu\nu}$), and

$$\mathcal{L} = iG_V\cos\theta\,(\sqrt{2}\,C_\pi)\partial_\mu\pi^-\bar{l}\gamma_\mu(1+\gamma_5)\nu \tag{2.3}$$

(for K, replace C_π by C_K, π by K, and $\cos\theta$ by $\sin\theta$). In (2.3) $G_V = G/\sqrt{2}$. (For example, cf. Sakurai.[5]) The resulting widths are

$$\Gamma(\rho^0 \to e^+e^-) = \frac{4\pi\alpha^2}{3}\,\frac{m_\rho}{f_\rho^{\,2}} , \tag{2.4}$$

$$\Gamma(\pi^- \to \mu^-\bar{\nu}) = \frac{G_V^{\,2}}{4\pi}\,(\sqrt{2}\,C_\pi)^2 m_\pi \cos^2\theta\, m_\mu^{\,2}\left(1-\frac{m_\mu^{\,2}}{m_\pi^{\,2}}\right), \tag{2.5}$$

with the obvious analogs for ω and K.

On the other hand, Van Royen and Weisskopf derived the following matrix element[6]:

$$A \simeq \psi(0)(2\pi)^3 \sum_{r,s} \phi(r,s)\langle 0|\mathcal{H}_{\rm int}\, a_r^*(0) b_s^*(0)|0\rangle , \tag{2.6}$$

where $\psi(0)$ is the meson wave function at the origin, a^* and b^* are the creation operators of a quark and an antiquark, and $\mathcal{H}_{\rm int}$ describes the q-$\bar{q}$ annihilation process into a photon or a lepton pair (see Figs. 1 and 2). The corresponding widths are

$$\Gamma(\rho^0 \to e^+e^-) = \frac{8\pi\alpha^2}{3}\,\frac{1}{m_\rho^{\,2}}\,|\psi_\rho(0)|^2 , \tag{2.7}$$

$$\Gamma(\pi^- \to \mu^-\bar{\nu}) = \frac{1}{\pi}\,|\psi_\pi(0)|^2\, G_A'^2 \cos^2\theta\, m_\mu^{\,2}\left(1-\frac{m_\mu^{\,2}}{m_\pi^{\,2}}\right)^2 , \tag{2.8}$$

and the analogous formulas for ω and K.

In (2.8), G_A' is the quark axial-vector weak-coupling constant, fitted to give $G_A/G_V = 1.2$ through the additivity relations in neutron β decay:

$$G_A' = \tfrac{3}{5}G_A , \quad G_V' = G_V , \quad G_A' = 0.7G_V .$$

The comparison between the two expressions for the widths yields

$$f_\rho = \frac{m_\rho^{3/2}}{\sqrt{2}\,|\psi_\rho(0)|} , \quad f_\omega = \frac{m_\omega^{3/2}}{3\sqrt{2}\,|\psi_\omega(0)|} , \tag{2.9}$$

$$C_\pi = \frac{3\sqrt{2}}{5}\left(\frac{G_A}{G_V}\right)\frac{|\psi_\pi(0)|}{m_\pi^{1/2}} , \quad C_K = \frac{3\sqrt{2}}{5}\left(\frac{G_A}{G_V}\right)\frac{|\psi_K(0)|}{m_K^{1/2}} . \tag{2.10}$$

Comparing these results with experiment, Van Royen and Weisskopf derived the well-known empirical formulas ("Van Royen and Weisskopf paradox"):

$$|\psi_\pi(0)|^2 \simeq \tfrac{1}{2}m_\pi^{\,3} , \tag{2.11}$$

$$\frac{m_\pi^{1/2}}{|\psi_\pi(0)|} \simeq \frac{m_K^{1/2}}{|\psi_K(0)|} \simeq \frac{m_\rho^{1/2}}{|\psi_\rho(0)|} \simeq \frac{m_\omega^{1/2}}{|\psi_\omega(0)|} . \tag{2.12}$$

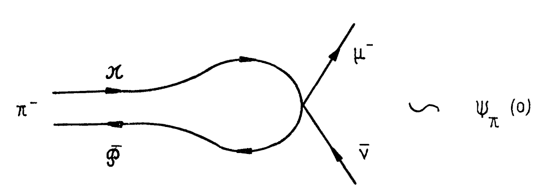

FIG. 1. The model of Van Royen and Weisskopf for π weak decay.

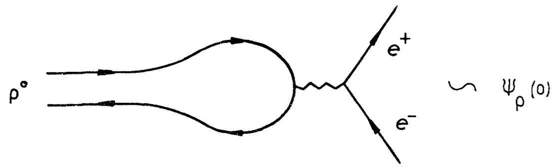

FIG. 2. The model of Van Royen and Weisskopf for ρ electromagnetic decay.

Relations (2.12) follow from the experimental values given by Sakurai[5]:

$$C_\pi = 94 \text{ MeV}, \quad \frac{C_K}{C_\pi} \simeq 1.3, \tag{2.13}$$

assuming $\theta_A = \theta_V$, and from the Orsay data:

$$\frac{f_\rho^2}{4\pi} = 2.10 \pm 0.11, \quad \frac{f_\omega^2}{4\pi} = 14.8 \pm 2.8. \tag{2.14}$$

In a previous paper[3] using a semirelativistic quark model, we showed that the Van Royen and Weisskopf $|\psi_\rho(0)|$ and $|\psi_\omega(0)|$ are compatible with a meson Regge slope 1 GeV^{-2}, but we did not explain the breaking among $|\psi_\pi(0)|$, $|\psi_K(0)|$, $|\psi_V(0)|$.

B. Strong-Coupling Constants Via VMD and PCAC

Let us write the current-field identity of the isovector part of the electromagnetic current:

$$j_\mu^\alpha = \frac{m_\rho^2}{f_\rho} \rho_\mu^\alpha, \tag{2.15}$$

which yields the so-called universality relation

$$\begin{aligned} f_{\rho NN} &= f_{\rho\pi\pi} \\ &= f_{\rho\rho\rho} \\ &= f_\rho, \end{aligned} \tag{2.16}$$

where the strong coupling constants of the ρ are defined through the Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{int}} = [&if_{\rho NN} \bar{N} \gamma_\mu (\tfrac{1}{2}\vec{\tau}) N - f_{\rho\pi\pi} \vec{\pi} \times \partial_\mu \vec{\pi} \\ &- f_{\rho\rho\rho} \vec{\rho}_\nu \times \partial_\mu \vec{\rho}_\nu] \cdot \vec{\rho}_\mu . \end{aligned} \tag{2.17}$$

More explicitly (2.16) through relation (2.9) means that the strong coupling constants of the ρ are built up through the quark binding in a way that we shall discuss later. From (2.16) and (2.9) we have

$$\begin{aligned} f_{\rho NN} &= f_{\rho\pi\pi} \\ &= f_{\rho\rho\rho} \\ &= \frac{m_\rho^{3/2}}{\sqrt{2}\,|\psi_\rho(0)|} . \end{aligned} \tag{2.18}$$

In a similar way, using PCAC[5]:

$$\partial_\mu j_{5\mu}^\alpha = C_\pi m_\pi^2 \pi^\alpha, \tag{2.19}$$

we get the Goldberger-Treiman relation:

$$C_\pi = \left(\frac{G_A}{G_V}\right) \frac{m_N}{G_{\pi NN}}, \tag{2.20}$$

where $G_{\pi NN}$ is defined through

$$\mathcal{L}_{\text{int}} = iG_{\pi NN} \bar{N} \gamma_5 \vec{\tau} N \cdot \vec{\pi}, \quad \frac{G_{\pi NN}^2}{4\pi} = 14.6 . \tag{2.21}$$

From (2.10) and (2.20) we find a relation similar to (2.18):

$$G_{\pi NN} = \frac{5}{3\sqrt{2}} m_N \frac{m_\pi^{1/2}}{|\psi_\pi(0)|} . \tag{2.22}$$

From relations (2.18) and (2.22) and the decomposition of the nucleon into quarks, we deduce the corresponding $f_{\rho qq}$ and $G_{\pi qq}$ coupling constants:

$$\begin{aligned} \frac{G_{\pi qq}}{2m_q} &= \frac{3}{5} \frac{G_{\pi NN}}{2m_N}, \\ f_{\rho qq} &= f_{\rho NN}, \end{aligned} \tag{2.23}$$

$$\frac{G_{\pi qq}}{2m_q} = \frac{m_\pi^{1/2}}{2\sqrt{2}\,|\psi_\pi(0)|}, \tag{2.24}$$

$$f_{\rho qq} = \frac{m_\rho^{1/2}}{\sqrt{2}\,|\psi_\rho(0)|}, \tag{2.25}$$

$G_{\pi qq}$ and $f_{\rho qq}$ being defined by Lagrangians similar to (2.17) and (2.21).

We have thus shown that the strong-coupling constants can be expressed in terms of (i) the masses and (ii) the nonrelativistic wave functions of mesons.[8,9] These formulas go one step further than the usual additive quark model, which provides only relations between coupling constants but not the absolute value.

One can now wonder how such formulas could be obtained by a strong-interaction quark model without the byway through leptonic decays.

III. QUARK-PAIR-CREATION MODEL OF STRONG-INTERACTION VERTICES

In Sec. II, we obtained an expression for strong-interaction coupling constants in terms of hadron masses and wave functions, i.e., quantities which can be completely derived from a Hamiltonian such as the well-known oscillator quark model. We also saw that within quark-model concepts, the coupling constants are fully explained. However, we are not completely satisfied; first, because the preceding deduction is rather involved and includes a detour through weak and electromagnetic currents, the role of which is not at all clear; second, because one has therefore no idea of the dynamical processes which take place in $\rho \to \pi\pi$, $\Delta \to N\pi$, etc. The aim of this section is to elucidate the pure strong-interaction mechanism which could lead to relations such as (2.18) and (2.22). Here, evidently, what we look for is a basic "pure quark" interaction in contrast with the elementary meson emission model which leaves us with undetermined constants such as $g_{qq\pi}$ and $f_{qq\rho}$ (see the Introduction). We propose as a candidate the *vacuum pair-creation model,* which is very simple and appealing.

The model is summed up in Figs. 3 and 4 for three-meson vertices and meson-baryon vertices, respectively. It is a rearrangement model with creation of a quark-antiquark pair; instead of being created from quark lines, the pair is created anywhere within the hadronic matter and since the two ingoing quarks suffer no change of their quantum numbers, the created pair evidently bears the quantum numbers of the vacuum. The model will be shown to give, among various interesting consequences, relations closely analogous to those of Sec. II. By comparison with simple additivity, it will be shown to explain some yet unsolved difficulties of previous quark-model calculations (Sec. IV).

What about other possible models? We discard local four-fermion interactions such as the one introduced by Marshak and Okubo[10], however interesting they may be, because they would yield, at least to first order, a coupling constant proportional to $\psi(0)$ (Fig. 5) (the argument goes the same as for the quark-lepton local couplings) in contrast with the formulas of Sec. II. Note, on the contrary, that the interaction of the quark-pair-creation model is nonlocal. Another type of model is the trilinear interaction between quarks and some fundamental meson field, such as the vector model of Fujii.[11] Such an interaction generates nonlocal forces between two quark lines and would not be incompatible with the relations of Sec. II. However, we are not able at present to discuss this model.

The general graphical form of Figs. 3 and 4 was suggested by Zweig[12] from the selection rules $\phi \not\to \rho\pi$ and $g_{\phi NN}=0$. (According to these rules, the quarks of the initial state seemed to be "conserved".) It was also much emphasized by Iizuka[13]

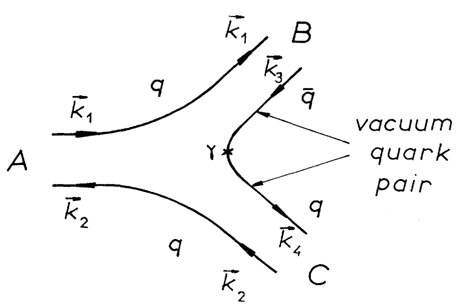

FIG. 3. The three-meson vertex in the QPC model.

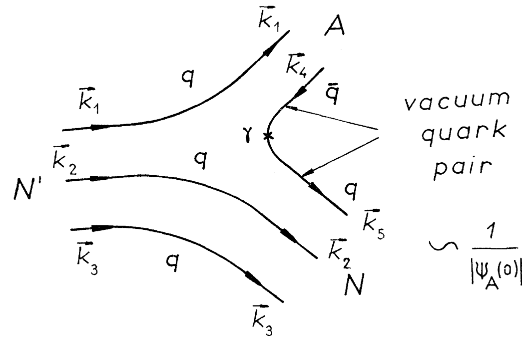

FIG. 4. The baryon-baryon-meson vertex in the QPC model.

as a consequence of the neutral-vector model and of pair-suppression hypothesis. (However, as we shall see, the pair-creation strength comes out to be very large.) It was extended to the four-point amplitude by Harari, by Rosner, and by Matsuoka *et al.*[14] in the interpretation of duality properties. Such diagrams are well known by now and have been used by many people either for selection rules or as a basis for calculations.

What we are doing here specifically is to consider these diagrams in a "naive" way, as reflecting a true quark-pair creation out of the hadronic vacuum, and to treat this creation process in the spirit of the usual additive quark model with spectator quarks, and tridimensional wave functions to describe the binding. This has been done also by Micu,[1] who, however, did not use an explicit set of wave functions, but rather fitted the various spatial integrals. He thus obtained a fit to decay widths but did not discuss the polarization phenomena which, in our opinion, are the most sensitive tests of this kind of model.

Carlitz and Kislinger[1] have also considered a real quark pair-creation process with $^3P_0$ structure, but they have neglected the internal momentum distributions, and, consequently, they recover no more than $SU(6)_W$ predictions.

Kitazoe and Teshima, Böhm and Gudehus, Kaiba *et al.*,[15] and Colglazier and Rosner[4] have considered "dual" quark diagrams with covariant treatment

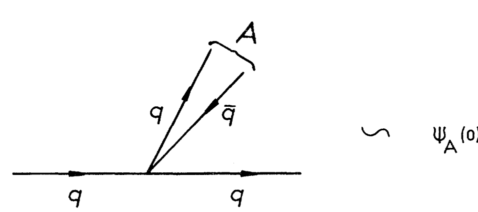

FIG. 5. The local four-quark interaction of Marshak and Okubo.

(Bethe-Salpeter amplitudes, etc.). This is quite a different spirit from the naive quark model. The first three papers do not explain the polarization phenomena.[15] Colglazier and Rosner can obtain a fit to the critical B and A_1 polarizations, but at the cost of superposing in an *ad hoc* manner various couplings which, in fact, are no longer described by the simple graphical structure of Figs. 4 and 5 (for further discussion see Sec. IV).

A. General Formula for the Matrix Element in the Present Model

We define the $\hat{T}$ matrix

$$S = 1 - 2\pi i \delta(E_f - E_i)\hat{T}\,, \tag{3.1}$$

and write (see Fig. 5 for illustration)

$$\hat{T} = I_1 \otimes I_2 \otimes I_3 \otimes \hat{T}_{\text{vac}}\,. \tag{3.2}$$

The I_i's are the identity matrices reflecting the quasi-free propagation of quarks in the same sense as for quark spectators in usual additivity. To get the matrix element, one must sandwich $\hat{T}$ between the hadron states.

Let us now discuss $\hat{T}_{\text{vac}}$. The $q\bar{q}$ pair must be created in a $^3P_0$ state[1] due to P- and C-parity conservation, $P = -(-1)^L$, $C = (-1)^{L+S}$, and it must be a SU(3) singlet. Thus we write

$$\langle \bar{q}_4 q_5 | \hat{T}_{\text{vac}} | 0 \rangle = \delta(\vec{k}_4 + \vec{k}_5)\gamma \sum_m \langle 1\,1\,m\; -m\,|00\rangle \times \mathcal{Y}_1^m(\vec{k}_4 - \vec{k}_5)\chi_1^{-m}\phi_0\,, \tag{3.3}$$

where χ_1^{-m} are the spin-triplet wave functions of two spins $\frac{1}{2}$ and

$$\phi_0 = \frac{1}{\sqrt{3}}(\mathcal{P}\bar{\mathcal{P}} + \mathcal{N}\bar{\mathcal{N}} + \lambda\bar{\lambda})\,.$$

γ is the dimensionless pair-creation constant in the hadronic matter; γ still bears a phenomenological character. We use it as a tool to investigate the structure of hadronic vertices. Its possible dependence on the particle involved in the vertex will be discussed later. As to the dynamical origin of pair creation, it will not be discussed in detail in this paper.

We now write the matrix element between hadron states

$$\langle BC | \hat{T} | A \rangle = \gamma \sum_m \langle 1\,1\,m\; -m\,|00\rangle \langle \Phi_B \Phi_C | \Phi_A \Phi_{\text{vac}}^{-m}\rangle \times I_m(A;B,C)\,, \tag{3.4}$$

where the Φ's are the SU(6) wave functions,

$$\Phi_{\text{vac}}^{-m} = \chi_1^{-m}\phi_0\,, \tag{3.5}$$

and $I_m(A;B,C)$ is the spatial integral. For mesons (see Fig. 4) it becomes

$$I_m(A;B,C) = \int d\vec{k}_1 d\vec{k}_2 d\vec{k}_3 d\vec{k}_4 \delta(\vec{k}_1 + \vec{k}_2 - \vec{k}_A)\delta(\vec{k}_2 + \vec{k}_3 - \vec{k}_B)\delta(\vec{k}_4 + \vec{k}_1 - \vec{k}_C)\delta(\vec{k}_3 + \vec{k}_4)\mathcal{Y}_1^m(\vec{k}_3 - \vec{k}_4) \times \tilde{\psi}_A(\vec{k}_1, \vec{k}_2)\tilde{\psi}_B(\vec{k}_2, \vec{k}_3)\tilde{\psi}_C(\vec{k}_4, \vec{k}_1)\,. \tag{3.6}$$

The $\tilde{\psi}$'s are the Fourier transforms of the meson spatial wave functions:

$$\tilde{\psi}_A(\vec{k}_1, \vec{k}_2) = \frac{1}{(2\pi)^{3/2}} \int d\vec{r}_1 d\vec{r}_2 \delta(\tfrac{1}{2}(\vec{r}_1 + \vec{r}_2))\psi_A(\vec{r}_1, \vec{r}_2)\exp[i(\vec{k}_1\cdot\vec{r}_1 + \vec{k}_2\cdot\vec{r}_2)]\,,$$

with

$$\int d\vec{r}_1 d\vec{r}_2\, \delta(\tfrac{1}{2}(\vec{r}_1 + \vec{r}_2))|\psi_A(\vec{r}_1, \vec{r}_2)|^2 = 1\,, \qquad \int d\vec{k}_1 d\vec{k}_2\, \delta(\vec{k}_1 + \vec{k}_2)|\tilde{\psi}_A(\vec{k}_1, \vec{k}_2)|^2 = 1\,.$$

For baryons, we have analogous formulas (see Fig. 5). Now eliminating the δ functions in the integral, and defining

$$\tilde{\psi}_A(\vec{k}_1 - \vec{k}_2) \equiv \tilde{\psi}_A(\vec{k}_1, \vec{k}_2),$$

we get for mesons in the center-of-mass system

$$I_m(A;B,C) = \tfrac{1}{8}\delta(\vec{k}_B + \vec{k}_C)\int d\vec{k}\,\mathcal{Y}_1^m(\vec{k}_B - \vec{k})\tilde{\psi}_A(\vec{k}_B + \vec{k})\tilde{\psi}_B(-\vec{k})\tilde{\psi}_C(\vec{k})\,. \tag{3.7}$$

For baryons, defining

$$\tilde{\psi}_N(\vec{k}_\rho, \vec{k}_\lambda) \equiv \tilde{\psi}_N(\vec{k}_1, \vec{k}_2, \vec{k}_3)\,,$$

where

$$\vec{k}_\rho = \frac{1}{\sqrt{2}}(\vec{k}_1 - \vec{k}_2)\,, \qquad \vec{k}_\lambda = \frac{1}{\sqrt{6}}(\vec{k}_1 + \vec{k}_2 - 2\vec{k}_3)\,,$$

we get

$$I_m(N';N,A)=\frac{1}{3\sqrt{3}}\delta(\vec{k}_N+\vec{k}_A)\int d\vec{k}_\rho d\vec{k}_\lambda \mathcal{Y}_1^m(-2(\vec{k}_A+(\tfrac{2}{3})^{1/2}\vec{k}_\lambda))\tilde{\psi}_{N'}(\vec{k}_\rho,\vec{k}_\lambda)\tilde{\psi}_N(\vec{k}_\rho,(\tfrac{2}{3})^{1/2}\vec{k}_A+\vec{k}_\lambda)\,\tilde{\psi}_A(\vec{k}_A+2(\tfrac{2}{3})^{1/2}\vec{k}_\lambda)\,. \tag{3.8}$$

If we now use Gaussian wave functions for the ground state, i.e., for mesons:

$$\tilde{\psi}_A(\vec{k}_1,\vec{k}_2)=\left(\frac{R_A^2}{\pi}\right)^{3/4}\exp\left[-\frac{(\vec{k}_1-\vec{k}_2)^2R_A^2}{8}\right]\,,\qquad \psi_A(\vec{r}_1,\vec{r}_2)=\frac{1}{(\pi R_A^2)^{3/4}}\exp\left[-\frac{(\vec{r}_1-\vec{r}_2)^2}{2R_A^2}\right]\,, \tag{3.9}$$

and baryons:

$$\begin{aligned}\tilde{\psi}_N(\vec{k}_1,\vec{k}_2,\vec{k}_3)&=\left(\frac{3R_N^2}{\pi}\right)^{3/2}\exp\left[-\frac{R_N^2}{6}\sum_{i<j}(\vec{k}_i-\vec{k}_j)^2\right]\,,\\ \psi_N(\vec{r}_1,\vec{r}_2,\vec{r}_3)&=\frac{1}{(3\pi R_N^2)^{3/2}}\exp\left[-\frac{\sum_{i<j}(\vec{r}_i-\vec{r}_j)^2}{6R_N^2}\right]\,,\end{aligned} \tag{3.10}$$

then the integrals amount to

$$I_m(A;B,C)=\left(\frac{3}{4\pi}\right)^{1/2}\frac{1}{\pi^{3/4}}(\vec{\epsilon}_m\cdot\vec{k}_C)\left(\frac{2R_AR_BR_C}{R_A^2+R_B^2+R_C^2}\right)^{3/2}\left(\frac{2R_A^2+R_B^2+R_C^2}{R_A^2+R_B^2+R_C^2}\right)\exp\left[-\frac{\vec{k}_C^2}{8}\cdot\frac{R_A^2(R_B^2+R_C^2)}{R_A^2+R_B^2+R_C^2}\right]\delta(\vec{k}_B+\vec{k}_C)\,, \tag{3.11}$$

$$I_m(N';N,A)=-\left(\frac{3}{4\pi}\right)^{1/2}\frac{1}{\pi^{3/4}}(\vec{\epsilon}_m\cdot\vec{k}_A)\left(\frac{3R_N^2R_A}{3R_N^2+R_A^2}\right)^{3/2}\left(\frac{4R_N^2+R_A^2}{3R_N^2+R_A^2}\right)\exp\left[-\vec{k}_A^2\,\frac{12R_N^2+5R_N^2R_A^2}{24(3R_N^2+R_A^2)}\right]\delta(\vec{k}_N+\vec{k}_A)\,. \tag{3.12}$$

B. Calculation of Some Typical Coupling Constants

We now identify the $\hat{T}$-matrix elements with those obtained from phenomenological Lagrangians. We consider $f_{\rho\pi\pi}$ as an example. Factorizing $2\pi i\delta(E_f-E_i)$, we get in the center-of-mass system of $\rho\to\pi\pi$ for the special choice $\rho^+(j_z=0)\to\pi^+\pi^0$ (Oz axis is in $\vec{k}_\pi$ direction)

$$(2\pi)^3\delta(\vec{k}_\pi+\vec{k}_\pi')f_{\rho\pi\pi}\frac{1}{(2\pi)^{9/2}}\frac{1}{(2m_\rho)^{1/2}}\frac{1}{m_\rho}2k_\pi=\delta(\vec{k}_\pi+\vec{k}_\pi')\gamma\left(-\frac{1}{3\sqrt{2}}\right)\left(\frac{3}{4\pi}\right)^{1/2}\frac{1}{\pi^{3/4}}\left(\frac{2R_\rho R_\pi^2}{2R_\pi^2+R_\rho^2}\right)^{3/2}\left(\frac{2(R_\pi^2+R_\rho^2)}{2R_\pi^2+R_\rho^2}\right)\times\exp\left[-\frac{\vec{k}_\pi^2}{4}\frac{R_\pi^2R_\rho^2}{2R_\pi^2+R_\rho^2}\right]\,.$$

In the left-hand side, we have set $2E_\pi=m_\rho$. In the right-hand side, we multiply the spatial integral by the factor $-1/3\sqrt{2}$ which comes from the L-S Clebsch-Gordan coefficient, and the spin-SU(3) matrix element of the created pair. This last matrix element would come out to be zero, if the relative sign of pion momenta were omitted (see Fig. 6). Then

$$f_{\rho\pi\pi}=\gamma\frac{4}{\sqrt{3\pi}}\pi^{3/4}m_\rho^{3/2}\left(\frac{R_\rho R_\pi^2}{2R_\pi^2+R_\rho^2}\right)^{3/2}\left(\frac{R_\pi^2+R_\rho^2}{2R_\pi^2+R_\rho^2}\right)\exp\left[-\frac{\vec{k}_\pi^2}{4}\frac{R_\pi^2R_\rho^2}{2R_\pi^2+R_\rho^2}\right]\,;$$

the exponential can be neglected in a rough approximation.

In the same way, considering ρ or π emission by nucleon in the initial nucleon rest-frame and taking π and ρ to be slow, we get, omitting the exponential (which is approximately correct),

$$\begin{aligned}\frac{G_{\pi NN}}{2m_N}&=\gamma\frac{5}{3\sqrt{6\pi}}\pi^{3/4}m_\pi^{1/2}\left(\frac{3R_N^2R_\pi}{3R_N^2+R_\pi^2}\right)^{3/2}\left(\frac{4R_N^2+R_\pi^2}{3R_N^2+R_\pi^2}\right)\,,\\ f_{\rho NN}&=\gamma\left(\frac{2}{3\pi}\right)^{1/2}\pi^{3/4}m_\rho^{3/2}\left(\frac{3R_N^2R_\rho^2}{3R_N^2+R_\rho^2}\right)^{3/2}\left(\frac{4R_N^2+R_\rho^2}{3R_N^2+R_\rho^2}\right)\,.\end{aligned} \tag{3.13}$$

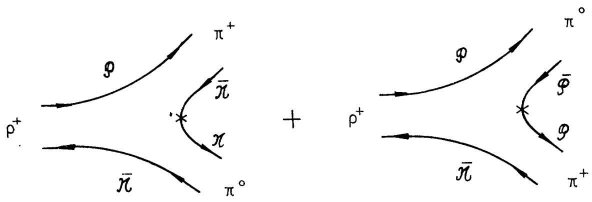

FIG. 6. These two diagrams are to be added when calculating the $\rho^+ \to \pi^+\pi^0$ vertex.

C. COMMENTS

A remarkable feature of the expression for the coupling constants is that they are quite similar to those obtained from consideration of leptonic decays in Sec. II. Let us, for instance, write $f_{\rho NN}$ in the following form:

$$f_{\rho NN} = \gamma \left(\frac{2}{3\pi}\right)^{1/2} \frac{m_\rho{}^{3/2}}{|\psi_\rho(0)|} \times \left[\left(\frac{3R_N{}^2}{3R_N{}^2 + R_\rho{}^2}\right)^{3/2} \frac{4R_N{}^2 + R_\rho{}^2}{3R_N{}^2 + R_\rho{}^2}\right]. \qquad (3.14)$$

In the case of $f_{\rho NN}$, we have good confidence in the Gaussian wave functions and, moreover, we have a good estimate of the R value. (See our paper in Ref. 3.) We find that the factor in brackets is indeed close to one and thus

$$f_{\rho NN} \simeq \gamma \left(\frac{2}{3\pi}\right)^{1/2} \frac{m_\rho{}^{3/2}}{|\psi_\rho(0)|}, \qquad (3.15)$$

to be compared with (2.18). Evidently the contents of (3.15) and (2.18) are not fully identical; the appearance of the arbitrary constant in the QPC model seems to weaken the interest of the model. However,one should not underestimate the success performed in formulating a simple description of all the strong interaction vertices, with the help of only basic hadron quantities. In this respect, we have a much more general description than in Sec. II, not restricted to 0^- and 1^- emission, but valid for any type of emission. In one formula we recover the various relationships which were previously obtained for coupling constants from universality, additivity, and SU(6) symmetry (Fig. 7). Indeed, if we take the limit where the ρ and π have the same mass and wave function, we get from (3.13) the Gürsey, Pais, Radicati relation.[16]

The way to recover additivity will be seen in Sec. IV. The main weakness of the QPC model is that at present one cannot take into account the symmetry breaking affecting the wave functions and, moreover, the emitted hadrons are treated in a nonrelativistic way. Thus, one cannot expect any accuracy for the magnitude of transition amplitudes. This has led us to look for predictions,

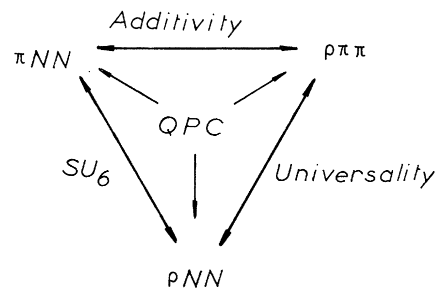

FIG. 7. We indicate here which coupling constants may be related using, respectively, additivity, universality, SU(6), and QPC.

which would not be sensitive to those effects, but would rather test the fundamental structure of the model. We formulate such predictions in Sec. IV and compare the QPC model to the old additive quark model.

IV. QUARK-PAIR-CREATION MODEL AND ADDITIVITY

A. Comparison with Elementary Pion Emission: Deduction of the Recoil Term of Mitra and Ross

In the quark model, one usually treats the processes $A \to B + \pi$, where A and B are any hadrons, by pion emission from a single quark,

$$\mathcal{H}(i) = \frac{G_{\pi qq}}{2m_q}[\vec{\sigma}(i)\cdot\vec{k}_\pi][\vec{\tau}(i)\cdot\vec{\pi}], \qquad (4.1)$$

where π is the pion field,

$$\frac{G_{\pi qq}}{2m_q}$$

being related to $G_{\pi NN}$ by (2.23). This $\vec{\sigma}\cdot\vec{k}_\pi$ coupling fails especially for ρ or ω polarization in A_1 and B decay; the experimental result is roughly inverse to the theoretical prediction.

Mitra and Ross[2] have introduced a recoil term:

$$\mathcal{H}(i) = \frac{G_{\pi qq}}{2m_q}\vec{\sigma}(i)\cdot\left(\vec{k}_\pi - \frac{E_\pi}{M_q}\vec{k}_i\right)\vec{\tau}(i)\cdot\vec{\pi}. \qquad (4.2)$$

It adds an isotropic contribution equal for the three polarization states of ρ or ω.[7]

Let us now compare (4.2) with the QPC model. In Eq. (3.4), we make $C = \pi$, and consider baryons. We write Φ_π and Φ_{vac}^{-m} with the help of σ and τ matrices and get

$$\langle N\pi|\hat{T}|N'\rangle = -\frac{\gamma}{\sqrt{3}}\, 3 \left\langle \Phi_N \left| \frac{\sigma_{-m}(3)\,\vec{\tau}(3)\cdot\vec{\phi}}{2\sqrt{6}} \right| \Phi_{N'} \right\rangle \times I_m(N';N\pi).$$

$\sigma_{-m}(3)$ and $\vec{\tau}(3)$ denote the spin and isospin Pauli matrices associated with the third quark. The factor 3 is for the number of quarks; $\vec{\phi}$ is the pion isovector.

We have defined

$$\sigma_{+1}=\frac{\sigma_x+i\sigma_y}{\sqrt{2}}\ ,\quad \sigma_{-1}=\frac{-\sigma_x+i\sigma_y}{\sqrt{2}}\ ,\quad \sigma_0=\sigma_z\ ,$$

and

$$I_m(N';N\pi)=\delta(\vec{k}_N+\vec{k}_\pi)\int d\vec{k}_1 d\vec{k}_2 d\vec{k}_3\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)\tilde{\psi}_N(\vec{k}_1,\vec{k}_2,\vec{k}_3-\vec{k}_\pi)\{\mathcal{Y}_1^m(-2(\vec{k}_\pi-\vec{k}_3))\tilde{\psi}_\pi(\vec{k}_\pi-2\vec{k}_3)\}\tilde{\psi}_{N'}(\vec{k}_1,\vec{k}_2,\vec{k}_3)\ .$$

Writing

$$\mathcal{Y}_1^m(-2(\vec{k}_\pi-\vec{k}_3))=-2\vec{\epsilon}_m\cdot(\vec{k}_\pi-\vec{k}_3)\left(\frac{3}{4\pi}\right)^{1/2}\ ,$$

we get the operator to be taken between $\Phi_{N'}$ and Φ_N:

$$\left(\frac{3}{8\pi}\right)^{1/2}\gamma\delta(\vec{k}_N+\vec{k}_\pi)\int d\vec{k}_1 d\vec{k}_2 d\vec{k}_3\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)\tilde{\psi}_N(\vec{k}_1,\vec{k}_2,\vec{k}_3-\vec{k}_\pi)[\vec{\sigma}(3)\cdot(\vec{k}_\pi-\vec{k}_3)(\vec{\tau}(3)\cdot\vec{\phi})\tilde{\psi}_\pi(\vec{k}_\pi-2\vec{k}_3)]$$
$$\times\tilde{\psi}_{N'}(\vec{k}_1,\vec{k}_2,\vec{k}_3)\ . \tag{4.3}$$

Between the brackets, we recognize an operator similar to the one of Mitra and Ross.[2] It is however multiplied by the function $\tilde{\psi}_\pi(\vec{k}_\pi-2\vec{k}_3)$, which expresses the composite character of the pion, and the nonlocal character of the interaction. In the limit where the pion radius is very small, one fully recovers the elementary pion emission of Mitra and Ross [assuming in (4.2) $E_\pi\approx M_q$, i.e., an effective quark mass].

$$\left(\frac{3}{8\pi}\right)^{1/2}\gamma\tilde{\psi}_\pi(0)\delta(\vec{k}_N+\vec{k}_\pi)\int d\vec{k}_1 d\vec{k}_2 d\vec{k}_3\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)\tilde{\psi}_N(\vec{k}_1,\vec{k}_2,\vec{k}_3-\vec{k}_\pi)[\vec{\sigma}(3)\cdot(\vec{k}_\pi-\vec{k}_3)\vec{\tau}(3)\cdot\vec{\phi}]\tilde{\psi}_{N'}(\vec{k}_1,\vec{k}_2,\vec{k}_3)\ , \tag{4.4}$$

where we have approximated $\tilde{\psi}_\pi(\vec{k}_\pi-2\vec{k}_3)$ by $\tilde{\psi}_\pi(0)$ (very small). This is a remarkable achievement of the QPC model since the interaction (4.2) is satisfactory for strong decays of mesons and baryons.[2,17] Thus the additivity (in the form assumed by Mitra and Ross[2]) appears as a limit of the QPC model when one meson is considered as being pointlike. But at the same time, the compositeness of the meson is still reflected in the $\vec{k}_i$ term, i.e., the quark momentum, which cannot be, in general, expressed in terms of hadron momenta.

Let us now see what is the effect of the finite pion radius. We set $\tilde{\psi}_\pi(\vec{k}_\pi-2\vec{k}_3)\to c\tilde{\psi}_\pi(0)$, where c is now <1. We get

$$\frac{G_{\pi qq}}{2m_q}=\frac{\sqrt{m_\pi}}{2\sqrt{2}\ \psi_\pi(0)}\frac{4c}{\sqrt{3\pi}}\gamma\ , \tag{4.5}$$

to be compared with (3.13). We see that

$$c=\tfrac{1}{2}\left(\frac{3R_N^2}{3R_N^2+R_\pi^2}\right)^{3/2}\frac{4R_N^2+R_\pi^2}{3R_N^2+R_\pi^2}\ , \tag{4.6}$$

when $R_\pi^2\to 0$, $c\to\frac{2}{3}$ and not 1. This shows that c also takes into account effects other than $R_\pi^2\neq 0$; it takes into account the effect of the $\vec{k}_3$ term.

B. A Calculation of Polarization in B and A_1 Decays

To emphasize the effect of the $\vec{\sigma}\cdot(\vec{k}_\pi-\vec{k}_3)$ coupling to Eq. (3.3), let us consider the typical effect on ρ or ω polarization in A_1, B decays.

In the case of ground-state mesons ($L=0$), the $\vec{\sigma}\cdot\vec{k}_3$ term is, after integration, simply proportional to $\vec{\sigma}\cdot\vec{k}_\pi$. This is not true for excited mesons, for which the $\vec{\sigma}\cdot\vec{k}_3$ term yields an additional isotropic term coming from integration over quadratic polynomials in $\vec{k}_3$.

We get

$$\left[\frac{M(\pm 1)}{M(0)}\right]_{A_1\to\rho\pi}=\frac{I_1-I_0}{2I_3}\ , \tag{4.7}$$

$$\left[\frac{M(0)}{M(\pm 1)}\right]_{B\to\omega\pi}=-\frac{I_0}{I_1}\ , \tag{4.8}$$

where

$$I_m=\tfrac{1}{8}\int d^3\vec{k}\ \mathcal{Y}_1^m(\vec{k}_\pi-\vec{k})\tilde{\psi}_\pi(\vec{k})\tilde{\psi}_V(-\vec{k})\tilde{\psi}_1^{-m}(\vec{k}_\pi+\vec{k})\ . \tag{4.9}$$

In (4.9) $\tilde{\psi}_1^{-m}(\vec{k}_\pi+\vec{k})$ is the Fourier transform of the $L=1$ spatial wave function

$$\tilde{\psi}_1^m(\vec{k})=i(\tfrac{2}{3})^{1/2}\frac{R^{5/2}}{\pi^{1/4}}\mathcal{Y}_1^m(\vec{k})\exp\left(-\frac{\vec{k}^2R^2}{8}\right)\ .$$

Then, explicitly in terms of the wave-function radii,

$$\left[\frac{M(\pm 1)}{M(0)}\right]_{A_1\to\rho\pi} = \frac{8/(R_A{}^2+R_\rho{}^2+R_\pi{}^2) - \vec{k}_\pi{}^2[1-R_A{}^4/(R_A{}^2+R_\rho{}^2+R_\pi{}^2)^2]}{8/(R_A{}^2+R_\rho{}^2+R_\pi{}^2)}\,, \tag{4.10}$$

$$\left[\frac{M(0)}{M(\pm 1)}\right]_{B\to\omega\pi} = \frac{4/(R_B{}^2+R_\omega{}^2+R_\pi{}^2) - \vec{k}_\pi{}^2[1-R_B{}^4/(R_B{}^2+R_\omega{}^2+R_\pi{}^2)^2]}{4/(R_B{}^2+R_\omega{}^2+R_\pi{}^2)}\,. \tag{4.11}$$

Independent of the accurate value of the radii, $\vec{k}_\pi{}^2$ being very small for A_1 decay, we get

$$[M(0)]_{A_1\to\rho\pi} \sim [M(\pm 1)]_{A_1\to\rho\pi}$$

in absolute value and sign. For B decay, $\vec{k}_\pi{}^2$ being large, we obtain an amplitude $[M(0)]_{B\to\omega\pi}$ of the same sign as $[M(\pm 1)]_{B\to\omega\pi}$ but smaller in absolute value.

With the three radii equal to the value of $R_\rho{}^2$ determined from the ρ-Regge slope,[3,18] $R_\rho{}^2 = 8$ GeV^{-2}, we get

$$\left[\frac{M(\pm 1)}{M(0)}\right]_{A_1\to\rho\pi} = 1 - \tfrac{1}{3}R_\rho{}^2\vec{k}_\pi{}^2 = 0.87\,,$$

$$\left[\frac{M(0)}{M(\pm 1)}\right]_{B\to\omega\pi} = 1 - \tfrac{2}{3}R_\rho{}^2\vec{k}_\pi{}^2 = 0.36\,.$$

Let us now compare with the experimental data. For $B\to\omega\pi$, experiments in $\pi^\pm p \to \pi^\pm p$ (see Refs. 19, 20) give the following value for the above ratio:

$$\left[\frac{|M(0)|^2}{\sum_\lambda |M(\lambda)|^2}\right] \simeq 0.1\,, \quad \left|\frac{M(0)}{M(\pm 1)}\right| = 0.47^{+0.20}_{-0.30}\,.$$

Our prediction is compatible with this experimental value. Moreover, we predict a ratio

$$\left(\frac{D}{S}\right)_{B\to\omega\pi} = \frac{1-[M(0)/M(\pm 1)]_{B\to\omega\pi}}{\sqrt{2}+(\frac{1}{2})^{1/2}[M(0)/M(\pm 1)]_{B\to\omega\pi}} \tag{4.12}$$

real and *positive*, which is not in contradiction with the experimental bound for the D/S phase $\phi < 45°$.[20] At the Kiev conference, Ascoli *et al.* reported a new experimental value,[21]

$$\left|\frac{M(0)}{M(\pm 1)}\right|_{B\to\omega\pi} = 0.68 \pm 0.12\,,$$

rather larger than our prediction.

For $A_1\to\rho\pi$, we agree very well with the result[22]:

$$\left|\frac{M(\pm 1)}{M(0)}\right|_{A_1\to\rho\pi} = 0.89^{+0.13}_{-0.06}\,.$$

However, the SLAC value is lower[23]:

$$\left|\frac{M(\pm 1)}{M(0)}\right|_{A_1\to\rho\pi} = 0.48 \pm 0.13.$$

Colglazier and Rosner[4] have reported a careful description of the experimental situation, pointing out from BNL data that if the ratio $[M(\pm 1)/M(0)]_{A_1\to\rho\pi}$ is real, then

$$\left[\frac{M(\pm 1)}{M(0)}\right]_{A_1\to\rho\pi} = 0.95\,,$$

in remarkable agreement with our prediction in absolute value and sign.

We must emphasize that the qualitative agreement of relations (4.10), (4.11), or more generally (4.7) and (4.8), is more important than a quantitative agreement with a given piece of data. The presence of the isotropic term in (4.10) and (4.11) is strongly supported by the data. It subtracts from the $\vec{k}_\pi{}^2$ depending term for $\lambda_\rho = \pm 1$ in $A_1\to\rho\pi$ and for $\lambda_\omega = 0$ in $B\to\omega\pi$ (this difference between B and A_1 is due to their different L-S quantum numbers).

This success gives us strong confidence in the quark model classification of parity + meson states as well as in our dynamical model of strong interaction vertices described in Sec. III.

C. More Comments on the Recoil Term

To our knowledge, the "recoil" term was introduced by Mitra and Ross[2] mainly to cure the failure of the $\vec{\sigma}\cdot\vec{k}_\pi$ term in certain decay widths close to threshold, for instance, $N(1535)\to\eta N$. However, they could not predict the ratio of the two contributions, since they did not use a definite set of wave functions, such as harmonic-oscillator states; rather, they fitted two separate form factors for the direct and recoil terms.

Dalitz[24] commented on this recoil term in relation with $L=1$, $I=1$ meson widths, the A_1, A_2, and B being predicted too small with only the direct term. The recoil term which he gave using a heavy (bare) quark mass was too small to account for the experimental widths. However, in view of the tight quark binding, he suggested fitting a coefficient in front of the recoil term; but then the small $\delta\to\eta\pi$ width remained unexplained. Now, we think that the small upper limit of the δ width (5 MeV) considered by Dalitz is not clearly supported by present experimental data.[25]

Feynman, Kislinger, and Ravndal[7] have introduced π emission in their "quadratic" covariant harmonic oscillator quark model. In this scheme the (mass)^2 operator is described by a four-dimensional oscillator:

$$M^2 \sim \sum_i \left(p_\mu(i) p^\mu(i) + \omega^2 q_\mu(i) q^\mu(i) \right),$$

the Regge trajectories coming out to be linear. Their model treats the pseudoscalar meson as an elementary quantum field, proportional to the divergence of some axial current, introduced in the M^2 operator through a minimal prescription, as in electromagnetism. Because of the quadratic character of the M^2 operator, the interaction term built in this way shows a dependence in the internal quark momenta very similar to that of Mitra and Ross[26] and to ours:

$$\vec{\sigma}(i) \cdot \left[\vec{k}_\pi + O\left(\frac{\vec{k}_\pi^{\,2}}{(M+m)^2} \right) - \left(\frac{M-m}{\frac{1}{3}M} \right) \vec{k}_i \right]. \qquad (4.13)$$

Note that

$$\left(\frac{M-m}{\frac{1}{3}M} \right) \sim \frac{E_\pi}{M_q} \sim 1,$$

M_q being an effective quark mass. The term of the order $\vec{k}_\pi^{\,2}/(M+m)^2$ is a relativistic correction to the dominant terms. Of course, the results for ω and ρ polarizations in B and A_1 decays of Feynman *et al.* are very close to our calculations.

We wonder why apparently disconnected approaches such as the following: (i) Galilean invariance in the static phenomenological elementary π emission, (ii) a specific prescription for elementary π emission in the quadratic model of Feynman *et al.*, and (iii) the *composite* π emission in the QPC model proposed by us, lead to similar effective interactions, which depend on the internal quark momenta $\vec{k}_i$.

Colglazier and Rosner[4] have studied the parity+ meson decays in the frame of a covariant QPC model. They note that if the transverse orbital angular momentum of the created quark pair is allowed, then the predictions of $SU(6) \otimes O(2)_{L_z}$ (or equivalently in our language, of the interaction term $\vec{\sigma} \cdot \vec{k}_\pi$) are considerably improved. They construct independent-invariant couplings by coupling the orbital angular momentum of the initial meson to (i) the external independent momentum and (ii) the orbital angular momentum of the $q\bar{q}$ pair; this last coupling leads to a term which contributes to all polarizations. The various couplings are affected by different parameters. In our notation, their coupling is equivalent, in a quantitative sense, to a coupling of the form

$$\sim \vec{\sigma}(i) \cdot (\vec{k}_\pi - \lambda \vec{k}_i), \qquad (4.14)$$

where λ is an arbitrary parameter to be fitted. λ is fixed to be 1 in our model. Note that from our expressions (4.7) and (4.8) we find, in the exact SU(6) limit, the Colglazier and Rosner relation:

$$2\left[\frac{M(\pm 1)}{M(0)} \right]_{A_1 \to \rho\pi} = \left[\frac{M(0)}{M(\pm 1)} \right]_{B \to \omega\pi} + 1. \qquad (4.15)$$

Although they apparently lead to similar results, the two models are not equivalent. What they have in common is a representation of the decay by a pair creation plus a rearrangement process. But the spirit is quite different. In the naive model, there is an explicit consideration of the quark motion. We can construct unambiguously the quark pair-creation amplitude with the help of the spins and momenta of the quark and antiquark only. The breaking of $SU(6)_W$ is a natural outcome of the presence of transverse quark momenta. In Ref. 4, quarks are only formally present; they are mere indices; the "wave function" matrices involve only the momentum and angular momentum of the hadron itself. One is thus compelled to reintroduce more structure through *extra* couplings between the "pair" and the incoming meson. But the various couplings depend on the nature of the hadron states and may be weighted by different arbitrary coefficients for different hadrons. There may be a different set of coefficients for each L (of the incoming meson). We are thus left with a host of arbitrary coefficients. On the contrary, in the naive model, the $^3P_0$ structure describes any decay process, whatever the hadrons may be. Of course one could note that we have also to introduce parameters for the wave functions. But this is quite different; these are parameters concerning the hadron in itself and not the decay process as in Ref. 4. Moreover with the oscillator model—apart from further complications of the spectrum—there is only one parameter for the whole set of wave functions, and it has a direct physical meaning (it is proportional to the Regge slope, as emphasized in Ref. 3).

Finally let us say some words on the relativistic quark model of Mitra and collaborators.[27] Their model is formally covariant and treats one meson as elementary. The couplings are constructed by starting from the "direct" term $\vec{\sigma}(i) \cdot \vec{k}_\pi$. The dependence of the coupling in the masses and energies is derived from empirical considerations. A term having a similar effect to the "recoil term" is obtained in this relativistic version from the direct term through the correspondence $\vec{k}_\pi^{\,2} \to k_\mu k^\mu$. However the model destroys the good properties of the naive recoil term $(E_\pi/m_q)\vec{\sigma}(i) \cdot \vec{k}_i$ in the description of A_1 and B decay polarization.

D. Comparison with Elementary Vector Emission

Just in the same way as one considers elementary pion emission, it is possible to use, in the

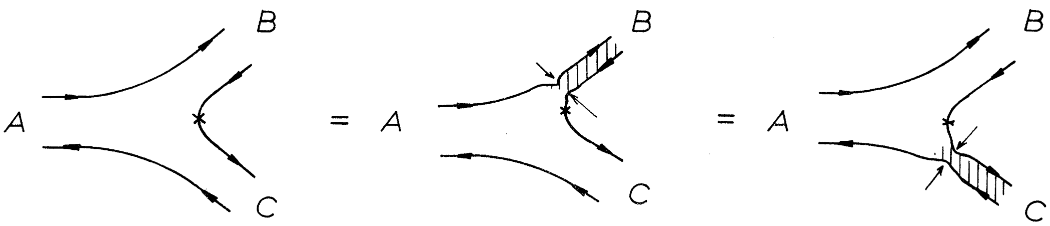

FIG. 8. The two possible limits of QPC model considering either B or C as elementary.

ordinary quark model, an elementary ρ or vector-meson emission (or γ emission). The corresponding single-quark interaction Hamiltonian is

$$\mathcal{H}_i = f_{\rho q q}\left[\frac{\vec{k}_\rho \cdot \vec{\epsilon}_\rho}{k_0} + \frac{\vec{k}_\rho \cdot \vec{\epsilon}_\rho}{2m_q} - \frac{\vec{k}_i \cdot \vec{\epsilon}_\rho}{m_q} + i\,\frac{\vec{\sigma}(i)\cdot(\vec{k}_\rho \times \vec{\epsilon}_\rho)}{2m_q}\right]\tfrac{1}{2}\vec{\tau}(i)\cdot\vec{\rho}\,, \qquad (4.16)$$

the first term coming from the Coulomb interaction.

Uretsky has introduced this vector-meson emission Hamiltonian for $A_1 \to \rho\pi$, $A_2 \to \rho\pi$, $B \to \omega\pi$ decays, because of the failure of the pion emission $\vec{\sigma}\cdot\vec{k}_\pi$ interaction. He proposed to consider both pion and vector-meson emission (taking the average), since in $\rho \to \pi\pi$ averaging of the two π's is successful.

Our model, being "dual" in the sense described above, solves in a very simple manner the problem of combining pion and vector emission. *Our unique amplitude $A \to P + V$ may be interpreted as either pion or vector emission, whether we single out the pion or the vector particle* (see Fig. 8).

At the same time, our model yields automatically a recoil term in pion emission, thus explaining correctly the decay polarizations.

Let us now write the result of singling out the ρ emission for $NN\rho$ coupling [considering only the operator to be taken between nucleon SU(6) wave functions]:

$$\left(\frac{3}{8\pi}\right)^{1/2}\gamma\delta(\vec{k}_\rho + \vec{k}_N)\int d\vec{k}_1 d\vec{k}_2 d\vec{k}_3\,\delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)\,\tilde{\psi}_N(\vec{k}_1, \vec{k}_2, \vec{k}_3 - \vec{k}_\pi)$$
$$\times\{\tfrac{1}{2}[(\vec{k}_\rho - \vec{k}_3)\cdot\vec{\epsilon}_m^{\,*}] + \tfrac{1}{2}i\vec{\sigma}(3)\cdot[(\vec{k}_\rho - \vec{k}_3)\times\vec{\epsilon}_m^{\,*}]\}\,[\vec{\tau}(3)\cdot\vec{\phi}]\,\tilde{\psi}_\rho(\vec{k}_\rho - 2\vec{k}_3)\,\tilde{\psi}_{N'}(\vec{k}_1, \vec{k}_2, \vec{k}_3)\,. \qquad (4.17)$$

It corresponds to an electric and a magnetic term, although there is only approximate correspondence with (4.16). One major feature is the $\vec{k}_3$ term, analogous to the recoil term of pion emission. In contrast with Uretsky's treatment, it gives the correct polarization.

This should *not* be added to the pion emission term; in fact, we have

$$\langle \pi | \mathcal{H}(\rho) | A_1 \rangle = \langle \rho | \mathcal{H}(\pi) | A_1 \rangle \qquad (4.18)$$

and the same for B, A_2.

V. CONCLUSION

We have presented a model describing in a very simple manner all strong-interaction vertices with only two parameters: R^2, which is the strength parameter of the oscillator simply related to the Regge slope, and an over-all quark-pair-creation constant which multiplies every amplitude. As is well known, a model for strong-interaction vertices gives a grasp over a large class of strong-interaction phenomena besides decay phenomena, through the particle exchange and resonance interpretation of two-body and multibody processes.

The QPC model not only includes many previously known relationships between the strong coupling constants, it also makes definite new predictions. The main predictive achievement of the model is the presence of the recoil term, giving a good polarization for $A_1 \to \rho\pi$ and $B \to \omega\pi$. We emphasize this achievement because it tests just the general structure of the model and is obtained without any *ad hoc* prescription.

The QPC model in its present state still suffers two main drawbacks. First, we have not recovered the full content of the relations of Sec. II; the general structure of the formulas for the coupling constants is the same, but the derivation through VMD and PCAC fixes γ, which is left undetermined in the QPC model. Second, our model treats the three hadrons of a given vertex in a nonrelativistic way; therefore, we cannot expect any accuracy in the calculation of absolute values (in contrast with obtaining qualitative relationships), and we cannot treat the effects of mass breaking.

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[9]Note that the Van Royen-Weisskopf "paradox" expressed by formula (2.12) is equivalent, through (2.9), (2.10), (2.18), and (2.22), (i) to the KSRF (Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin) relation (Ref. 5) coming from SU(2)⊗SU(2) algebra, for the π and ρ mesons:

$$f_\rho^2 = m_\rho^2/2c_\pi^2,$$

(ii) to the SU(6) relation between strong coupling constants of Gürsey, Pais, and Radicati:

$$\frac{G_{\pi NN}}{m_N} = \frac{5}{3}\frac{f_{\rho NN}}{m_\rho}$$

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