ily correct) way to resolve this formal field theory ambiguity. However, we have applied his method to this problem and find that it gives results for A_{10} and D_{10} which are twice as large as those given by dispersion theory. What is worse, it does not generate the PC spurion vertex $l=2$ suppression factor $(m_D - m_B)$ in B_{10} and C_{10}.

[33]This gives a sensitive test of our choice of dispersion covariants for the spurion process. Schnitzer (Ref. 9) and Chan (Ref. 10) choose axial vertex covariants $g_{\mu\nu}$ and three others which are orthogonal to q_ν. This choice contains kinematic zeros which eventually destroy the needed $m_D - m_B$ suppression at the $l=2$ PC spurion vertex.

[34]We take the results of J. Mathews, Phys. Rev. 137, B444 (1965). For a review of the somewhat confusing literature on this subject, see H. F. Jones and M. D. Scadron, Imperial College report, 1972, Ann. Phys. (N.Y.) (to be published).

[35]The sign of C_3 and C-parity phase of DB is consistent with Refs. 29 and 30. The magnitude of C_3 and of g_2 should be reduced somewhat due to corrections to the narrow-width approximation at threshold. See Refs. 7 and 8 and G. Höhler *et al.*, Nucl. Phys. B39, 237 (1972). In practice this will simply scale up our fitted values for h_2 and h_3 but will not alter our fits to the data in any way.

[36]G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. 106, 1345 (1957).

[37]Reference 7 finds that corrections to the narrow-width approximation (38) reduces (40) to 22% below the experimental value for κ_p.

[38]The $U=1$ state Σ_3 will rule out possible Y_0^* resonant contributions.

[39]It should be noted that the various measurements of the $\Sigma^+ \to p\gamma$ branching ratio are somewhat inconsistent, but that the experiment with the smallest errors gives $|C|^2 + |D|^2 = (2.7 \pm 0.4)\times 10^{-20}$ MeV^{-2}, closer to our theoretical prediction.

[40]Y. Chiu, J. Schechter, and Y. Ueda, Phys. Rev. 150, 1201 (1966).

[41]K. Gavroglu and M. D. Scadron, Imperial College report, 1972 (unpublished).

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From a Hard-Pion Model to a Phenomenological Lagrangian

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A hard-pion model is presented for a general symmetry. In the case of strong partially conserved currents the proper vertices are assumed to be analytical. Ward-like identities are obtained for the contact terms. For the soft-pion process in the symmetry limit of SU(2) × SU(2) a phenomenological Lagrangian is built which reproduces the results of the hard-pion model. As an illustration we deal with the K_{l3} problem, and build an amplitude for the photoproduction process $\gamma N \to \pi N$.

I. INTRODUCTION

Hard pions (H.P.) and the phenomenological Lagrangians (P.L.) have been successfully used in describing various low-energy phenomena.[1,2] It has been pointed out that both methods give similar results. More systematic proof of the equivalence of the results given by P.L. with those obtainable from current algebra was given by Dashen and Weinstein.[3] The equivalence is clear, in particular for soft-pion emissions in the symmetry limit. Our aim is to present a general H.P. model which in some sense is a generalization of the Gell-Mann–Oakes–Renner (GMOR) model[4] for symmetry breaking and the work of Gerstein *et al.*[5] on the structure of 3-point functions.

The main advantage of the H.P. model presented is the possibility of extracting the scalar (pseudoscalar) contribution from the nonconserved currents. Having constructed the H.P. model, we shall discuss soft-pion processes in the symmetry limit. We shall show that the same results may be obtained using a P.L. in the tree approximation. As an illustration for the use of the H.P. model we shall deal with the K_{l3} problem and with the photoproduction process $\gamma N \to \pi N$ in the $P_\gamma < 1$ GeV/c region.

II. THE HARD-PION MODEL

The construction of the H.P. amplitudes will be achieved in four steps.

Step 1: The symmetry structure. The symmetry structure of the model is defined by the following commutation relations between the currents and their divergences[5]:

$$[J_a^0(x), J_b^\mu(y)]\,\delta(x^0 - y^0) = iC_{abc}J_c^\mu(x)\delta^4(x-y) + \text{S.T.}, \tag{1a}$$

$$\partial_\mu J^\mu_a(x) = \epsilon_j T^a_{jk}\phi_k(x), \tag{1b}$$

$$[J^0_a(x), \phi_k(y)]\delta(x^0 - y^0) = -iT^a_{kj}\phi_j(x)\delta^4(x-y). \tag{1c}$$

(We sum over repeated indices.) C_{abc} are the structure constants of the Lie algebra defined by the charges of the currents. S.T. is the Schwinger term, for which we assume we have at most a first-order derivative of the δ function. ϵ_j are the symmetry-breaking parameters, and the fields $\phi_j(x)$ are scalars [pseudoscalar according to the nature of the current $J^\mu_a(x)$] forming an irreducible representation of the Lie algebra. Equation (1b) may be obtained for example in a GMOR[4] model in which the symmetry-breaking part of the Hamiltonian, $H_{\rm SB}$, is given by

$$H_{\rm SB}(x) = -\epsilon_i\phi_i(x).$$

For complete specification of the symmetry structure one should also specify the symmetry of the vacuum, which might be broken "spontaneously," i.e., there are some fields $\phi_i(x)$ such that $\langle 0|\phi(0)|0\rangle = \lambda_i \neq 0$. Such fields would give rise to asymmetric vacuum, i.e., there are some charges Q_a such that $Q_a|0\rangle \neq 0$. The following constraints are caused by the symmetries G_H and G_V of the Hamiltonian and of the vacuum:

$$Q_a \in G_H \Rightarrow \epsilon_j T^a_{jk} = 0, \quad \forall k, \tag{2a}$$

$$Q_a \in G_V \Rightarrow T^a_{jk}\lambda_k = 0, \quad \forall j. \tag{2b}$$

The symmetry structure will be used through Ward identities applied to the covariant time-ordered products of currents and their divergences. The existence of such time-ordered products was proved by various authors.[6]

Step 2: The external singularities. Let us define the Fourier transform of the vacuum expectation value of the covariant time-ordered products of n fields $A_1(x), \ldots, A_n(x)$:

$$\int e^{-ip_1x_1\cdots -ip_{n-1}x_{n-1}}\langle 0|T^*(A_1(x_1)\cdots A_{n-1}(x_{n-1})A_n(0))|0\rangle dx_1\cdots dx_{n-1} \equiv \langle A_1(p_1)\cdots A_n(p_n)\rangle, \quad p_1 + p_2 + \cdots p_n = 0.$$

It is well known that if the field $A(x)$ is an extrapolating field for a particle of mass m (no mass-zero particles), then the amplitude

$$(p_1{}^2 - m^2)\langle A_1(p_1)\cdots A_n(p_n)\rangle$$

is analytic in the vicinity of $p_1{}^2 = m^2$.

In what follows we shall define the analytic structure of our amplitude in the external momenta. Let $A_i(x)$ be fields describing particles in the theory (stable or unstable). Let us define

$$\langle J^\mu_a(p_1)\cdots\rangle = \Delta^\mu_{ai}(p_1)\langle \overline{A}_i(p_1)\cdots\rangle, \tag{3}$$

where

$$\Delta^\mu_{ai}(p) = \int e^{-ipx}\langle 0|T^*(J^\mu_a(x)A_i(0))|0\rangle.$$

We assume that the amplitude $\langle\overline{A}_i(p_1)\cdots\rangle$ is analytic in the variable $p_1{}^2$. The extraction (3) is to be performed on all currents $J^\mu_a(x)$, and on the fields $\phi_i(x)$ appearing in the above amplitude. We shall call this amplitude a *reduced amplitude*. This amplitude corresponds on the mass shell to the scattering amplitude of the particles described by the fields $A_i(x)$.

As the singularity structure in the external momenta was supposed to be given entirely by the 2-point functions, we present below all 2-point functions (covariant and satisfying Ward identities) of interest:

$$\int e^{-ipx}\langle 0|T^*(J^\mu_a(x)J^\nu_b(0))|0\rangle \equiv \Delta^{\mu\nu}_{ab}(p) = -(g^{\mu\nu}p^2 - p^\mu p^\nu)\Delta^{ab}_T(p) + g^{\mu\nu}\Delta^{ab}_1(p), \tag{4a}$$

where

$$\Delta^{ab}_T(p) = \frac{1}{2\pi i}\int\frac{\rho^{ab}_2(s)ds}{s(s-p^2-i\epsilon)}, \qquad \Delta^{ab}_1(p) = \frac{-1}{2\pi i}\int\frac{\rho^{ab}_1(s)-\rho^{ab}_2(s)}{s-p^2-i\epsilon}ds,$$

$$-g^{\mu\nu}\rho^{ab}_1(p^2) + \frac{p^\mu p^\nu}{p^2}\rho^{ab}_2(p^2) = (2\pi)^4\sum_n\langle 0|J^\nu_b(0)|n\rangle\langle n|J^\mu_a(0)|0\rangle\delta(p-p_n),$$

$$\int e^{-ipx}\langle 0|T^*(J^\mu_a(x)\phi_i(0))|0\rangle \equiv \Delta^\mu_{ai}(p) = -\frac{p^\mu}{2\pi}\epsilon_k T^a_{kj}\int\frac{\rho^0_{ij}(s)}{s(s-p^2-i\epsilon)}ds, \tag{4b}$$

$$\int e^{-ipx}\langle 0|T^*(\phi_i(x)\phi_j(0))|0\rangle \equiv \Delta_{ij}(p) = \frac{1}{2\pi i}\int \frac{\rho^0_{ij}(s)}{s-p^2-i\epsilon}ds\,, \tag{4c}$$

where

$$\rho^0_{ij}(s) = (2\pi)^4 \sum_n \langle 0|\phi_j(0)|n\rangle\langle n|\phi_i(0)|0\rangle \delta(p-p_n) \qquad (s=p^2).$$

Equation (4b) was obtained by using the Ward identities, which also enables us to write

$$s[\rho_1^{ab}(s) - \rho_2^{ab}(s)] = -\epsilon_i T^a_{ik}\epsilon_j T^b_{jl}\rho^0_{kl}(s)\,. \tag{4d}$$

In the model of current algebra defined by Eq. (1) the fields $J^\mu_a(x)$ have the quantum numbers $J^P = 1^{+,-}$. Therefore the fields $A_i(x)$ appearing in Eq. (3) have the quantum numbers $J^P = 1^{+,-}$ and also $J^P = 0^{-,+}$ for nonconserved currents. Let us suppose that in the symmetry limit to each $J^\mu_a(x)$ (conserved in this case) there corresponds one particle for which $J^\mu_a(x)$ may be taken as an extrapolating field. The same assumption is made about the fields $\phi_i(x)$. We therefore assume [in the SU(2)×SU(2) case and the $(\frac{1}{2},\frac{1}{2})$ representation of $\phi_i(x)$] the existence of ρ, A_1, π, and σ. When the symmetry is broken by the parameters $\epsilon_i \neq 0$, mixing will occur in the first order in ϵ between some states, and some mass shifts will occur in the multiplets. These facts may be taken into account, introducing the renormalization constants Z_{ij} and g_{ab}, by

$$\langle 0|J^\mu_a(0)|\lambda, p, b\rangle = \epsilon^\mu_\lambda(p) g_{ab}/(2\pi)^{3/2}\,, \tag{5a}$$

$$\langle 0|\phi_i(0)|p,j\rangle = a_{ij}/(2\pi)^{3/2}\,. \tag{5b}$$

(λ is the helicity of the vector state, p is the corresponding momentum, and a, b, i, j are the symmetry indices.)

As $\det(a_{ij}) \neq 0$, let us define $a^2 = Z^{-1}$ (in matrix notation) and the renormalized fields $\phi^r_j(x) = (Z^{1/2})_{jk}\phi_k(x)$.

For $\phi^r_k(x)$ we shall have

$$\langle 0|\phi^r_i(0)|p,j\rangle = \delta_{ij}/(2\pi)^{3/2}\,.$$

We may keep the relations (1b), (1c), (2a), and (2b) formally unchanged if we define the "renormalized constants"

$$\epsilon^r_i = \epsilon_j (Z^{1/2})_{ji}\,,$$

$$\lambda^r_i = (Z^{1/2})_{ij}\lambda_j\,,$$

$$T_{ij}^{r,a} = (Z^{-1/2})_{ik} T^a_{kl} (Z^{1/2})_{lj}\,.$$

Similar renormalization may be performed on the currents $J^\mu_a(x)$, changing the structure constants C_{abc} and $T_{ij}^{r,a}$. However, such a procedure would be unsatisfactory for nonconserved currents, and we would lose the identification of the currents with the weak and electromagnetic currents, as is the case in SU(3)×SU(3). We therefore do not perform this type of renormalization on currents, but rather use Eq. (3) to extract the 1^+ (1^-) and 0^- (0^+) parts from the currents:

$$\langle J^\mu_a(p)\cdots\rangle = V^{\mu\nu}_{ab}\langle \bar{J}^\nu_b(p)\cdots\rangle + \Delta^\mu_{aj}\langle\bar{\phi}_j(p)\cdots\rangle\,, \tag{6a}$$

$$\langle \phi_i(p)\cdots\rangle = \Delta_{ij}(p)\langle\bar{\phi}_j(p)\cdots\rangle\,, \tag{6b}$$

where

$$V^{\mu\nu}_{ab}(p) = \frac{-1}{2\pi i}\int \frac{\rho_1^{ab}(s)(g^{\mu\nu}p^\mu p^\nu/s)}{s-p^2-i\epsilon}ds\,.$$

In Eq. (6a) the amplitude $\langle \bar{J}^\nu_b(p)\cdots\rangle$ is analytic in p^2 and void of 0^+ (0^-) contributions, i.e., they have only ρ, (A_1), etc. contributions. Equations (6a) and (6b) define for us the analytic structure in the external mometa. In the next step we shall reduce the number of independent amplitudes.

Step 3: The primitive amplitudes. We recall the relation (1b),

$$\partial_\mu J^\mu_a(x) = \epsilon_i T^a_{ij}\phi_j(x)\,,$$

which may be used [whenever the fields $\phi_i(x)$ can be expressed in terms of current divergences] to express the amplitudes of fields $\phi_i(x)$ as divergences of the reduced amplitudes $\langle \bar{J}^\mu_a(p)\cdots\rangle$. We shall call *primitive* amplitudes the amplitudes containing the fields $\bar{J}^\mu_a(x)$ as well as the fields $\bar{\phi}_i(x)$ for which the relation (1b) is not invertible. We shall describe now a procedure to express all amplitudes in terms of the primitive ones.

The Ward identity may be written as

$$ip_\mu\langle J^\mu_a(p)\cdots\rangle = \langle \partial_\mu J^\mu_a(p)\cdots\rangle + \langle \hat{J}^0_a(p)\cdots\rangle\,. \tag{7}$$

$\langle \hat{J}^0_a(p)\cdots\rangle$ denotes all the terms which arise from commutation relations. For instance,

$$\langle \hat{J}^0_a(p_1)J^\mu_b(p_2)\phi_j(p_3)\rangle = iC_{abc}\langle J^\mu_a(p_1+p_2)\phi_j(p_3)\rangle - iT^a_{jk}\langle J^\mu_b(p_2)\phi_k(p_1+p_3)\rangle\,.$$

Let us substitute in Eq. (7) the decompositions (6a), (6b):

$$ip_\mu[V^{\mu\nu}_{ab}(p)\langle\bar{J}^\nu_b(p)\cdots\rangle + \Delta^\mu_{aj}(p)\langle\bar{\phi}_j(p)\cdots\rangle] = \epsilon_i T^a_{ij}\Delta_{jk}(p)\langle\bar{\phi}_k(p)\cdots\rangle + \langle\hat{J}^0_a(p)\cdots\rangle\,. \tag{8}$$

Introducing the matrix notations and using the abbreviations

$$\Delta_{ij}(p_1) \to \Delta_1\,,$$

$$V^{\mu\nu}_{ab}(p_1) \to V_1\,,$$

$$ip_\mu V^{\mu\nu}(p_1) = F_1 = p_1{}^2\Delta_L\,,$$

$$\Delta_L = \frac{-1}{2\pi}\int \frac{\rho_1(s)}{s}ds\,,$$

$$R^a_k = \epsilon_i T^a_{ij}\Delta_{jk}(0) \to R\,,$$

we write Eq. (8) as

$$R_1\langle\bar{\phi}_1(p_1)\cdots\rangle = F_1\langle\bar{J}_1(p_1)\cdots\rangle - \langle\bar{J}_1\cdots\rangle\,. \tag{9}$$

We note that if R_1 has an inverse [in the SU(2)×SU(2) case it is the number F_π/i for pions], then $\langle\bar{\phi}_1(p_1)\cdots\rangle$ may be expressed in terms of a primitive amplitude (of index 1) and amplitudes of lower order. For $R_1=0$, we obtain the so-called vector constraints on primitive amplitudes. The above procedure should be repeated for all (nonprimitive) fields $\phi_i(x)$ appearing in the amplitudes. The situation is more complicated for the case when we have two or more nonprimitive fields, but a final expression may be obtained (see Appendix A):

$$R_1\cdots R_k V_{k+1}\cdots V_n\langle\bar{\phi}_1(p_1)\cdots\bar{\phi}_k(p_k)\bar{J}_{k+1}(p_{k+1})\cdots\bar{J}_n(p_n)\rangle = F_1\cdots F_k V_{k+1}\cdots V_n\langle\bar{J}_1(p_1)\cdots\bar{J}_n(p_n)\rangle - \sum_{l=1}^{k} S_k\, i^{l-1} p_1^{\mu_1}\cdots p_{l-1}^{\mu_{l-1}} A_l\,, \tag{10}$$

where

$$A_l = \left\langle\left[\prod_{j=1}^{l-1}(J_j^{\mu_j}X_j\phi)\right]\hat{J}_l\left[\prod_{k\geq j\geq l}(R_l^a(\Delta^{-1})_{lk}\phi_k)\right]\left[\prod_{j>k}(J_j^{\mu_j}X_j\phi)\right]\right\rangle,$$

$$X_j\phi = X^{\mu_j}_{a_j k}\phi_k$$
$$= \Delta^{\mu_j}_{a_j l}(p_j)(\Delta^{-1})_{lk}\phi_k\,.$$

S_k denotes the symmetrizer on the first k indices $1\cdots k$ [$S_k=\sum(\text{permut.})/k!$]. As an example we demonstrate the use of Eq. (10) for the amplitude $\langle\bar{\phi}\bar{\phi}\bar{J}\rangle$:

$$R_1R_2V_3\langle\bar{\phi}_1\bar{\phi}_2\bar{J}_3^\mu\rangle = F_1F_2V_3\langle\bar{J}_1\bar{J}_2\bar{J}_3\rangle - \tfrac{1}{2}[R_1\Delta_1^{-1}\langle\phi_1\hat{J}_2,\,(J_3^\mu - X_3^\mu\phi)\rangle + R_2\Delta_2^{-1}\langle\hat{J}_1,\,\phi_2,\,(J_3^\mu - X_3^\mu\phi)\rangle]$$
$$-\tfrac{1}{2}i[p_1^\nu\langle(J_1^\nu - X_1^\nu\phi),\hat{J}_2,\,(J_3^\mu - X_3^\mu\phi)\rangle + p_2^\nu\langle\hat{J}_1,\,(J_2^\nu - X_2^\nu\phi),\,(J_3^\mu - X_3^\mu\phi)\rangle]\,. \tag{11}$$

Step 4: The proper vertices and contact terms. In Step 3 we were concerned with the analytic structure in the external momenta. As an n-point function has, for $n>3$, $3n-10$ other independent variables, we wish to specify also the analytic structure in the remaining variables.

First we shall define the proper vertices by the diagramatic decomposition shown in Fig. 1, in which (b) denotes the proper vertex. The amplitude (a) with k inside the circle denotes a reduced amplitude of order $n-k+1$, having in the tree decomposition of Eq. (11) proper vertices of at least order k. The wavy line denotes the possible propagator which may connect two proper vertices [they may have $J^P=1^+$ (1^-) and 0^- (0^+)]; $\sum$ stands for the sum of all possible permutations on the external legs, giving rise to different singularities in the propagators.

In the perturbation approach the proper vertices would correspond as usual to vertices which cannot be divided into two parts connected by a single propagator. As our approach is not perturbative, the decomposition of Fig. 1 states that the proper vertices do not contain singularities associated with single propagators (factorizable singularities). An example of the decomposition of Fig. 1 for $n=4$, 5 may be seen in Figs. 2(a) and 2(b). Having defined the proper vertices, we would like to get the constraints imposed on them

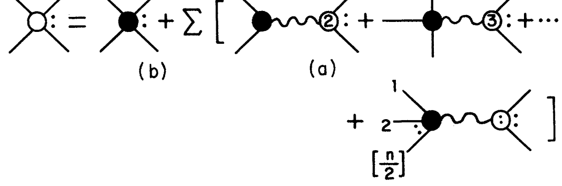


FIG. 1. Decomposition of a reduced amplitude to proper vertices.

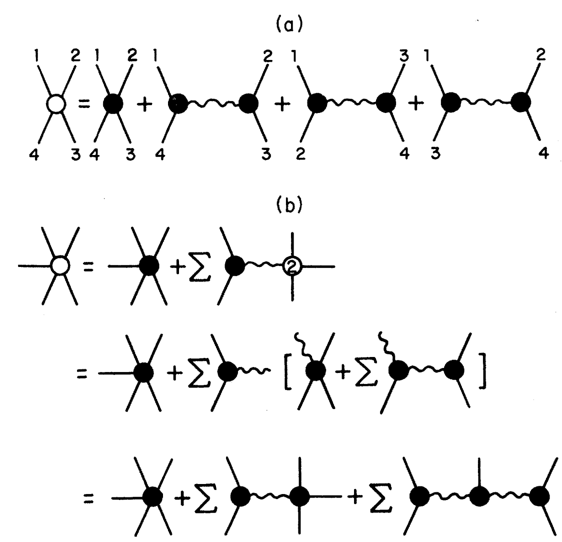


FIG. 2. Example of Fig. 1 for (a) $n=4$, and (b) $n=5$.

by the Ward identities. For conserved currents only, one can easily prove that the proper vertices obey the usual Ward identities. For the algebra defined by Eq. (1) the problem is more complicated, and one obtains simple relations only in the case of strong P.C.C. (partially conserved currents). Expressed differently, we obtain the relations stated below only under the assumption that the propagators of the fields appearing in Eq. (1b) are of the form $1/i(m^2 - p^2)$ (i.e., single-particle-approximated). In the P.C.C. assumption the Ward identity reads in short notation (see Appendix B for proof)

$$R_1\langle\bar{\phi}_1(p_1)\bar{J}_2^\mu(p_2)\bar{\phi}_3(p_3)\cdots\rangle_{\text{prop}} = \Delta_L p_1^\nu\langle\bar{J}_1^\nu(p_1)\bar{J}_2^\mu(p_2)\bar{\phi}_3(p_3)\cdots\rangle_{\text{prop}}$$
$$-iC_{12a}\langle\bar{J}_a^\mu(p_1+p_2)\bar{\phi}_3(p_3)\cdots\rangle_{\text{prop}} + iT_{3k}^1\langle\bar{J}_2^\mu(p_2)\bar{\phi}_k(p_1+p_3)\cdots\rangle_{\text{prop}}$$
$$-(1/\mu_2{}^2)[p_3^\nu\epsilon_i T_{i3}^\beta C_{1\alpha\beta} + (p_1+p_3)^\nu\epsilon_i T_{ij}^\alpha T_{j2}^1]\langle\bar{J}_2^\mu(p_2)\bar{J}_\alpha^\nu(p_1+p_3)\cdots\rangle_{\text{prop}}\,. \quad (12)$$

In Eq. (12) we have denoted by $\langle\cdots\rangle_{\text{prop}}$ the proper vertices, and have written down only the typical contributions coming from vector (axial-vector) and scalar (pseudoscalar) external legs in the proper vertices. Equation (12) may be used to reduce the number of independent proper vertices and to obtain the vector constraints on them.

Now comes the *crucial* assumption in the H.P. model. The proper vertices are assumed to be analytic in all variables. We shall call such vertices *contact terms*.

A few remarks are in order. The definition of the proper vertices has nothing to do with the model. If we avoid P.C.C. we still obtain Ward identities, which are more complicated than Eq. (12). In this case we cannot make the "analytic assumption," as the Ward identities will connect the proper vertices to others, multiplied by propagator and their inverse. The singularities of these propagators are canceled under the P.C.C. assumption. Still, we do not have to assume the single-particle approximation for the vector (axial-vector) propagators.

All four steps were carried out in the case of $SU(2)\times SU(2)$ and $n=4$ by Gerstein and Schnitzer,[7] and for general symmetry and $n=3$ (the proper vertices being the reduced amplitudes) in Ref. 5.

So far we have not included in the H.P. model any fields besides $J_a^\mu(x)$, $\phi_i(x)$. It turns out that a more general model can be built which includes other fields as well, such as a nucleon field for example. An attempt in this direction was made by Osypowski.[8] We shall not pursue this line of approach here, but rather study in the next section the connection of the H.P. model to a phenomenological Lagrangian.

III. FROM HARD PIONS TO A PHENOMENOLOGICAL LAGRANGIAN

In Sec. II we have seen that the H.P. model has a serious drawback. First, the procedure is very tedious; second, one needs the P.C.C. assumption. Making the latter assumption, one cannot expect the model to describe correctly scattering processes for all ranges of external momenta. We rather hope that the model is a good approximation for low-energy phenomena, such as soft-pion emission, determination of scattering length, etc., i.e., processes in which the momenta (external, internal) are not much higher than the masses involved. There is an alternative way of describing the above-mentioned phenomena. Among them we find the effective and phenomenological Lagrangians.[9,10,3] We shall attempt here to obtain a P.L. which will reproduce the result of H.P. described in Sec. II, at least for soft pions in the symmetry limit of $SU(2)\times SU(2)$.

Let us therefore study the process $\alpha\to\beta+n\pi$, i.e., the emission of n soft pions in the process $\alpha\to\beta$, where α and β are some hadronic states (which do not include soft pions). The T matrix for such a process is proportional to

$$\langle\alpha|\bar{\phi}_1(p_1)\cdots\bar{\phi}_n(p_n)|\beta\rangle\,,$$

where we used the definition of Sec. II, but $|0\rangle\to|\alpha\rangle$, $|\beta\rangle$. We assume the fields $\phi_i(x)$ to belong to the $(\frac{1}{2},\frac{1}{2})$ representation of $SU(2)\times SU(2)$. In this case we have $R^\alpha = F_\pi/i$, where $\alpha=1,2,3$ are the axial-vector indices and F_π is the weak-decay constant of the charged pions ($F_\pi\simeq 94$ MeV). $X_{\alpha j}^\mu(p) = -i\delta_{\alpha j}F_\pi p^\mu$, $\epsilon=\epsilon_0=F_\pi\mu^2$, and μ is the mass of the meson π. $C_{abc}=\epsilon_{abc}$ is the antisymmetric tensor of third order; $A_\alpha^\mu(x)$ are the axial-vector currents, $\partial_\mu A_\alpha^\mu(x)=F_\pi\mu^2\phi_\alpha(x)$; $V_\alpha^\mu(x)$ are the vector currents, conserved; $p_l=\xi Q_l$ are the external momenta, and $\xi\to 0$. Using Eq. (10) we obtain

$$(F_\pi/i)^n\langle\alpha|\bar{\phi}_1(p_1)\cdots\bar{\phi}_n(p_n)|\beta\rangle$$
$$=(\Delta_L)^n p_1\cdots p_n\langle\alpha|\bar{A}_1(p_1)\cdots\circ\bar{A}_n(p_n)|\beta\rangle$$
$$+\,(\text{commutation relations}). \quad (13)$$

The first term on the right-hand side of Eq. (13) is is of the order ξ^0 (the Adler consistency condition) and obtains contributions whenever the currents $A^\mu_\alpha(x)$ (axial-vector) can be attached to the external legs [it would be of the order ξ^1 if $|\alpha\rangle$ (or $|\beta\rangle$) were a vacuum state].

In the "commutation relation" we distinguish among three classes of contribution, which stem from the following typical commutators:

$$\begin{aligned} &\textit{class a: } R\Delta^{-1}[A^0_\alpha, \phi_l], \\ &\textit{class b: } p\cdot X[A^0_\alpha, \phi_l], \\ &\textit{class c: } p^\mu[A^0_\alpha, A^\mu_\beta]. \end{aligned} \tag{14}$$

In the limit of $\epsilon\to 0$ ($\mu^2\to 0$), $\xi\to 0$, we have $p\cdot X \to p^2=\mu^2=O(\epsilon)$ and $R\Delta^{-1}\to(F_\pi/i)(\mu^2-p^2)\to 0$. Therefore only class c will contribute to order $1(\epsilon)$. It can be shown by induction that a typical term arising from class c will be of the form

$$A_m p^{\mu_1}\cdots p^{\mu_m} C_1\cdots C_m\langle\alpha|\bar{J}^{\mu_1}(\bar{p}_1)\cdots\bar{J}^{\mu_m}(\bar{p}_m)|\beta\rangle, \tag{15}$$

where A_m is a numerical coefficient. $\{\bar{p}_1\},\ldots,\{\bar{p}_m\}$ is a decomposition of the set $\{p_1,\ldots,p_n\}$ into disjoint sets; $\bar{p}_i$ denotes also the sum of all momenta in the set i, $p^{\mu_i}\in\{\bar{p}_i\}$. The coefficients C_i are the numerical coefficients obtained by successive commutations of currents, i.e., in the expression

$$[A^0_{a1}(p_1),\ldots A^0_{a_{m-1}}(p_{m-1}), A^0_{a_m}(p_m)];\quad p_j\in\{\bar{p}_j\}.$$

The currents $\bar{J}^\mu_a$ are vector currents $\bar{V}^\mu_a$ or axial-vector currents $\bar{A}^\mu_\alpha$ (free of pion poles).

Formally, such terms may be generated by the functional

$$F(\varphi)=T^*\exp[iB(\varphi)],$$

where

$$B(\varphi)=\int D(\varphi_t)A^\mu_\alpha(\vec{x},t)D^{-1}(\varphi_t)\partial_\mu\varphi^\alpha(\vec{x},t)dx,$$

$$D(\varphi_t)=\exp\left[i\int G(\varphi^2)A^0_\alpha(\vec{x},t)\varphi^\alpha(\vec{x},t)d^3x\right].$$

Here the field φ has to be understood as an external pion field, and the amplitude for pion emission is obtained by functional differentiation and the Fourier transform.

In the case of SU(2)×SU(2), which we treat here, the functional may be brought to a more compact form by introducing a rotation around the "direction" $\vec{\varphi}(x)$ by an angle $\theta=\pm|G(\varphi^2)(\varphi^2)^{1/2}|$.[3] The result is

$$\begin{aligned} B(\varphi)=\int\Big\{&\sin(\theta)\vec{\bar{V}}^\mu(y)\cdot\left[\frac{1}{(\varphi^2)^{1/2}}\vec{\varphi}(y)\times\partial_\mu\vec{\varphi}(y)\right] \\ &+\cos(\theta)\vec{\bar{A}}^\mu(y)\cdot\partial_\mu\vec{\varphi}(y) \\ &+\frac{1-\cos(\theta)}{\varphi^2(y)}[\vec{\varphi}(y)\cdot\partial_\mu\vec{\varphi}(y)][\vec{\bar{A}}^\mu(y)\cdot\vec{\varphi}(y)]\Big\}dy. \end{aligned} \tag{16}$$

Here the current $\vec{\bar{A}}^\mu(y)$ is an axial-vector current void of pion poles. The exact expression for $G(\varphi^2)$ [which determines $B(\varphi)$] would be obtained by counting the numerical coefficients A_m appearing in (15). In Eq. (16) $B(\varphi)$ should be thought of as representing a phenomenological Lagrangian describing the coupling of currents $\vec{\bar{V}}^\mu(x)$ and $\vec{\bar{A}}^\mu(x)$ to the pionic field $\vec{\varphi}(x)$. The currents $\vec{\bar{V}}^\mu(x)$ and $\vec{\bar{A}}^\mu(x)$ are phenomenological ones built from the fields appearing in the states α and β, since in the limit as $\epsilon\to 0$, and $\xi\to 0$, only the phenomenological structure of the currents is important.[11] In the calculations of diagrams using the Lagrangian (16) only tree diagrams should enter, as these were built in the H.P. model we started with.

It should be noted that we have omitted the case of $|\alpha\rangle=|\beta\rangle=|0\rangle$, i.e., the pion scattering. We treat this case separately. We shall see that the process is of the order ξ^2; therefore, in the expansion given by Eq. (10) only terms where at most two p's appear will contribute. As we do not calculate the exact numerical coefficients, we wish to indicate how different contact terms and tree diagrams arise in this case. We start with Eq. (10). On the right-hand side we keep only two types of contributions (the others are of orders less than ξ^2):

$$p_2^\mu\langle\hat{A}_1^0,(A_2^\mu-X_2^\mu\phi), R_3\Delta_3^{-1}\phi, R_4\Delta_4^{-1}\phi,\ldots\rangle, \tag{17a}$$

$$p_2^\mu p_3^\nu\langle\hat{A}_1^0,(A_2^\mu-X_2^\mu\phi),(A_3^\nu-X_3^\nu\phi), R_4\Delta_4^{-1}\phi,\ldots\rangle. \tag{17b}$$

In both terms only the commutator between the currents will contribute to the leading order. The reason is as follows: $p^\mu X^\mu=O(\epsilon)$, $R\Delta^{-1}\propto(p^2-m^2)$, while $[A^0,\varphi]\propto\sigma$, and the σ particle has mass different from zero. Therefore the pole coming from the σ commutator will not cancel the zero of Δ^{-1} in the limit of $\xi\to 0$, $\epsilon\to 0$. In other words, in the above-mentioned limits we shall not have any σ contributions. From Eq. (17) we therefore obtain typical contributions of the form

$$p_2^\mu\langle V_1^\mu(p_1+p_2), R_3\bar{\phi}_3, R_4\bar{\phi}_4,\ldots\rangle, \tag{18a}$$

$$p_2^\mu p_3^\nu\langle V_1^\mu(p_1+p_2),\bar{A}^\nu(p_3), R_4\bar{\phi}_4,\ldots\rangle. \tag{18b}$$

In the next step [using Eq. (10)] we shall obtain typical terms of the types

$$p_2^\mu \langle A^\mu(p_1+p_2+p_3), R_4\bar{\phi}_4, \ldots \rangle, \qquad (19a)$$

$$p_2^\mu p_3^\nu \langle V^\mu(p_1+p_2), V^\nu(p_3+p_4), R_5\bar{\phi}_5, \ldots \rangle, \qquad (19b)$$

$$p_2^\mu p_3^\nu \langle A^\mu(p_1+p_2+p_4), \bar{A}^\nu(p_3), R_5\bar{\phi}_5, \ldots \rangle. \qquad (19c)$$

Expression (19a) may be rewritten, using Eq. (6a), as

$$\Delta_L\, p_2^\mu \langle \bar{A}^\mu(p_1+p_2+p_3), R_4\bar{\phi}_4, \ldots \rangle + p_2^\mu \Delta^\mu(p_1+p_2+p_3)\langle \bar{\phi}(p_1+p_2+p_3), R_4\bar{\phi}_4, \ldots \rangle. \qquad (20)$$

$p_2^\mu \Delta^\mu(p_1+p_2+p_3)$ has a pole at $(p_1^2+p_2^2+p_3^2)=\mu^2$ and is of the order ξ^0. These terms correspond therefore to the tree diagrams of the form shown in Fig. 3. The above extraction of the pion poles from the currents $A_a^\mu(x)$ will in general generate for us all the different tree diagrams which may occur, while the other terms will contribute to the contact term of order n which will at the end be the sum of terms of the form

$$Cp_1 \cdot p_2(\delta_{13}\delta_{24}\delta_{56}\cdots).$$

As we have not counted the numerical coefficients C, we will write down only the formal expression which generates such terms:

$$\mathcal{L}(\varphi) = G_1(\varphi^2)(\partial_\mu \vec{\varphi}\cdot\vec{\varphi})^2 + G_2(\varphi^2)(\partial_\mu\varphi)^2. \qquad (21)$$

The process of pion scattering is described by the above Lagrangian, using only tree diagrams (of course one has to check whether the numerical coefficients in the tree diagrams and contact terms are correct).

We note then that in the above Lagrangian the field σ does not appear. As we started with the chiral-invariant theory and the above Lagrangian was obtained in the limit of $\epsilon\to 0$, $p\to 0$, the Lagrangian should be invariant under chiral transformations. Thus the fields $\varphi_i(x)$ have to transform nonlinearly.

The transformation properties of the fields $\varphi_i(x)$ are defined by the functionals $G_1(\varphi^2)$, $G_2(\varphi^2)$. Using Eqs. (47) and (45) of Weinberg[2] we should identify

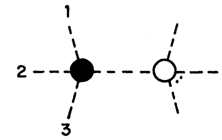


FIG. 3. The pion pole in a tree diagram.

$$G_2(\varphi^2) \propto [f^2(\varphi^2)+\varphi^2]^{-1},$$
$$G_1(\varphi^2) \propto \{[f^2(\varphi^2)+\varphi^2]^{-1}[f^1(\varphi^2)-\tfrac{1}{2}V(\varphi^2)]\}^2\varphi^2 + [f^2(\varphi^2)+\varphi^2]^{-3/2}[f^1(\varphi^2)-\tfrac{1}{2}V(\varphi^2)]. \qquad (22)$$

The transformation properties of $\varphi_i(x)$ under axial transformation will be given by

$$[Q_a, \varphi_b(x)] = -i[\delta_{ab}f(\varphi^2) + \varphi_a\varphi_b g(\varphi^2)].$$

[The functions $g(\varphi^2)$, $V(\varphi^2)$, $f(\varphi^2)$ are defined by Weinberg.[2]]

IV. APPLICATION

(a) K_{l3}. Although the main purpose of this paper was to give a unified H.P. model and to link it to a phenomenological Lagrangian, we shall give some application of the H.P. model presented. We have chosen a simple problem of finding the function $f_+(t)$ in the K_{l3} process. This process was successfully treated by Arnowitt *et al.*[12] using the effective-Lagrangian approach, and by Ecker[13] using a noncovariant H.P. model. Our results will be closely related to the latter.

The amplitude $f_+(t)$ is defined by

$$\langle \pi^0(p_2)|V_\alpha^\mu(0)|K^+(p_1)\rangle = \frac{i}{2(2\pi)^3}[(p_1-p_2)^\mu f_-(t) + (p_1+p_2)^\mu f_+(t)],$$
$$V_\alpha^\mu = \frac{1}{\sqrt{2}}(V_4^\mu - iV_5^\mu). \qquad (23)$$

Applying the reduction technique and Eq. (11) (in this case $R_1 = F_\pi/i$, $R_2 = F_K/i$), we arrive at

$$(-)F_\pi F_K \langle \bar{\pi}^0(-p_2)\bar{K}^+(p_1)V^\mu\rangle = F_1^\nu F_2^\lambda \langle \bar{A}_0^\nu(-p_2)\bar{A}_K^\lambda(p_1)V_\alpha^\mu(p_3)\rangle + \text{commutation relations}. \qquad (24)$$

We may now extract, according to Eq. (6a), the K^* and κ contributions to V_α^μ. But it is easy to see that the κ will contribute to the $f_-(t)$ only. Collecting all the terms proportional to $(p_1+p_2)^\mu$, we obtain

$$F_K F_\pi f_+(t) = \tfrac{1}{2}(F_K^2 + F_\pi^2 - C_v) + \tfrac{1}{2}\int \frac{\rho_1^{K^*}(a)}{2\pi(a-t)}da + [(p_1+p_2)^\mu \text{ coefficient in } F_1^\nu F_2^\lambda V_3^{\mu\epsilon}\langle \bar{A}_0^\nu(-p_1)\bar{A}_K^\lambda(p_2)\bar{V}_{K^*}^\epsilon(p_3)\rangle],$$
$$t = p_3^2, \quad C_v = \int \frac{\rho_2^{K^*}(a)\,da}{2\pi a}. \qquad (25)$$

We now need some estimation of the last terms in Eq. (25).

Nutbrown[14] gave an interesting integral representation for a 3-point function in the H.P. model in the SU(2)×SU(2) case. Let us make the crude approximation of SU(3) symmetry on the reduced 3-point functions of currents. More specifically, we assume that $\rho_1^{A_1}(a)=\rho_1^{K_A}(a)$. Therefore, using an equation similar to (4.2) of Ref. 14 we obtain for the coefficient $(p_1+p_2)^\mu$ the expression

$$K(t)=\tfrac{1}{2}Ct\int\frac{\rho_1^{K^*}(a)da}{a(a-t)2\pi}\,. \tag{26}$$

The constant C contains information on the anomalous magnetic moment of K_A and on the spectral function $\rho_1^{K_A}(a)$, but it need not be specified for our purpose. Let us use the result[13]

$$t|f_+(t)|^2\underset{t\to\infty}{\longrightarrow}0,$$

and suppose that Eq. (25) represents $f_+(t)$ for all values of t. (From the H.P. model one expects this to be correct for small values of t.) Therefore we obtain the sum rule

$$F_K^{\,2}+F_\pi^{\,2}-C_v=CC_v^1,\qquad C_v^1=\int\frac{\rho_1^{K^*}(a)}{2\pi a} \tag{27}$$

(in contrast to the sum rule $F_\pi^{\,2}+F_K^{\,2}=C_v$ in Ref. 13). Introducing for small values of t the usual parametrization

$$f_+(t)=f_+(0)\left(1+\frac{\lambda_+}{m_\pi^{\,2}}\,t\right),$$

we obtain

$$f_+(0)=\frac{1}{2F_\pi F_K}\,(F_K^{\,2}+F_\pi^{\,2}-C_v+C_v^1)=\frac{1}{2F_\pi F_K}\,(F_K^{\,2}+F_\pi^2-F_\kappa^{\,2}), \tag{28}$$

which is the Glashow-Weinberg[15] value.

$$\lambda_+=\frac{m_\pi^2}{2f_+(0)F_\pi F_K}\,(\tilde{C}_v+C\tilde{C}_v),\qquad \tilde{C}_v=\int\frac{\rho_1^{K^*}(a)da}{2\pi a^2}\,. \tag{29}$$

From the sum rule (27) and Eq. (28) we get

$$(C+1)C_v^1=2f_+(0)F_\pi F_K,$$

and therefore

$$\frac{\lambda_+}{m_\pi^{\,2}}=\frac{\tilde{C}_v}{C_v^1}\,. \tag{30}$$

In the pole approximation we have the well-known result

$$\lambda_+=\frac{m_\pi^{\,2}}{m_K^{\,2}}\simeq 2.5\times10^{-2},$$

which is consistent with experiment.[16]

How strongly does our result depend on P.C.C.? The answer is that it does not, as a pole approximation on the mass shell ($p_1^{\,2}=m_\pi^{\,2}$, $p_2^{\,2}=m_K^{\,2}$) is a correct one and the κ component in $V^\mu_{4+i5}(x)$ does not contribute to $f_+(t)$. The F_π, F_K, etc., should stand for $F_\pi m_\pi^{\,2}\Delta_\pi(0)$, $F_K m_K^{\,2}\Delta_K(0)$, etc. [see Eqs. (8) and (9)]. On the other hand the result for the last term in Eq. (25) depends on the model used. Thus, taking our assumption of SU(3) symmetry of the *reduced* 3-point function literally and still using the Nutbrown representation, we would obtain instead of the result (26) the expression

$$K(t)=C\int\frac{\rho_1^{K^*}(a)da}{2\pi(a-t)}\left[\int\frac{\rho_1^\rho(a)da}{(a-t)2\pi}\right]^{-1}\int\frac{\rho_1^\rho(a)da}{2\pi a(a-t)}\,,$$

and therefore get a different result for λ_+. (Both results are equal in the pole approximation.)

What we know quite generally on $K(t)$ is that it is bound by a constant at $t\to\infty$ and is an analytic function on the cut plane from $t=(m_\pi+m_K)^2$ to $t=\infty$. Different assumptions on its spectral representation may lead to different results for $f_+(0)$ and λ_+.[13]

(b) $\gamma N\to\pi N$. In the first example we dealt with a 3-point function only. To demonstrate fully the use of the pole extraction and of the contact terms we have chosen the photoproduction process $\gamma N\to\pi N$. This process is described by the following T matrix:

$$T\propto\lim_{\substack{p_{1,2}\to M\\ q^2\to0\\ k^2\to\mu^2}}\bar{u}(p_2)\epsilon^\mu(q)[(-\not{p}_2+M)\langle J^\mu_{\rm em}(q)\varphi_\alpha(k)\psi(p_2)\bar{\psi}(p_1)\rangle(-\not{p}_1+M)(\mu^2-k^2)]u(p_1)\,. \tag{31}$$

To explore in our method the expression $\langle J_{\rm em}\varphi\psi\bar{\psi}\rangle$ we have to specify the commutation relations of the currents involved $[J_{\rm em}=J_3+(1/\sqrt{3})J_8]$ with the nucleon fields (resonances). This commutation relations are model-dependent. We adopt similar commutation relations to those of Osypowski[8] [SU(2)×SU(2)]:

$$\delta(x_0)[V_\alpha^0(x),\psi_r(0)]=-T_\alpha^r\psi_r(0)\delta^4(x),\qquad \delta(x_0)[A_\alpha^0,\psi_r(0)]=+T_\alpha^r\gamma_5\psi_r(0)\delta^4(x)\,. \tag{32}$$

The index r labels the isospin of the field. T_α^r is the matrix representing the rotation around the α direc-

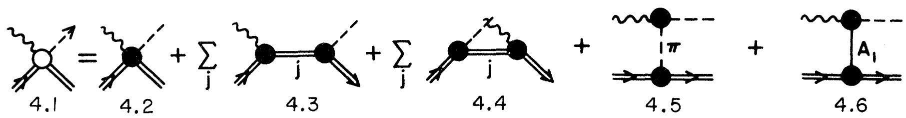


FIG. 4. Tree decomposition for $\gamma N \rightarrow \pi N$.

tion in the corresponding isospin space (i.e., $T^{1/2}_{\alpha} = \tau_{\alpha}$)

$$\delta(x_0)[J^0_{\rm em}(x), \psi_r(0)] = (T^r_3 + \tfrac{1}{2})\psi_r(0)\delta^4(x) . \tag{33}$$

In the first step we write the decomposition of the reduced amplitude into the tree diagrams. This step is shown in Fig. 4. In Fig. 4 $\sum_j$ stands for the sum over πN resonances (including N) having different (I, J^P) assignments [i.e., N and $P_{11}(1970)$ are included in the same diagram]. We need now to parametrize the different independent contact terms. The 3-point functions including only mesons (the one independent is the $\langle \overline{V}\overline{A}\overline{A}\rangle$ vertex) may be taken from Ref. 1 or Ref. 14, introducing one unknown parameter δ (the anomalous magnetic moment of A_1) for which the most probable experimental as well as theoretical value is $\delta = -\frac{1}{2}$. To parametrize the $\langle J_{\rm em}\overline{A}\psi\overline{\psi}\rangle_{\rm cont}$ term [to which $\langle J_{\rm em}\varphi\psi\overline{\psi}\rangle_{\rm cont}$ is connected by Eq. (12)] we adopt the old hard-pion hypothesis and assume that it is of zeroth order in the current momenta. Consequently the 3-point functions will be of first order in the current momentum. This sort of approximation we expect to be reasonable in the low region of the photon beam momentum, say $P_\gamma < 1$ GeV/c. This limitation is also expected from the following considerations. Adopting the Rarita-Schwinger representation for high-spin fields, the electromagnetic form factor (between nucleon and resonance of spin J) is [12]

$$\langle V^{\mu}(q)\psi_J^{\lambda_1\cdots\lambda_{J-1/2}}(p_2)\overline{\psi}(p_1)\rangle = q^{\lambda_2}q^{\lambda_3}\cdots q^{\lambda_{J-1/2}}(A_1^J\gamma^{\nu} + A_2^J p_1^{\nu} + A_3^J p_2^{\nu})(q^{\nu}g^{\lambda_1\mu} - q^{\lambda_1}g^{\mu\nu})\gamma_5{}^{a} , \tag{34}$$

where $a = 0$ (1) for natural (unnatural) parity of the resonance. This form is obviously of order q^2 at least for $J \geq \frac{5}{2}$. Therefore in this approximation the $F_{15}(1688)$ decouples, which is inconsistent with experiment; therefore the above approximation cannot hold within the energy range where the $F_{15}(1688)$ may be produced, i.e., $P_\gamma > 1$ GeV/c. Limiting ourselves to the $P_\gamma < 1$ GeV/c we have the four important resonances, $P_{11}(1470)$, $D_{13}(1520)$, $S_{11}(1535)$, and $P_{33}(1236)$. Let us begin with

$$\langle \overline{V}^{\mu}_{\alpha}(q)\overline{A}^{\nu}_{\beta}(k)\psi(p_1)\overline{\psi}(p_2)\rangle_{\rm cont}, \quad \alpha = 3, 8.$$

We shall have to deal separately with the isoscalar and isovector parts of the electromagnetic current. The independent tensors in both cases are

$$\gamma_5(g^{\mu\nu}, \sigma^{\mu\nu}, P^{\mu}P^{\nu}, \gamma^{\mu}P^{\nu}, P^{\mu}\gamma^{\nu}, \ldots), \quad P = p_1 + p_2 .$$

Making isospin decomposition in the t channel, we may use "G parity" to reduce the number of independent parameters. The restrictions from G parity are that for the isoscalar case the possible tensors are

$$\gamma_5(g^{\mu\nu}\not{P}, P^{\mu}\gamma^{\nu}, \gamma^{\mu}P^{\nu}, \sigma^{\mu\nu}, \ldots),$$

and for the isovector case the possible tensors are

$I_t = 1$: $\gamma_5(g^{\mu\nu}, P^{\mu}P^{\nu})$,

$I_t = 0$: as in the isoscalar case.

We know that the $\langle \overline{A}^{\mu}_{\alpha}(k)\psi(p_2)\overline{\psi}(p_1)\rangle_{\rm cont}$ has to be of first order in k. We have also the vector constraints

$$a_{\rho}q^{\mu}\langle V^{\mu}_{\alpha}(q)\overline{A}^{\nu}_{\beta}(k)\psi(p_2)\overline{\psi}(p_1)\rangle_{\rm cont} = i\epsilon_{\alpha\beta\gamma}\langle A^{\nu}_{\gamma}(k-q)\psi(p_2)\overline{\psi}(p_1)\rangle - \tfrac{1}{2}\tau_{\alpha}\langle A^{\nu}_{\beta}(k)\psi(p_2-q)\overline{\psi}(p_1)\rangle$$
$$+ \langle A^{\nu}_{\beta}(k)\psi(p_2)\overline{\psi}(p_1+q)\rangle\tfrac{1}{2}\tau_{\alpha}, \tag{35}$$

$$a_{\omega}q^{\mu}\langle V^{\mu}_{8}(q)A^{\nu}_{\beta}(k)\psi(p_2)\overline{\psi}(p_1)\rangle_{\rm cont} = -\tfrac{1}{2}\sqrt{3}\,\langle A^{\nu}_{\beta}(k)\psi(p_2-q)\overline{\psi}(p_1)\rangle + \tfrac{1}{2}\sqrt{3}\,\langle A^{\nu}_{\beta}(k)\psi(p_2)\overline{\psi}(p_1+k)\rangle . \tag{36}$$

Parametrizing the 3-point functions to first order in the axial momentum, we finally obtain from (35) and (36) that there are only three independent constants and

$$\langle A^{\mu}_{\alpha}(k)\psi(p_2)\overline{\psi}(p_1)\rangle = \gamma_5\tau_{\alpha}(\gamma^{\mu}G_1 + k^{\mu}G_2 + i\sigma^{\mu\nu}P^{\nu}G_3), \tag{37}$$

$$\langle V^{\mu}_{3}(q)A^{\nu}_{\alpha}(k)\psi(p_2)\overline{\psi}(p_1)\rangle = \gamma_5(G_2\delta_{3\alpha}i\sigma^{\mu\nu} - G_3\tfrac{1}{2}[\tau_3, \tau_{\alpha}]g^{\mu\nu}), \tag{38}$$

$$\langle V^{\mu}_{8}(q)A^{\nu}_{\alpha}(k)\psi(p_2)\overline{\psi}(p_1)\rangle = \gamma_5 G_2\tau_{\alpha}i\sigma^{\mu\nu}\sqrt{3} . \tag{39}$$

The constants G_1 and G_3 are connected to the weak decay constant of $n \rightarrow pe\nu$, $2F_A$ by $G_1 - 2MG_3 = 2F_A = 1.18$. For the S_{11}, D_{13} resonance the contact terms have five constants:

$$\langle A_\alpha^\mu(k)\psi_{S_{11}}(p_2)\bar\psi(p_1)\rangle = \tau_\alpha(\gamma^\mu D_1 + k^\mu D_2 + P^\mu D_3 + \sigma^{\mu\nu}k^\nu D_4 + \sigma^{\mu\nu}P^\nu D_5), \tag{40}$$

$$\langle A_\alpha^\mu(k)\psi_{D_{13}}(p_2)\bar\psi(p_1)\rangle = \tau_\alpha(\gamma^\mu E_1\cdots). \tag{41}$$

In the canonical basis $(T_\alpha)ij = \langle 1-\alpha|\frac{3}{2}, -i; \frac{1}{2}, j\rangle$. For the P_{33} we have satisfying our assumptions eight tensors, which reduce to only two on the mass shell (of the nucleon and the resonance). [Schematically $g^{\mu\lambda}(1, \not k, \not p)$, $\gamma^\mu(P^\lambda, k^\lambda)$, $\gamma^\lambda(P^\mu, k^\mu)$, $\sigma^{\mu\lambda} \to (g^{\mu\lambda}, \gamma^\mu k^\lambda)$.] We have no theoretical reason to abandon any of the tensors, but in the actual fit the probable most predominant contribution to the over-all amplitude will be from the s-channel pole in the vicinity of the pole, where only one coupling (in πNN^*) will be important. From this point of view we may be satisfied with one coupling only.

The propagators are taken to be of the form

$$\Lambda/(p^2 - M^{*2} + iM\Gamma)$$

where Λ is an appropriate projection operator. The S_{11} propagator [which should include the nucleon and the $S_{11}(1535)$] may be approximated by

$$S_F(p) = \frac{\not p + M}{p^2 - M^2 + i\epsilon} + a\frac{\not p + M^*}{p^2 - M^{*2} + iM\Gamma}, \tag{42}$$

and the $\langle \phi, \psi, \bar\psi\rangle$ is found from

$$(F_\pi/i)\langle\phi_\alpha(k)\psi(p_2)\bar\psi(p_1)\rangle = \Delta_A^L k^\mu\langle\bar A_\alpha^\mu(k)\psi(p_2)\bar\psi(p_1)\rangle + \tau_\alpha\gamma_5\langle\psi(p_2-k)\bar\psi(p_1)\rangle - \langle\psi(p_2)\bar\psi(p_1+k)\rangle\gamma_5\tau_\alpha, \tag{43}$$

which is found to be *not* smooth. [It is interesting to note that $S_F(p)$ may be found by solving an integral equation emerging from the unitarity condition (the continuum approximated by the πN state) and using Eq. (43).] We need some extrapolation in p_1^2, p_2^2 in the form factor:

$$\langle J_{\rm em}^\mu(q)\psi(p_1)\bar\psi(p_2)\rangle = \tfrac{1}{2}(\tau_3+1)\gamma^\mu + [\tfrac{1}{2}(\tau_3+1)\mu_p + \tfrac{1}{2}(\tau_3+1)\mu_N]\sigma^{\mu\nu}q^\lambda. \tag{44}$$

The simplest way which will take into account the experimental fact that the $P_{11}(1470)$ couples weakly to the electromagnetic current will be to multiply Eq. (44) by

$$\frac{(p_1^2 - M^{*2})(p_2^2 - M^{*2})}{(M^{*2} - M^2)^2}.$$

(p_1^2 or p_2^2 will actually be equal to M^2.) We may as well try the form $a + bp^2$ and leave the constants to be found by the numerical fit. We have thus finished building the complete amplitude for photoproduction in the $P_\gamma < 1$ GeV/c region, using the hard-pion approach. We do not attempt an actual numerical fit to the data, as this is out of the scope of this paper (and will be done elsewhere), but conclude this section with a few remarks.

The main feature of the amplitude is the resonances which are built in. In this respect it resembles strongly the old isobar model,[18,19] but the t-u channel structure is different. In the u channel appear all the resonances which contribute to the s channel; in the t channel we have the additional pole due to A_1. All current-algebra constraints have been built in systematically. Some of the couplings are connected directly to the physical values of the width of resonances. We have also given the specific form of the background contribution (in the low partial waves) which is due to the 4-point contact term. (This may explain the "additional contribution"[19] in Walker's terminology.)

V. SUMMARY

In the first part of this work we built a hard-pion model for a general symmetry scheme. Using the strong-P.C.C. assumption we were able to obtain Ward-like identities for the contact terms, thus reducing the number of unknown amplitudes and obtaining some constraints on the others. It has to be noted that the H.P. model is not compatible with unitarity. We observe from Eq. (10) (for conserved currents) that for $n = 2$

$$p_1^\mu\langle\bar J^\mu(p_1)\bar J^\nu(p_2)\bar J^\lambda(p_3)\rangle_{\rm prop} \propto [\Delta^{\nu\lambda}(p_3)]^{-1} - [\Delta^{\nu\lambda}(p_2)]^{-1},$$

and therefore $[\Delta^{\nu\lambda}(p)]^{-1}$ has to be analytic, which is in direct contradiction to the existing cuts in the spectral decomposition. One may approximate those cuts by a finite or an infinite number of poles. In the former we shall obtain that [see Eq. (10)] an integer k exists such that all contact terms of order greater than k will vanish. The converse is also true, i.e., the existence of the above k will force the $[\Delta^{\nu\lambda}(p)]^{-1}$ to be a finite polynomial in p^2. The approximation of the cut by an infinite number of poles seems therefore more natural in the H.P. model.

In the second part of this work we studied the processes of soft-pion emission in the symmetry limit. We obtained that to order $1(\epsilon)$ the processes $\alpha \to \beta + n\pi$ are of the order ξ^0 for $|\alpha\rangle = |\beta\rangle = |0\rangle$, of the order ξ^1 if $|\alpha\rangle = |0\rangle$ (or $|\beta\rangle = |0\rangle$), and of the order ξ^2 for $|\alpha\rangle=|\beta\rangle=|0\rangle$. In this limit we pointed out how a phenomenological Lagrangian arises, which, in the tree approximation, reproduces the same results as the H.P. model does.

In the last section we applied the H.P. model together with the suggestion of Nutbrown for 3-point functions to find the form factor $f_+(t)$ in the K_{l3} process. We recovered the Glashow-Weinberg value for $f_+(0)$. In the pole approximation for the vector spectral function (H.P. is not bound, as has been shown, to this approximation) we got

$$\lambda_+ \simeq m_\pi^2/m_K^3 \simeq 2.5\times10^{-2}.$$

The high-energy contribution will probably lower this value. We also built a hard-pion amplitude for the photoproduction process $\gamma N\to\pi N$. With the aid of current algebra and partial conservation of axial-vector current (PCAC), we have been able to reduce considerably the number of independent coupling constants. The amplitude obtained resembles the isobar model, but we have been able to give a specific form to the "additional" background contributions which are not due to s, t, u poles. Adopting the "smoothness assumption" for the 4-point function, we were forced to limit ourselves to the $P_\gamma<1$ GeV/c region. We have also seen that by releasing the propagator from the one-particle approximation the 3-point function becomes "unsmooth."

APPENDIX A

We prove Eq. (10) by induction. From Eq. (9) and Eq. (3) we obtain $[\Delta^\mu_{ai}(p_a)\to D(a)]$

$$R_1\langle\bar\phi_1, J_2\cdots\rangle = F_1\langle\bar J_1, J_2\cdots\rangle - \langle\hat J_1, J_2\cdots\rangle$$

$$\Rightarrow R_1[V(2)\langle\bar\phi_1,\bar J_2\cdots\rangle + D(2)\langle\bar\phi_1\bar\phi_2\cdots\rangle] = F_1[V(2)\langle\bar J_1,\bar J_2\cdots\rangle + D(2)\langle\bar J_1,\bar\phi_2\cdots\rangle] - \langle\hat J_1, J_2\cdots\rangle, \tag{A1}$$

$$R_1\Delta(2)\langle\bar\phi_1\bar\phi_2\cdots\rangle = F_1\Delta(2)\langle\bar J_1,\bar\phi_2\cdots\rangle - \langle\hat J_1,\phi_2\cdots\rangle$$

$$\Rightarrow R_1\langle\bar\phi_1\bar\phi_2\cdots\rangle = F_1\langle\bar J_1,\bar\phi_2\cdots\rangle - \Delta(2)^{-1}\langle\hat J_1,\phi_2\cdots\rangle\,. \tag{A2}$$

Multiplying Eq. (A2) by $D(2)$ and subtracting the result from Eq. (A1) lead to

$$R_1V(2)\langle\bar\phi_1,\bar J_2\cdots\rangle = F_1V(2)\langle\bar J_1,\bar J_2\cdots\rangle - \langle\hat J_1,(J_2-x_2\phi)\cdots\rangle\,. \tag{A3}$$

Let us use Eq. (10) (by induction) and Eq. (3) to obtain

$$R_1\cdots R_kV(k+2)\cdots V(n)\langle\bar\phi_1\cdots\bar\phi_k,\phi_{k+1},\bar J_{k+2}\cdots\bar J_n\rangle$$
$$= F_1\cdots F_kV(k+2)\cdots V(n)\langle\bar J_1\cdots\bar J_k\phi_{k+1},\bar J_{k+2}\cdots\bar J_n\rangle + \sum(\text{commut. relations})$$
$$\Rightarrow R_1\cdots R_kR_{k+1}V(k+2)\cdots V(n)\langle\bar\phi_1\cdots\bar\phi_{k+1},\bar J_{k+2}\cdots\bar J_n\rangle$$
$$= F_1\cdots F_kR_{k+1}V(k+2)\cdots V(n)\langle\bar J_1\cdots\bar J_k\bar\phi_{k+1},\bar J_{k+2}\cdots\bar J_n\rangle - R_{k+1}\Delta^{-1}(k+1)\sum(\text{commut. relations}). \tag{A4}$$

From Eq. (A3) we have

$$V(1)\cdots V(k)R_{k+1}V(k+2)\cdots V(n)\langle\bar J_1\cdots\bar J_k,\bar\phi_{k+1},\bar J_{k+2}\cdots\bar J_n\rangle$$
$$= V(1)\cdots V(k)F_{k+1}V(k+2)\cdots V(n)\langle\bar J_1\cdots\bar J_{k+1}\cdots\bar J_n\rangle - \langle(J_1-x_1\phi)\cdots\hat J_{k+1}\cdots(J_n-x_n\phi)\rangle\,. \tag{A5}$$

Multiplying Eq. (A5) by $p_1^{\mu_1}\cdots p_k^{\mu_k}$, subtracting the result from (A5), and then performing symmetrization, we obtain the desired result in Eq. (10).

APPENDIX B

Equation (13) will be proved by induction. As we shall represent the proof diagrammatically, let us use the following abbreviations: A dashed (solid) line labeled with indices $k, 1$ ($\alpha, \mu, 1$) represents a scalar (vector) external leg (or internal leg, then representing a propagator) of symmetry index k (α), momentum p_1 (Lorentz index μ) ($p_{1+2}\equiv p_1+p_2$).

Let us write the decomposition of Fig. 1 in a different way, as shown in Fig. 5, where we have classified different types of diagrams in which the external scalar leg labeled by index 1 appears. In class a it appears in a 3-point function; the other two legs are also scalars (pseudoscalars). Class b differs from class a by the vector internal leg instead of the scalar one. In class c the external leg numbered 1 cou-

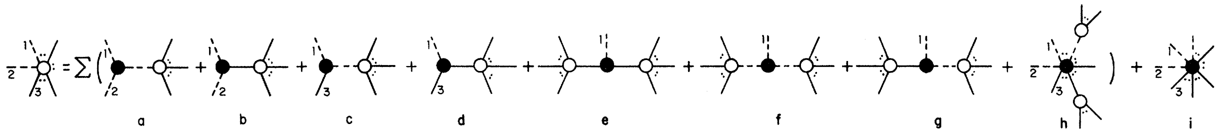


FIG. 5. Explicit appearance of 3-point functions in Fig. 1.

ples with the external vector and the internal scalar. In class d the external leg (1) couples with two vectors. In class e, as in class d, it couples with two vectors, but both are internal, and in class f it appears with two internal scalars. In class g it couples with the internal vector and the internal scalar. In class h it couples with a contact term of order greater than 3 and less than n. Class i is the contact term of order n.

Let us start with Ward identities of 3-point contact terms. Using Eq. (10) we obtain

$$\begin{aligned} R_1^{\alpha}\langle\bar{\phi}_1(p_1)\bar{J}^{\mu_2}_{\alpha_2}(p_2)\bar{J}^{\mu_3}_{\alpha_3}(p_3)\rangle &= \Delta_L p_1^{\mu}\langle J^{\mu}_{\alpha_1}(p_1)\bar{J}^{\mu_2}_{\alpha_2}(p_2)\bar{J}^{\mu_3}_{\alpha_3}(p_3)\rangle - \langle\hat{J}_1, V^{-1}(J_2-X_2\phi), V^{-1}(J_3-X_3\phi)\rangle \\ &= \Delta_L p_1^{\mu}\langle\bar{J}^{\mu}_{\alpha_1}(p_1)\bar{J}^{\mu_2}_{\alpha_2}(p_2)\bar{J}^{\mu_3}_{\alpha_3}(p_3)\rangle \\ &\quad - i[(C_{\alpha_1\gamma\beta}(V^{-1})_{\gamma\alpha_2}\langle J^{\mu_2}_{\beta}(p_1+p_2), (V^{-1})_{\alpha_3\delta}(J_\delta-X_\delta\phi)\rangle \\ &\qquad + (V^{-1})_{\alpha_2\gamma}(p_2)X_{\gamma i}(p_2)T^{\alpha_1}_{ik}\langle\phi_k(p_1+p_2), V^{-1}(J-X\phi)\rangle) + (2\leftrightarrow 3)]\,. \end{aligned} \tag{B1}$$

Let us use Eqs. (4a)–(4c) to rewrite the term appearing in the square bracket of the right-hand side of Eq. (B1):

$$\begin{aligned} C_{\alpha_1\gamma\beta}(V^{-1})^{\mu_2\nu}_{\gamma\alpha_2}(V^{-1})^{\mu_3\lambda}_{\alpha_3\delta}\Big[(p_3^\nu p_3^\lambda - g^{\nu\lambda}p_3^2)\frac{1}{2\pi i}\int\frac{\rho_2^{\beta\delta}(s)ds}{s(s-p_3^2-i\epsilon)} - g^{\nu\lambda}\int\frac{(\rho_1-\rho_2)^{\beta\delta}(s)ds}{s-p_3^2-i\epsilon} \\ + \frac{p_3{}^\lambda}{2\pi}\epsilon_k T^{\delta}_{kn}\int\frac{\rho^0_{nj}(s)}{s(s-p_3^2-i\epsilon)}ds\,(\Delta^{-1})_{jf}(p_3)\frac{(-)p_3^\nu}{2\pi}\epsilon_k T^{\beta}_{kn}\int\frac{\rho^0_{nf}(s)ds}{s(s-p_3^2-i\epsilon)}\Big] + [(2\leftrightarrow 3)]\,. \end{aligned} \tag{B2}$$

From Eq. (4d) it follows that

$$\frac{1}{2\pi i}\int\frac{\rho_2^{\alpha\beta}(s)ds}{s(s-p^2-i\epsilon)} = \epsilon_i T^{\alpha}_{ik}\epsilon_j T^{\beta}_{jl}\frac{1}{2\pi i}\int\frac{\rho^0_{kl}(s)ds}{s^2(s-p^2-i\epsilon)} + \frac{1}{2\pi i}\int\frac{\rho_1^{\alpha\beta}(s)ds}{s(s-p^2-i\epsilon)}\quad . \tag{B3}$$

Therefore for (B2) we arrive at

$$\begin{aligned} C_{\alpha_1\gamma\beta}(V^{-1})^{\mu_2\nu}_{\gamma\alpha_2}(V^{-1})^{\mu_3\lambda}_{\delta\alpha_3}\Big[g^{\nu\lambda}\int\frac{\rho_2^{\beta\delta}(s)}{s}ds + V^{\nu\lambda}_{\beta\delta}(p_3) + \epsilon_i T^{\beta}_{ik}\epsilon_i T^{\delta}_{il}\frac{1}{2\pi i}\int\frac{\rho^0_{kl}(s)}{s^2(s-p^2-i\epsilon)}ds\,p_3^\nu p_3^\lambda \\ + p_3^\nu p_3^\lambda\frac{1}{2\pi}\epsilon_i T^{\delta}_{ip}\int\frac{\rho^0_p(s)}{s(s-p_3^2-i\epsilon)}ds\,\Delta^{-1}(p_3)\epsilon T^{\beta}\frac{1}{2\pi}\int\frac{\rho^0(s)}{s(s-p_3^2-i\epsilon)}ds\Big] + (2\leftrightarrow 3)\,. \end{aligned} \tag{B4}$$

In (B4) we have used some obvious abbreviations and the definition

$$V^{\mu\nu}_{\alpha\beta}(p) = \frac{-1}{2\pi i}\int\frac{(g^{\mu\nu}-p^\mu p^\nu/s)\rho_1^{\alpha\beta}(s)}{s-p^2-i\epsilon}ds\,.$$

Now, because of the antisymmetry property $C_{\alpha_1\gamma\beta} = -C_{\alpha_1\beta\gamma}$,

$$\int\frac{\rho_2^{\beta\delta}(s)}{s}ds$$

does not contribute to the over-all result.

Let us recall that

$$\Delta_{ij}(p) = \frac{1}{2\pi i}\int\frac{\rho^0_{ij}(s)}{s-p^2-i\epsilon}ds\,;$$

therefore the terms involving $\rho^0_{ij}(s)$ in (B4) may be written symbolically as

$$\epsilon T\left\{\frac{1}{p_3{}^2}\left(\frac{\Delta(p_3)-\Delta(0)}{p_3{}^2} - \int\frac{\rho^0(s)ds}{s^2}\right) - \frac{1}{(p_3{}^2)^2}[\Delta(p_3)-\Delta(0)]\Delta^{-1}(p_3)[\Delta(p_3)-\Delta(0)]\right\}T\epsilon\,. \tag{B5}$$

In (B5) we have used the obvious relations

$$\begin{aligned} p^2\int\frac{\rho^0(s)ds}{s(s-p^2-i\epsilon)} - \int\frac{\rho^0(s)ds}{s-p^2-i\epsilon} &= -\int\frac{\rho^0(s)ds}{s} \\ \Rightarrow \frac{1}{2\pi i}\int\frac{\rho^0(s)ds}{s(s-p^2-i\epsilon)} &= \frac{\Delta(p)-\Delta(0)}{p_3{}^2}. \end{aligned}$$

Remembering that $\Delta(p)$ is a matrix in the symmetry space, we obtain the expression in the square bracket of (B5) as

$$\frac{\Delta(0)\Delta^{-1}(p_3)\Delta(0)}{(p_3{}^2)^2}-\frac{1}{2\pi i}\int\frac{\rho^0(s)ds}{p_3{}^2 s}+\frac{\Delta(0)}{(p_3{}^2)^2}\ . \tag{B6}$$

This expression vanishes only if $\Delta^{-1}(p)$ is of the form

$$\Delta(0)\Delta^{-1}(p)\Delta(0)=C_1p^2+C_2\,,$$

$$C_1=\frac{1}{2\pi i}\int\frac{\rho^0(s)ds}{s^2}\,,$$

$$C_2=\Delta(0),$$

i.e., only if

$$\Delta(p)=\frac{A}{p^2-B}\ . \tag{B7}$$

We arrive therefore at the P.C.C. assumption. In what follows we assume P.C.C., and Eq. (B1) finally becomes

$$\begin{aligned}R_1^{\alpha_1}\langle\bar\phi(p_1)\bar J^{\mu_2}_{\alpha_2}(p_2)\bar J^{\mu_3}_{\alpha_3}(p_3)\rangle &=-iC_{\alpha_1\gamma\alpha_3}(V^{-1})^{\mu_2\mu_3}_{\gamma\alpha_2}(p_2)-iC_{\alpha_1\gamma\alpha_2}(V^{-1})^{\mu_2\mu_3}_{\gamma\alpha_3}(p_2)\\&\quad+\Delta_L p_1^\mu\langle\bar J^\mu_{\alpha_1}(p_1)\bar J^{\mu_2}_{\alpha_2}(p_2)\bar J^{\mu_3}_{\alpha_3}(p_3)\rangle\,.\end{aligned} \tag{B8}$$

The second 3-point contact term is $\langle\bar\phi_1\bar\phi_2\bar J_3\rangle$, for which we obtain

$$\begin{aligned}R_1^{\alpha_1}\langle\bar\phi_1(p_1)\bar\phi_k(p_2)\bar J^\mu_{\alpha_3}(p_3)\rangle&=R_1^{\alpha_1}\langle\bar\phi_1(p_1)(\Delta^{-1})_{ki}(p_2)\phi_i(p_2),\bar J^\mu_{\alpha_3}(p_3)\rangle\\&=\Delta_L p_1^\nu\langle\bar J^\nu_{\alpha_1}(p_1)\bar\phi_2(p_2)\bar J^\mu_{\alpha_3}(p_3)\rangle-\langle\hat J_1,(\Delta^{-1})_{ki}(p_2)\phi_i(p_2),V^{\mu\lambda}_{\alpha_3\gamma}(p_3)[J^\lambda_\gamma(p_3)\\&\qquad-X^\lambda_{\gamma i}(p_3)\phi_i(p_3)]\rangle\\&=\Delta_L p_1\langle\bar J_1\bar\phi_2\bar J_3\rangle-iC_{\alpha_1\beta\gamma}(V^{-1})^{\mu\nu}_{\beta\alpha_3}(p_3)-(\Delta^{-1})_{kl}(p_2)\frac{ip_2^\nu}{\mu_s{}^2}\epsilon_jT^\gamma_{js}\Delta_{sl}(p_2)\\&\quad+(V^{\mu\nu}_{\beta\alpha_3}(p_3))^{-1}\frac{ip_3^\nu}{\mu_s{}^2}\epsilon_kT^\beta_{ks}\Delta_{sf}(p_3)\Delta_{ft}(p_3)(i)T^\alpha_{tj}\Delta_{jn}(p_2)(\Delta^{-1})_{nk}(p_2)\\&\equiv\Delta_L p_1\langle\bar J_1\bar\phi_2\bar J_3\rangle+A^\nu_{\alpha_1k\beta}(V^{-1})^{\nu\mu}_{\beta\alpha_3}(p_3)\,,\end{aligned} \tag{B9}$$

where

$$A^\nu_{\alpha_1k\beta}=\frac{p_2^\nu}{\mu_2{}^2}C_{\alpha_1\beta\gamma}\epsilon_jT^\gamma_{jk}-\frac{p_3^\nu}{\mu_2{}^2}\epsilon_iT^\beta_{ij}T^{\alpha_1}_{jk}\ .$$

The first 3-point contact term gives

$$R_1^\alpha\langle\bar\phi_1(p_1)\bar\phi_{k_2}(p_2)\bar\phi_{k_3}(p_3)\rangle=\Delta_Lp_1^\mu\langle\bar J^\mu_\alpha(p_1)\bar\phi_{k_2}(p_2)\bar\phi_{k_3}(p_3)\rangle+iT^\alpha_{nk_3}(\Delta^{-1})_{k_2n}(p_2)+iT^\alpha_{nk_2}(\Delta^{-1})_{k_3n}(p_3)\,. \tag{B10}$$

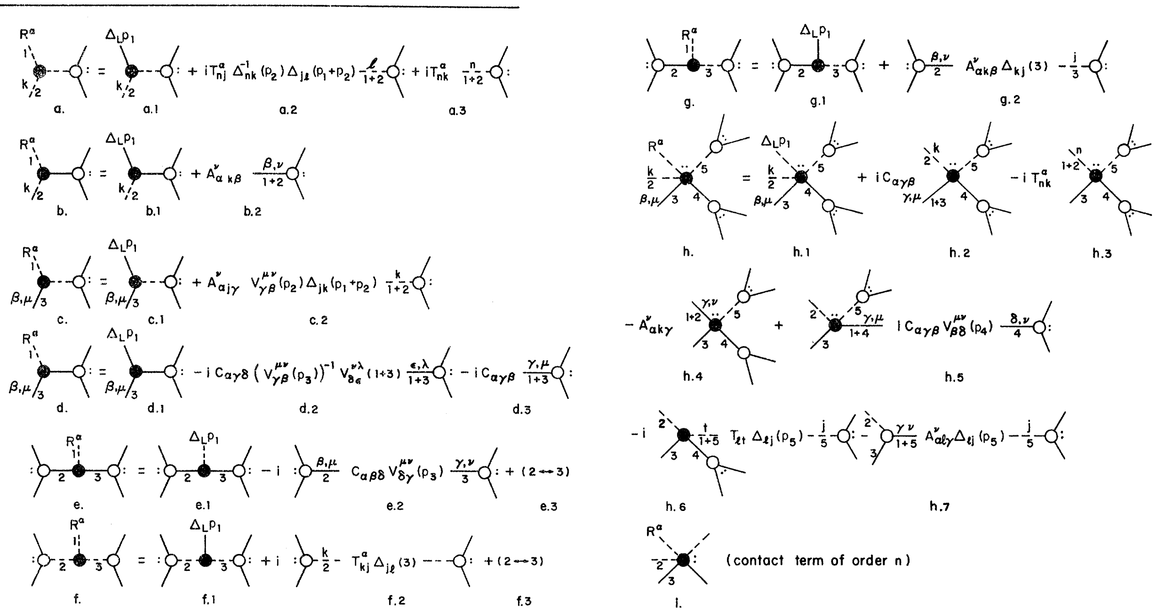
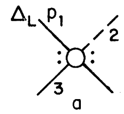


FIG. 6. Ward identities presented diagrammatically.

We now apply the results (B8), (B9), (B10), and Eq. (12) (by induction) to the different classes of graphs. The result we present diagrammatically in Fig. 6. The right-hand side of Eq. (10) may be presented diagrammatically as shown in Fig. 7.

When expressed in terms of tree diagrams (in Fig. 7) the a.1 contact terms will cancel all the diagrams a.1, b.1, c.1, . . . , h.1 (in Fig. 6) except the n-point contact term in a.1, which does not have a counterpart on the right-hand side of Fig. 6. We perform the commutations in b.2 and express the results in terms of tree diagrams. Then b.2 will cancel all the terms a.2, b.2, c.2, d.2. We note that different diagrams coming from class h [h.2 , . . . , h.7] will cancel all other diagrams except for a.3, b.2, d.3 [and i.] when the reduced amplitude is a contact term of order $n-1$. We therefore obtain the diagrammatic equality



$$-\langle \hat{J}_1\, \Delta^{-1}(2)\,\phi(2), V^{-1}(3)\,(J_3 - X_3\,\phi)\cdots\rangle$$

b

FIG. 7. Right-hand side of Eq. (10) presented diagrammatically.

$$(\mathrm{i.}) + (\mathrm{a.3})_{n-1} + (\mathrm{b.2})_{n-1} + (\mathrm{d.3})_{n-1} = (\mathrm{A1})_n\,,$$

which is equivalent in the full form to Eq. (12).

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