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Unified Theory of Nonleptonic Hyperon Decays

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Inclusion of baryon resonances in the current-algebra analysis of $B \to B'\pi$ and $B \to B'\gamma$ decays explains both the mismatch of the s- and p-wave pion amplitudes and the nonvanishing negative asymmetry parameter for $\Sigma^+ \to p\gamma$.

I. INTRODUCTION

Despite certain successes of the current-algebra approach to the nonleptonic hyperon decays,[1-3] two important problems remain unsolved. First, the $B \to B'\pi$ s-wave amplitudes are mismatched relative to the p-wave amplitudes, due to the s-wave Suzuki-Sugawara[1] current commutator and the p-wave baryon octet poles.[2,3] Second, the parity-violating amplitude is predicted to vanish for the weak radiative decay $\Sigma^+ \to p\gamma$,[4] in contradiction with the large measured asymmetry parameter.[5]

It is the purpose of this paper to solve *both* of these problems simultaneously by inclusion of decuplet intermediate states along with the baryon octet poles. Decuplet poles dominate the pion photoproduction background amplitudes at low energy and therefore account for most of the anomalous magnetic moment of the nucleon as expressed by the current-algebra low-energy theorem of Fubini, Furlan, and Rossetti (FFR).[6,7] The Adler-Weisberger[8] (AW) background amplitudes in low-energy pion-nucleon scattering are also dominated by decuplet states.[8,9] It is therefore not surprising that decuplet poles play an important role in weak hyperon decays. However, in contrast with AW, FFR, and the weak radiative decays $B \to B'\gamma$, the decuplet poles in $B \to B'\pi$ dominate the axial-vector current algebra background amplitudes.

Decuplet states have been considered before,[10] but not in the context of current algebra for both pion and radiative hyperon decays. By applying dispersion techniques to two different methods used to obtain current-algebra results, we settle certain ambiguities which arise in the off-shell decuplet propagator. Choice of dispersive covariants requires particular care, but the two methods lead to identical decuplet contributions which in all cases display the correct mass suppression factors $m_D - m_B$ $(m_D + m_B)$ corresponding to parity-conserving (-violating) spurion-baryon orbital angular momentum transitions $l=2$ $(l=1)$.

Furthermore, having included decuplet states (10–30% effects), we feel obliged to consider Y_0^* states (3–10% effects) and also 27-plet contributions to the weak Hamiltonian (2% effects in H_w). We do in fact obtain an almost perfect fit to all of these decays, but one might then claim that we are simply "parameter-fitting" the four different types of amplitudes in question. This is not the case for three reasons: (i) We make independent numerical estimates of the magnitudes expected for all nonleading effects, and we do not allow fits which require them to be unreasonably large (such estimates also show that these effects should not be neglected). (ii) The seven s waves, seven p waves, and two radiative measured amplitudes far overdetermine our additional parameters. (iii) Forcing these parameters to fit both the pion and radiative amplitudes requires a unique choice for their *sign* which is otherwise undetermined if pion or radiative decays are fitted separately.

One might further argue that there is no point in trying to obtain a perfect fit because the basic input of SU_3 conservation at the vertices may be broken by about 15%. We point out that all $\frac{1}{2}^+$, $\frac{3}{2}^+$, 0^-, 1^-, and 2^+ data given in the Particle Data Group tables[11] can now be fitted to within one standard deviation assuming SU_3 vertex conservation.[12] We therefore feel compelled to treat SU_3 as an exact vertex symmetry in these weak nonleptonic decays in an attempt to probe the scaling of the algebra of currents which occurs only in the s-wave, $B \to B'\pi$ decays. We conclude that the algebra of currents is indeed consistent with all of the measured nonleptonic $B \to B'\pi$ and $B \to B'\gamma$ decays.

In the past the mismatch between the s- and p-wave pion decays has led to alternative s-wave $(d_w/f_w = -0.30)$ and p-wave $(d_w/f_w = -0.865)$ fits to the data.[13] One usually argues that the s-wave solution is preferable because the resulting p-wave pole "model" is somewhat ambiguous in the soft $q_\pi \to 0$ limit. However, in a recent paper[14] we have used the hard-pion technique with $q_\pi{}^2 \to 0$ to demonstrate that in fact it is the *p-wave-octet poles* which closely approximate reality. It is then the s waves which are more appreciably modified by inclusion of resonance states.

The choice[14,15] of the meson channel K^* pole as saturating the s-wave background amplitude gives a fair s-wave fit in conjunction with the octet spurion as determined by the p-wave octet poles. This is in fact a correct procedure, as opposed to the K^* vector-dominance model of the entire s-wave amplitude,[16] but it is only after computing the s-wave background in the decuplet-dominance model that one appreciates the dual nature of the K^* "Regge" pole. In any case the decuplet saturation is more convincing than the K^* saturation because the choice of the *sign* of the decuplet s-wave contribution also gives a negative asymmetry parameter in $\Sigma^+ \to p\gamma$, in agreement with experiment. In contrast, the sign of the K^* pole in the s-wave pion decays is a free parameter and is chosen to reduce the magnitude of the fitted s waves. Furthermore, this baryon resonance approach definitely gives a better fit to all the pion decays than does the meson resonance, K^* approach.

We begin by summarizing the role of the current commutator and octet baryon poles in Sec. II for both $B \to B'\pi$ and $B \to B'\gamma$ decays. Then in Secs. III and IV we discuss the decuplet contributions to the pion and radiative amplitudes, respectively. In Sec. V we include the additional contributions due to the Y_0^* and the $\underline{27}$ part of H_w, and in Sec. VI we obtain our fit and also discuss various methods of analyzing the data. In the Appendix we list all relevant formulas for each of the various decays.

II. REVIEW—THE CURRENT COMMUTATOR AND OCTET POLES

A. Pion Decays

There are two methods to incorporate the content of the algebra of currents into the on-shell hyperon decays $B^i(p) \to B^f(p') + \pi^j(q)$. The hard-pion method[17,14] involves the divergence of the axial-vector amplitude

$$M_\mu = i\int e^{iq\cdot x}\,\theta(x_0)\langle B^f|[A_\mu^j(x), H_w]\,|B^i\rangle\, d^4x\,, \quad (1)$$

in conjunction with the (nonsoft) PCAC (partial conservation of axial-vector current) structure of the pion amplitude (where M_μ^x has the pion pole removed)

$$M_\pi = M_{cc} + (1/if_\pi)q^\mu M_\mu^x. \quad (2)$$

Because of the assumed current-current structure of H_w, the current commutator term can be directly evaluated at $q_\pi{}^2 = m_\pi{}^2$:

$$if_\pi M_{cc} = -\langle B^f | [F_5^j, H_w] | B^i \rangle$$
$$= if^{fjb}\langle B^b | H_w | B^i\rangle - if^{jia}\langle B^f | H_w | B^a\rangle . \tag{3}$$

Separating the amplitude into parity-violating (PV) and parity-conserving (PC) parts (between baryon spinors)

$$M_\pi = \langle B^f \pi^j | H_w^{\mathrm{PV}} + H_w^{\mathrm{PC}} | B^i \rangle = iA + \gamma_5 B , \tag{4}$$

$$\langle B^f | H_w | B^i \rangle = i\gamma_5 H_{\mathrm{PV}}^{fi} + H_{\mathrm{PC}}^{fi} , \tag{5}$$

we extract the octet poles in $q^\mu M_\mu^\pi$ with axial-vector coupling $\frac{1}{2} g_A i\gamma_\mu\gamma_5$ to find

$$A = A_{cc} + A_8 + \overline{A} , \tag{6}$$

$$B = B_{cc} + B_8 + \overline{B} , \tag{7}$$

where

$$A_8 = \frac{\Delta m}{2f_\pi}\sum_n\left(\frac{g_A^{fn} H_{\mathrm{PV}}^{ni}}{m_f + m_n} + \frac{H_{\mathrm{PV}}^{fn} g_A^{ni}}{m_n + m_i}\right) , \tag{8}$$

$$B_8 = -\frac{(m_f + m_i)}{2f_\pi}\sum_n\left(\frac{g_A^{fn} H_{\mathrm{PC}}^{ni}}{m_n - m_f} - \frac{H_{\mathrm{PC}}^{fn} g_A^{ni}}{m_n - m_f}\right), \tag{9}$$

with $\Delta m = m_i - m_f$. The background axial-vector amplitude can be parametrized as $\overline{M}_\mu = \overline{F}_1 \gamma_\mu + \overline{G}_A i\gamma_\mu\gamma_5$ so that $f_\pi \overline{A} = \Delta m \overline{F}_1$ and $f_\pi \overline{B} = (m_f + m_i)\overline{G}_A$.

In the soft-pion method,[2,3,15] one can work directly with the pion amplitude and separate off the rapidly varying poles; $M = M^P + \overline{M}$. One then evaluates M at the soft point $q_\pi = 0$ in order to determine the slowly varying background to find

$$M_\pi = M(0) + M_\pi^P - M_\pi^P(0) , \tag{10}$$

where $M(0) = M_{cc}$ and $M_\pi^P(0)$ are evaluated at the soft point $q_\pi = 0$. In terms of the $BB\pi$ coupling $g\gamma_5$, the $M_\pi^P - M_\pi^P(0)$ structure leads to the octet pole contributions

$$A_8 = -\sum_n \left[g^{fn}\left(\frac{1}{m_i + m_n} - \frac{1}{m_n + m_f}\right) H_{\mathrm{PV}}^{ni} + H_{\mathrm{PV}}^{fn}\left(\frac{1}{m_f + m_n} - \frac{1}{m_n + m_i}\right) g^{ni}\right] , \tag{11}$$

$$B_8 = -\sum_n \left[g^{fn}\left(\frac{1}{m_i - m_n} + \frac{1}{m_f + m_n}\right) H_{\mathrm{PC}}^{ni} + H_{\mathrm{PC}}^{fn}\left(\frac{1}{m_f - m_n} + \frac{1}{m_n + m_i}\right) g^{ni}\right] . \tag{12}$$

The Goldberger-Treiman relation (with $q^2 = 0$)

$$f_\pi g^{fi} = \tfrac{1}{2}(m_f + m_i) g_A^{fi} \tag{13}$$

then relates these two forms for the octet poles, (8) with (11) and (9) with (12). Our reason for discussing both formalisms is that the soft limit is sometimes extremely subtle and we prefer to demonstrate that this limit can be achieved in more than one way. For example, it is sometimes stated that the parity-conserving p-wave octet poles (12) are ambiguous because one is taking $q_\pi \to 0$ and the related covariant $\overline{u}(p')\gamma_5 u(p)$ is also vanishing, thus rendering this limit meaningless. However the hard-pion method $q^2 \to 0$ is not ambiguous and gives the same pole terms (9). It is therefore clear that the B_8 pole terms are extremely important in the current-algebra approach and *cannot* be ignored.

To proceed further, we make use of the symmetric structure of the current-current H_w to deduce that H_w ($\Delta Y = 1$) contains an octet part transforming like λ_6 and also a $\underline{27}$ part. This leads to the general conclusion that[18]

$$\langle \boldsymbol{B}^f | H_w^{\mathrm{PV}} | B^i \rangle = 0 . \tag{14}$$

We may also write

$$\langle B^f | H_w^{\mathrm{PC}}(\lambda_6) | B^i \rangle = h_1 (d_w d^{fi6} - f_w i f^{fi6}) , \tag{15}$$

with $d_w + f_w = 1$. Because all the $\Delta I = \frac{1}{2}$ and Lee-Sugawara[19] sum rules are nearly valid for s and p waves, we can neglect the $\underline{27}$ part of H_w to first approximation, but we shall return to it in detail in Sec. V.

Next we extract the decuplet intermediate states from the background amplitudes $\overline{A}$ and $\overline{B}$. The PV decuplet transition has a relative baryon-spurion angular momentum of $l = 1$ while the PC decuplet transition corresponds to $l = 2$. This indicates that $B_{10} \sim (m_D - m_B)$, whereas $A_{10} \sim (m_D + m_B)$. Combining these mass factors with the Δm octet baryon mass suppressions of (8) and (9), we rewrite (6) and (7) in a form which roughly estimates the size of the various contributions because $\Delta m/m \sim (m_D - m_B)/m \sim \frac{1}{5} - \frac{1}{10}$:

$$A = A_{cc}(1) + A_{10}(\Delta m/m) + \overline{A}' , \tag{16}$$

$$B = B_8(m/\Delta m) + B_{10}[(m_D - m_B)/m] + \overline{B}' . \tag{17}$$

The terms $\overline{A}'$ and $\overline{B}'$ will be discussed in Sec. V and are small. We are adhering to strict SU_3 conservation at the (hadronic) vertex, but even if we assumed mass breaking effects to violate (14) to $O(\Delta m/m)$ we would find that A_8 and B_{cc} are suppressed by $O((\Delta m/m)^2)$ relative to the dominant terms in (16) and (17) and therefore can be ignored.

To first approximation we can keep the leading terms in (16) and (17), A_{cc} and B_8, and try to fit both the s and p waves with one set of values for the parameters h_1 and d_w/f_w in the octet-dominated weak Hamiltonian (15). The mismatch between the s and p waves is then apparent because $d_w/f_w \sim -0.30$ for the s waves and $d_w/f_w \sim -0.86$ for the

p waves. The latter ratio is determined by the p-wave pole terms (12) with $d_P/f_P=1.7$, and gives s-wave amplitudes which are in proper proportion to one another but a factor of 2 larger than experiment. There are two possible variations in this scheme. First, we can use the p-wave pole terms (9) instead of (12), with the measured value of $d_A/f_A=1.7$ obtained from semileptonic hyperon decays.[20,21] Due to the relative minus sign between the s- and u-channel poles of (9) or (12), the mass factors in the Goldberger-Treiman relation (13) lead to somewhat different values for B_8 as given by (9) or (12). While the best fit to the axial poles, $d_w/f_w\sim-0.79$, is slightly closer to the s-wave value -0.30, the ratio A_{cc}/A is still a factor of 2 greater than experiment. As a second alternative we can again use the p-wave pion poles (12), but with a slightly larger value of d_P/f_P than the axial ratio 1.7. The best value for d_P/f_P obtained from low-energy scattering[22] and from Regge fits[23] is $d_P/f_P\sim2.1$. This has the effect of lowering A_{cc}/A_{exp} to about 1.5, with $d_w/f_w\sim-0.88$. Since we shall be using a dispersion theoretic approach, we must adopt the latter alternative which is formally derived from the on-shell version of the Goldberger-Treiman relation.[24,25] Thus, we shall use p-wave pion-pole terms (12) along with

$$d_P/f_P=2.1\,,\quad d_w/f_w=-0.88\,,\quad A_{cc}/A_{exp}\sim1.5\,. \tag{18}$$

B. Radiative Decays

The weak radiative decays $B^i(p)\to B^f(p')+\gamma(k)$ can be viewed in the same perspective as the pion decays. We write the amplitude as

$$T(B\to B'\gamma)=i\int e^{ik\cdot x}\theta(x_0)\langle B^f|[V_\mu^{3+8/\sqrt{3}},H_w]|B^i\rangle\epsilon^{\mu *}d^4x$$
$$=e(C+i\gamma_5 D)\tfrac{1}{2}[\gamma\cdot\epsilon^*,\gamma\cdot k]\,, \tag{19}$$

where C is the PC and D the PV amplitude. The octet pole terms are

$$C_8=-\sum_n\left[\frac{\kappa_{fn}H_{ni}^{PC}}{(m_f+m_n)(m_i-m_n)}-\frac{H_{fn}^{PC}\kappa_{ni}}{(m_n-m_f)(m_n+m_i)}\right], \tag{20}$$

$$D_8=\sum_n\left[\frac{\kappa_{fn}H_{ni}^{PV}}{(m_f+m_n)(m_i+m_n)}+\frac{H_{fn}^{PV}\kappa_{ni}}{(m_n+m_f)(m_n+m_i)}\right]. \tag{21}$$

It is clear that D_8, like A_8, vanishes by (14). Moreover, U-spin symmetry also suppresses C_8 for U^2-conserving transitions because these anomalous magnetic moments $\kappa/2m$ also become equal in the strict SU_3 limit. Fortunately, this suppression is only one order in $\Delta m/m$ since experimentally it seems that the dimensionless moments κ obey SU_3.[26] Finally, U-spin "G parity" suppresses the entire PV amplitude D for U^2-conserving transitions in accordance with the Hara theorem[4] to be crossing-odd in the variable[27] $\nu=(p'-p)\cdot(p'+p)=m_f^2-m_i^2$:

$$D_U(\nu)=-D_U(-\nu)\,. \tag{22}$$

This forces D_U to vanish in the SU_3 mass limit $\nu=0$, but for physical masses to be $O(\Delta m/m)$.[27] Our SU_3 expansion of the amplitude in powers of $\Delta m/m$ thus takes the form

$$C=C_8(m/\Delta m)+C_{10}((m_D-m_B)/m)\,, \tag{23}$$

$$D=D_{10}(1) \tag{24}$$

for the U^2-nonconserving transitions $\Lambda\to n\gamma$, $\Xi^0\to\Lambda\gamma$, $\Xi^0\to\Sigma^0\gamma$, and

$$C^U=C_8^U(1)+C_{10}^U((m_D-m_B)/m)\,, \tag{25}$$

$$D^U=D_{10}^U(\Delta m/m) \tag{26}$$

for the U^2-conserving transitions $\Sigma^+\to p\gamma$, $\Xi^-\to\Sigma^-\gamma$. The background corrections to (23)–(26) are taken to be small. Again, we have displayed the PC $l=2$ suppression by m_D-m_B in (23) and (25).

Experimentally the only available data are for[5,11] $\Sigma^+\to p\gamma$:

$$\Gamma=\frac{e^2}{8\pi}\left(\frac{m_i^2-m_f^2}{m_i}\right)^3(|C|^2+|D|^2)$$
$$=(1.02\pm0.15)\times10^{-14}\text{ MeV}, \tag{27}$$

$$\alpha=\frac{2\,\mathrm{Re}C^*D}{|C|^2+|D|^2}$$
$$=-1.03^{+0.52}_{-0.42}\,. \tag{28}$$

This large asymmetry parameter would seem to imply that $C^U\approx-D^U$, thus violating the SU_3 structure of (25) and (26) and the Hara theorem (22). This mismatch between C and D might indicate some new structure in weak radiative decays not accounted for by the above simple analysis.[28]

However, extreme measures are really not necessary. By comparing (26) with (25) we estimate $|D|\sim\frac{1}{5}|C|$ and therefore $|\alpha|\sim|2D/C|\sim0.4$, which is, after all, just one standard deviation from experiment. The important point is that the *sign* of α will be correctly determined by our decuplet fit to the s-wave pion decays.

III. DECUPLET STATES AND PION DECAYS

First we display the various couplings needed for decuplet transitions. The strong vertex can be written as[29] $(q=p'-p)$

$$\langle D|j^\pi|B\rangle=\frac{g_{DB\pi}}{\frac{1}{2}(m_D+m_B)}\overline{D}_\alpha q_\pi^\alpha B\,, \tag{29}$$

where the SU_3 structure of $g_{DB\pi}$ is given by

$g_2\bar{D}^{(abc)}\epsilon_{cde}B^e_b\pi^d_a$, leading to $g_{\Delta^{++}p\pi^+}=g_2$, $g_{\Delta^+p\pi^0} = -(\frac{2}{3})^{1/2}g_2$, etc. We are adhering to strict SU_3 conservation at the hadron vertex and extracting the mass factor $\frac{1}{2}(m_D+m_B)$ from the vertex guarantees that the dimensionless coupling constant $g_2=15.7$ leads to a perfect fit to the four $D\to B\pi$ decays with $\chi^2<1$.[12,30] Because the decuplet terms are already corrections to the leading octet or current-commutator terms in (16) and (17), we can safely ignore the $\underline{27}$ part of H_w and write

$$\langle D|H_w^{\rm PV}|B\rangle = i\bar{D}_\alpha p^\alpha B H_{DB}^{\rm PV}, \tag{30}$$

where p_α is the momentum of the baryon, and for $\Delta Y=1$ transitions the SU_3 structure of (30) is, for[31] $H_w\sim\lambda_6$,

$$H_{DB}^{\rm PV}=h_2\bar{D}^{(2bc)}\epsilon_{c3e}B^e_b, \tag{31}$$

$$H_{BD}^{\rm PV}=h_2\bar{B}^b_e\epsilon^{c2e}D_{(3bc)}. \tag{32}$$

The coupling constant h_2 is dimensionless and we infer that, like g_2, it is the correct SU_3-invariant vertex strength. Likewise, we replace $H_w^{\rm PV}$ by $H_w^{\rm PC}$, ip_α by p_α, and h_2 by h_3 in (30)–(32) in order to specify the PC BD transition. The actual magnitude of h_3 should not differ too greatly from h_2 because a theorem analogous to (14) does not exist for DB weak transitions.

The violation of angular momentum at the weak spurion vertex (30) implies that the divergence of the on-shell projection operator $\mathcal{P}_{\beta\alpha}p_\alpha$ vanishes for $p=p^D$, where

$$\mathcal{P}_{\beta\alpha}=-(\gamma\cdot p^D+m_D)\left[g_{\beta\alpha}-\tfrac{1}{3}\gamma_\beta\gamma_\alpha-\frac{1}{3m_D}(\gamma_\beta p^D_\alpha-p^D_\beta\gamma_\alpha)-\frac{2}{3m_D{}^2}p^D_\beta p^D_\alpha\right]. \tag{33}$$

The entire decuplet contribution then comes from off-shell corrections. If we were to use field theory we would not obtain reliable results, since the form of the off-shell spin-$\frac{3}{2}$ propagator is ambiguous.[32] Instead, we have used dispersion theory to evaluate both of the hard pion methods. In the first method, we write an unsubtracted fixed-t dispersion relation for the fictitious process $B^iH_w(q')\to B^fA_\mu(q)$ to obtain the coefficients of p_μ, $p_\mu[q\cdot\gamma, q'\cdot\gamma]$, q'_μ, $q'_\mu[q\cdot\gamma,q'\cdot\gamma]$, γ_μ, and $\gamma_\mu q'\cdot\gamma$ for the parity-violating case, and the same covariants with an extra γ_5 in the parity-conserving case. We use the narrow-resonance approximation. Taking the limit as $q'\to 0$ then determines M_μ, from which M_π can be found by Eq. (2), the answer appearing in terms of the axial-vector vertex $\langle D_\mu(p-q)|A_\nu|B(p)\rangle$. Of the four invariants corresponding to $g_{\mu\nu}$, $q_\mu p_\nu$, $q_\mu q_\nu$, and $q_\mu\gamma_\nu$, the last three contribute to the Goldberger-Treiman relation and to M_π with an extra p-wave suppression factor (m_D-m_B), as they must.[33] Hence, we may ignore all but the coefficient of $g_{\mu\nu}$, and the generalized Goldberger-Treiman relation determines $(M_\pi)_{10}$ in terms of the pion-baryon-decuplet vertex (29).

Our second method is the hard-pion technique of Okubo,[10] involving a once-subtracted dispersion relation for $B^iH_w(q')\to B^f\pi(q)$ with the covariants $1(1,\gamma_5)$, $[q\cdot\gamma,q'\cdot\gamma](1,\gamma_5)$. Results of the two methods are identical:

$$A_{10}(B_i\to B_f\pi)=\tfrac{1}{3}(m_{B_i}-m_{B_f})\left[g_{B_fD\pi}\frac{(m_D+m_{B_i})}{m_D{}^2}H_{DB_i}^{\rm PV}+H_{B_fD'}^{\rm PV}\frac{(m_{D'}+m_{B_f})}{m_{D'}{}^2}g_{D'B_i\pi}\right], \tag{34}$$

$$B_{10}(B_i\to B_f\pi)=\tfrac{1}{3}(m_{B_i}+m_{B_f})\left[g_{B_fD\pi}\frac{(m_D-m_{B_i})}{m_D{}^2}H_{DB_i}^{\rm PC}+H_{B_fD'}^{\rm PC}\frac{(m_{D'}-m_{B_f})}{m_{D'}{}^2}g_{D'B_i\pi}\right]. \tag{35}$$

These two decuplet contributions ought to be the same order of magnitude and, according to (16) and (17), no more than 40% of the s-wave $O(1)$ term A_{cc}. We can estimate the size of h_2 or h_3 by comparing, for example, the ratio $H_{\Delta\Sigma}^{\rm PV,PC}/H_{N\Sigma}^{\rm PC}$ to the strong-coupling-constant ratio $g_{\Delta\Sigma K}/g_{N\Sigma K}$. This leads to $h_{2,3}\sim 0.2\times10^{-6}$, corresponding to decuplet contributions which are 30% of A_{cc} and 10% of B_8. Our fits will, in fact, be consistent with this estimate.

IV. DECUPLET STATES AND RADIATIVE DECAYS

There are two $DB\gamma$ couplings which can be written as $[q=p_D-p_B,\ P=\frac{1}{2}(p_D+p_B)]$:

$$\langle D|J_\mu^{\rm em}|B\rangle = e\bar{D}_\beta[C_3(DB)(q_\beta\gamma_\mu-q\cdot\gamma g_{\beta\mu})+C_4(DB)(q_\beta P_\mu-q\cdot Pg_{\beta\mu})]\gamma_5B. \tag{36}$$

The $\Delta N\gamma$ transition has been analyzed in resonance photoproduction[34] and found to be almost purely magnetic dipole in nature with[35]

$$C_3(\Delta^+p)=-m_\Delta C_4(\Delta^+p)\cong 2.14\ {\rm GeV}^{-1}. \tag{37}$$

These couplings can be used to dynamically compute the baryon anomalous magnetic moments κ_B from the decuplet contribution to the first CGLN (Chew, Goldberger, Low, and Nambu) covariant[36] $\frac{1}{2}[\gamma\cdot k,\gamma_\mu]\gamma_5$ for the soft-pion photoproduction process $B\gamma\to B'\pi$:

$$A_{1,10}^{\mathrm{CGLN}}\,(q_\pi=0)=+2g_{DB\pi}C_3(DB)\frac{(m_D+m_B)}{6m_D{}^2}\,, \qquad (38)$$

which includes both C_3 and C_4 couplings in the magnetic dipole configuration. The dispersion-theoretic value (38) is unambiguous because of the particularly unique structure of the spin-flip covariant $\frac{1}{2}[\gamma\cdot k,\gamma_\mu]\gamma_5$. The soft-pion theorem of Fubini, Furlan, and Rossetti then implies that[6,7] $A_1^{\mathrm{total}}=A_{1,8}+\overline{A}_1\to 0$ as $q_\pi\to 0$, leading to

$$\overline{A}_1(q_\pi=0)\approx A_{1,10}(q_\pi=0)=\frac{-\kappa_B\,g_{BB\pi}}{2m_B{}^2}\,. \qquad (39)$$

Combining (38) and (39) we find for a proton target that

$$\kappa_p=\tfrac{2}{3}\left(\tfrac{2}{3}\right)^{1/2}\frac{m_N{}^2}{m_\Lambda{}^2}(m_\Delta+m_N)\frac{g_2}{g}C_3(\Delta^+p)\cong 2.2\,, \qquad (40)$$

which is close to the measured value $\kappa_p=1.79$. Neutron targets lead to $\kappa_p=-\kappa_n$ compared to $\kappa_n=-1.91$. We therefore conclude that decuplet saturation of background amplitudes as given by (38) is a reasonable dynamical approximation,[37] and at the very least the sign and approximate magnitude of $C_3(\Delta p)$ are indeed correct.

There is as yet no resonance photoproduction analysis off hyperon targets, but we could argue that SU_3 partners of (40) ought to be the dimensionless anomalous "transition" moments λ in analogy with the "elastic" moments κ (Ref. 26):

$$C_3(DB)=\lambda_{DB}/\tfrac{1}{2}(m_D+m_B)\,. \qquad (41)$$

The SU_3 $DB\gamma$ structure $\overline{D}^{(1bc)}\epsilon_{c1e}B_b^e$ then leads to the SU_3 relations

$$\lambda_{\Delta^+p}=\lambda_{\Delta^0n}=-\lambda_{\Sigma^{*+}\Sigma^+}=2\lambda_{\Sigma^{*0}\Sigma^0}$$
$$=-(2/\sqrt{3})\lambda_{\Sigma^{*0}\Lambda}=-\lambda_{\Xi^{*0}\Xi^0}\approx 2.34\,, \qquad (42)$$

with $\lambda_{DB}=\lambda_{BD}$.

On the other hand, it is remarkable that the SU_3 analog of the FFR relation (40) for various hyperon targets is *exactly* satisfied for $d_P/f_P=3$ and $d_{V_2}/f_{V_2}=3$, the latter ratio corresponding to anomalous moments with $\kappa_p=-\kappa_n$. For physical mass and $d_P/f_P=2.1$, these relations lead to

$$\begin{aligned}&-\lambda_{\Sigma^{*+}\Sigma^+}/\lambda_{\Delta^+p}\approx\kappa_{\Sigma^+}/\kappa_p\,,\\&2\lambda_{\Sigma^{*0}\Sigma_3}/\lambda_{\Delta^+p}\approx-0.7\kappa_{\Sigma_3}/\kappa_p\,,\\&\lambda_{\Xi^{*0}\Xi^0}/\lambda_{\Delta^+p}\approx+0.5\kappa_{\Xi^0}/\kappa_p\,,\end{aligned} \qquad (43)$$

where[38] $2\Sigma_3=\Sigma^0-\sqrt{3}\Lambda$. We might then conclude that on the average the heavier hyperon SU_3 transition moments are suppressed by 0.75 over SU_3-invariant moments as defined by (42). In any case it is clear that the hyperon λ_{DB} SU_3 couplings cannot be larger than predicted by (42).

Now we are prepared to analyze the various decuplet contributions to the PV and PC amplitudes of $D\to B\gamma$. The same dispersion-theoretic spin-$\frac{3}{2}$ projection operator used in $A_{1,10}^{\mathrm{CGLN}}$ immediately determines the decuplet amplitude D_{10} because both are coefficients of the same covariant: $\frac{1}{2}[\gamma_\mu,\gamma\cdot k]\gamma_5$. Treating the weak spurion as the analog of the soft pion in photoproduction, we find

$$D_{10}(B_i\to B_f\gamma)=\tfrac{1}{6}\left[\lambda_{B_fD}\frac{(m_D+m_{B_i})}{m_D{}^2}H_{DB_i}^{\mathrm{PV}}-H_{B_fD'}^{\mathrm{PV}}\frac{(m_{D'}+m_{B_f})}{m_{D'}{}^2}\lambda_{DB_i}\right]. \qquad (44)$$

Again we have used a pure magnetic dipole transition for the $DB\gamma$ vertex, and the only significant difference in the structure of (44) compared to (38) is the relative *minus* sign between the s and u channels in (44) because of the opposite SU_3 $\mathcal{C}$ parity of H_w^{PV} relative to π. This sign forces $D_{10}(\Sigma^+\to p\gamma)$ to zero in the equal-mass limit, thus respecting the Hara theorem.[4] However, for physical masses and the values of λ_{DB} as determined by (42) or (43), the u channel in (44) is suppressed relative to the s channel leading to $D_{10}(\Sigma^+\to p\gamma)\neq 0$. Similarly, for the PC decuplet amplitude we have

$$C_{10}(B_i\to B_f\gamma)=\tfrac{1}{6}\left[\lambda_{B_fD}\frac{(m_D-m_{B_i})}{m_D{}^2}H_{DB_i}^{\mathrm{PC}}+H_{B_fD'}^{\mathrm{PC}}\frac{(m_{D'}-m_{B_f})}{m_{D'}{}^2}\lambda_{DB_i}\right], \qquad (45)$$

where the plus sign between the two channels in (45) follows from $\mathcal{C}(H_w^{\mathrm{PC}})=1$.

V. OTHER CONTRIBUTIONS

We shall also consider other contributions which are less than 10% of the dominant A_{cc} or B_8 terms. These may become important when the leading terms interfere destructively with the secondary decuplet amplitudes. In particular we include $Y_0^*[\Lambda'(1405)]$ resonance states and contributions due to a small $\Delta I=\frac{3}{2}$ part in H_w.

The partial decay width of[11] 40 MeV for $\Lambda'\to\Sigma\pi$ leads to a coupling constant $g_{\Lambda'\Sigma\pi}\sim 0.84$. The weak vertex $\langle n|H_w^{\mathrm{PC}}|\Lambda\rangle\sim 40$ eV indicates that the weak Λ' transitions are also $\langle n|H_w^{\mathrm{PC,PV}}|\Lambda'\rangle\sim 40$ eV. With these magnitudes we find that

TABLE I. $10^6 B$ for p waves.

	$\underline{27}$	$\underline{8}$	$\underline{10}$	Y_0^*	Theory	Exp 1 [a]	Exp 2 [b]
Λ_0^0	−0.16	−1.32	−0.30	0	−1.78	-1.63 ± 0.10	-1.74 ± 0.16
Λ_-^0	0.05	1.87	0.42	0	2.34	2.27 ± 0.065	2.28 ± 0.07
Σ_0^+	0.03	3.09	−0.31	0	2.81	2.73 ± 0.22	$2.86^{+0.25}_{-0.19}$
Σ_+^+	0.15	4.02	−0.13	0.11	4.15	4.22 ± 0.035	$4.17^{+0.08}_{-0.21}$
Σ_-^-	−0.14	−0.36	0.31	0.11	−0.08	-0.14 ± 0.02	-0.14 ± 0.02
Ξ_-^-	0.13	2.00	−0.45	0	1.68	1.52 ± 0.12	1.52 ± 0.12
Ξ_0^0	0.20	−1.41	0.32	0	−0.89	-1.00 ± 0.23	-1.00 ± 0.24

[a] These values were calculated from the branching ratios and asymmetry parameters α by assuming A and B to be real. See O. E. Overseth, Appendix III of Ref. 11.

[b] These values represent the real parts of the amplitudes obtained by assuming time-reversal invariance and incorporating known information about final-state phase shifts into the data on α and ϕ in Ref. 11.

$$A_{\Lambda'}(\Sigma_+^+) = A_{\Lambda'}(\Sigma_-^-) = \frac{g_{\Lambda'\Sigma\pi}\langle n|H_w^{\mathrm{PV}}|\Lambda'\rangle}{(m_{\Lambda'} - m_n)}\frac{(m_\Sigma - m_n)}{(m_{\Lambda'} - m_\Sigma)} \sim 0.075\times 10^{-6}, \tag{46}$$

$$B_{\Lambda'}(\Sigma_+^+) = B_{\Lambda'}(\Sigma_-^-) = \frac{g_{\Lambda'\Sigma\pi}\langle n|H_w^{\mathrm{PC}}|\Lambda'\rangle}{(m_{\Lambda'} + m_n)}\frac{(m_\Sigma + m_n)}{(m_{\Lambda'} - m_\Sigma)} \sim 1.4\times 10^{-6}, \tag{47}$$

with undetermined signs. Again we expect our fits to respect these estimates.

The Λ' will make important contributions to the neutral radiative decays, especially the PV amplitudes, where they are not suppressed by angular momentum effects. (In the pion decays, these contributions are suppressed by the smallness of their coupling to the pion.) We estimate the $B\Lambda'\gamma$ coupling by including the Λ' in the FFR relation (39) for Σ^0 and Λ targets. The U-spin singlet target $\Sigma_1 = \frac{1}{2}\sqrt{3}\,\Sigma^0 + \frac{1}{2}\Lambda$ is decoupled from the Σ^{*0}, so that (39) becomes

$$\overline{A}_1(q_\pi = 0) \approx A_{1,\Lambda'}(q_\pi = 0) = \frac{-\sqrt{3}\, b g_{\pi\Lambda'\Sigma}}{m_{\Sigma_1} - m_{\Lambda'}} = \frac{d_{pK_n}}{2m_\Sigma{}^2} g_{BB\pi}, \tag{48}$$

where b is defined by

$$\langle \Lambda'|J_\mu^{\mathrm{em}}|\Sigma_1\rangle = eib\bar{u}_{\Lambda'}[q, \gamma_\mu]\gamma_5 u_{\Sigma_1}, \tag{49}$$

$q = p_{\Lambda'} - p_{\Sigma_1}$. (For simplicity, we here ignore the $\Lambda - \Sigma_0$ mass difference.) Equation (48) leads to the estimate

$$b \approx -1.0\,\mathrm{GeV}^{-1}, \tag{50}$$

which is to be compared with Eq. (37). Contributions to the C and D amplitudes are listed in the Appendix.

With regard to $\Delta I = \frac{3}{2}$ contributions in H_w, we note that the violations of the $\Delta I = \frac{1}{2}$ rule are

$$\Lambda_-^0 + \sqrt{2}\,\Lambda_0^0 = 0.002 \pm 0.010, \quad \Xi_-^- + \sqrt{2}\,\Xi_0^0 = 0.022 \pm 0.016, \tag{51}$$

$$\Sigma_+^+ - \Sigma_-^- - \sqrt{2}\,\Sigma_0^+ = {}^{0.030}_{0.015} \pm 0.034 \tag{52}$$

for s waves and

$$\Lambda_-^0 + \sqrt{2}\,\Lambda_0^0 = {}^{-0.03}_{-0.19} \pm 0.025, \quad \Xi_-^- + \sqrt{2}\,\Xi_0^0 = 0.11 \pm 0.44, \tag{53}$$

$$\Sigma_+^+ - \Sigma_-^- - \sqrt{2}\,\Sigma_0^+ = {}^{0.50 \pm 0.37}_{0.27 \pm 0.48} \tag{54}$$

for p waves, where the upper values correspond to Overseth's analysis[11] assuming no final-state interactions and the lower values correspond to possible final-state interactions as given by the raw data.[11] (See Tables I and II and discussion in Sec. VI.)

While the first two relations (51) must identically satisfy the $\Delta I = \frac{1}{2}$ rule in the context of current algebra, the last four relations can violate this rule. Although the errors of (52)–(54) mask any attempt to identify absolutely a $\Delta I = \frac{1}{2}$ violation, the $\underline{27}$ coupling coefficients

$$n\Lambda : p\Sigma^+ : n\Sigma^0 : \Lambda\Xi^0 : \Sigma^0\Xi^0 : \Sigma^-\Xi^- = \tfrac{1}{2}(\tfrac{3}{2})^{1/2} : 1 : 3/2\sqrt{2} : \tfrac{1}{2}(\tfrac{3}{2})^{1/2} : 3/2\sqrt{2} : 1 \tag{55}$$

lead to an H_w^{27} which agrees with the signs of *all four* central values (52)–(54). With the definition

$$\langle p|H_{w,27}^{\mathrm{PC}}|\Sigma^+\rangle = h_{27}, \tag{56}$$

TABLE II. $10^6 A$ for s waves.

	27	cc	10	Y_0^*	Theory	Exp 1	Exp 2
Λ_0^0	−0.006	−0.265	0.060	0	−0.211	-0.234 ± 0.005	-0.232 ± 0.005
Λ_-^0	0.008	0.374	−0.084	0	0.298	0.331 ± 0.002	0.330 ± 0.004
Σ_0^+	−0.009	−0.575	0.254	0	−0.330	-0.313 ± 0.020	$-0.303^{+0.020}_{-0.014}$
Σ_+^+	+0.033	0	−0.080	0.065	+0.018	0.013 ± 0.004	0.013 ± 0.004
Σ_-^-	−0.020	0.814	−0.439	0.065	0.420	0.426 ± 0.002	0.426 ± 0.003
Ξ_-^-	+0.008	−0.682	0.202	0	−0.472	-0.450 ± 0.004	-0.450 ± 0.005
Ξ_0^0	−0.006	0.482	−0.143	0	0.334	0.333 ± 0.008	0.333 ± 0.008

we find that h_{27} is negative with a magnitude between zero and 3% of the octet spurion.

VI. FITS

A straightforward method of obtaining experimental values for A and B is to neglect final-state interactions, perhaps justifiably on the grounds that the theory gives real values. On the other hand, in principle it would be better to use time-reversal invariance and known phase shifts and asymmetries to find the complex amplitudes, only then to compare the real parts with theory. To free our results from uncertainties in the handling of the second method, we use the first method for fitting the theory, and then compare with the numbers obtained by the second method. Final-state effects are greatest for the Σ_0^+ decays, but errors are largest there as well.

We have performed all our fits in such a way as to preserve the role of refinements as such, and not just as added free parameters. In fact, we regard the weak couplings for our baryon resonances and the 27-plet piece not as free but as constrained by the requirement that they contribute reasonably small corrections to A and B as indicated in the previous sections. Thus treated, they improve the fits without weakening the theory's predictive power for the pion decays.

In addition, we have strong cross checks on our important refinements. As noted in the previous section, all four relevant deviations from the $\Delta I=\frac{1}{2}$ rule for pion decays indicate the same sign for h_{27}, for either method of data analysis. Furthermore, our values for h_2 and h_3 determined from the pion decays correctly describe the radiative decay $\Sigma^+\to p\gamma$, and make testable predictions for the other radiative weak decays.

We begin by estimating h_{27}. We note that $h_{27}\neq 0$ will lead to a violation of the $\Delta I=\frac{1}{2}$ rule for the s-wave Σ decays via the dominant current-commutator contribution, and for the p-wave Σ, Λ, and Ξ decays via the baryon pole terms. We estimate from experiment that

$$h_{27}=-1.7\pm1.5 \text{ eV},$$

which is about a 2% addition to the octet part of the spurion.

For fitting the B amplitudes we have a choice between a theoretical expression involving the pion-baryon coupling and one using the axial-vector coupling constants for the octet baryons. We have found that fitting the p-wave decays with the second method requires unreasonably large decuplets. We have also discovered that the choice $(d/f)_P=2.1$ (see Sec. II) is an advantageous one for fitting—e.g., a choice of $(d/f)_P=2.3$ requires noticeably larger decuplets.

Although the B amplitudes are strongly dominated by the octet baryon poles, these are not quite so large as might be expected because of cancellations between the s and u channel poles.[3] The resulting delicacy of the fit allows a good determination of the octet spurion. Furthermore, these cancellations allow the decuplet contributions to be significant, despite their suppression by the p-wave factor

$$(m_D-m_B)(m_f-m_i)/m^2$$

relative to the octet poles. The same effect enhances the 27-plet contribution to B, although it is only 2% of H_w.

Taking the central value for h_{27}, we fit the B amplitudes by first including octet baryon and decuplet poles (with octet spurion). We find

$$h_3=(-0.40\pm0.03)\times10^{-6},$$

$$h_1d_w=-97.5\pm9 \text{ eV},$$

$$h_1f_w=111\pm3 \text{ eV},$$

where the errors for h_3 and h_1d_w have a strong direct correlation. The magnitude of h_3 is slightly larger than the estimate given in Sec. III but still acceptable. Finally, we find that the p-wave Σ_+^+ and Σ_-^- both benefit by adding a reasonable amount of $Y_0^*(\Lambda')$, in agreement with our estimates in Sec. V:

$$h_{\Lambda'}^{\mathrm{PC}} = 31 \text{ eV} .$$

The fit is displayed in Table I. The quality of the fit does not change much if h_{27} is varied, aside from $\Delta I = \frac{1}{2}$ rule effects.

Having now determined the current-commutator contribution to the A amplitudes, we fit the s-wave decays Λ_-^0, Σ_0^+, and Ξ_-^- by adding decuplets leading to

$$h_2 = (0.20 \pm 0.03) \times 10^{-6} ,$$

in which the error due to uncertainty of the weak spurion is included. Finally, a small amount of Y_0^* pole was added to Σ_+^+ and Σ_-^- to achieve the fit displayed in Table II:

$$h_{\Lambda'}^{\mathrm{PV}} = 31 \text{ eV} .$$

The quality of this fit is insensitive to the values of h_{27}, $h_1 d_w$, and $h_1 f_w$.

In spite of our use of seven parameters, it remains remarkable that the B's are quite well described by the octet spurion alone, as required by our expansion in mass differences (17), and that the spurion thus determined need be supplemented by only one parameter, h_2, to describe the s waves.

For the radiative decays we use the above parameters, and making the choice of electromagnetic couplings corresponding to (42) we find

$$C_{10}(\Sigma^+ \to p\gamma) = 0.22 \times 10^{-10} \text{ MeV}^{-1} ,$$

$$D_{10}(\Sigma^+ \to p\gamma) = -0.17 \times 10^{-10} \text{ MeV}^{-1} ,$$

$$C_8(\Sigma^+ \to p\gamma) = 1.15 \times 10^{-10} \text{ MeV}^{-1} .$$

Since $D_8(\Sigma^+ \to p\gamma) = 0$ and the Y_0^* does not couple to this decay, we find $|C|^2 + |D|^2 = 1.9 \times 10^{-20}$ MeV^{-2}, and $\alpha = -0.24$. If we use the extra suppression of the $\Sigma^+\Sigma^{*+}\gamma$ vertex indicated by our analysis of FFR in (43), we find $C_{10} = 0.18 \times 10^{-10}$ MeV^{-1} and $D_{10} = -0.33 \times 10^{-10}$ MeV^{-1}, so that $|C|^2 + |D|^2 = 1.88 \times 10^{-20}$ MeV^{-2}, and $\alpha = -0.46$. These predictions are to be compared with the $\Sigma^+ \to p\gamma$ experimental

TABLE III. Contributions to radiative decay amplitudes.

Process	C (10^{-10} MeV^{-1}) $\underline{8}$	$\underline{10}$ [a]	1 (Λ')	D (10^{-10} MeV^{-1}) 10 [a]	1 (Λ')
$\Sigma^+ \to p\gamma$	1.15	0.22	0	−0.17	0
$\Lambda \to n\gamma$	−0.66	−0.13	−0.13	0.33	0.7
$\Xi^0 \to \Lambda\gamma$	−2.06	−0.21	−0.11	−0.23	3.5
$\Xi^0 \to \Sigma^0\gamma$	−3.26	−0.18	−0.20	−0.25	6.1
$\Xi^- \to \Sigma^-\gamma$	−0.03	0	0	0	0

[a] We have used Eq. (42) in obtaining the decuplet contributions. Only D ($\Sigma^+ \to p\gamma$) is sensitive to this assumption.

results[11,39] $|C|^2 + |D|^2 = (3.07 \pm 0.5) \times 10^{-20}$ MeV^{-2} and $\alpha = -1 \pm 0.5$.[5] In either case the *sign* of α is determined by the pion decays ($h_2 > 0$) and is in agreement with experiment. The Hara theorem[4] really prevents us from obtaining $\alpha = -1$, but $\alpha = -0.46$ is within one standard deviation of the measured values. Furthermore, the *sign* of C_{10} is also determined from the pion decays ($h_3 < 0$). If, instead of adding to C_8, C_{10} had the opposite sign ($h_3 < 0$), then $|C|^2 + |D|^2 \to 1 \times 10^{-20}$ MeV^{-2}, which is far too small to be considered acceptable.

In order to see that the sign of $\alpha(\Sigma^+ \to p\gamma)$ is indeed correlated with the reduction of the $\Sigma^+ \to p\pi^0$ s-wave amplitude, we note that the expressions in the Appendix indicate

$$\mathrm{sign}(A_{cc}A_{10})_{\Sigma_0^+} = -\,\mathrm{sign}(h_1 h_2 g g_2) < 0 ,$$

$$\mathrm{sign}(C_8 D_{10})_{\Sigma^+ p\gamma} = -\,\mathrm{sign}(h_1 h_2\ \kappa_p \lambda_{p\Delta^+}) < 0 .$$

Because of the FFR relation (40) or the higher symmetry groups,[30] these two constraints are identical.

In Tables III and IV we display this result along with our predictions for the other four radiative decays. Note the large contributions of Y_0^* to the neutral D amplitudes. This is because of the smallness of the pole denominator $m_{\Lambda'} - m_{\Lambda,\Xi}$. However, the errors on $h_{\Lambda'}^{\mathrm{PV}}$ are large and our method for estimating b is purely theoretical.

TABLE IV. Predictions for radiative decays.

Process	C (10^{-10} MeV^{-1})	D (10^{-10} MeV^{-1})	Branching ratio	Asymmetry parameter
$\Sigma^+ \to p\gamma$	1.37	−0.17	0.78×10^{-3}	−0.2 to −0.5
$\Lambda \to n\gamma$	−0.92	1.0	1.5×10^{-3}	−0.995
$\Xi^0 \to \Lambda\gamma$	−2.38	3.3	1.5×10^{-2}	−0.95
$\Xi^0 \to \Sigma^0\gamma$	−3.64	5.8	1.0×10^{-2}	−0.90
$\Xi^- \to \Sigma^-\gamma$	−0.03	0	0	0

Lower values for these parameters could substantially reduce the asymmetries and branching ratios listed in Table IV.

VII. CONCLUSION

We have shown that the apparent mismatches between the pion decay amplitudes A and B and between the radiative amplitudes C and D are completely resolved in the framework of current algebra by inclusion of decuplet states. Further, the addition of the Y_0^* resonance and a $\Delta I = \frac{3}{2}$ piece in H_w refine the fits to the pion decays. Whereas the decuplets, Y_0^*, and $H_w(\Delta I = \frac{3}{2})$ contribute 30%, 10%, and 2% to A, they contribute 10%, 3%, and 6% to B, respectively. The decuplet states completely account for a negative $\Sigma^+ \to p\gamma$ asymmetry parameter of $\alpha \sim -0.2$ to -0.5, and the Y_0^* dominates the $\Xi^0 \to \Sigma^0\gamma$ and $\Xi^0 \to \Lambda\gamma$ decays.

Theoretically we conclude that the hyperon nonleptonic decays $B \to B'\pi$ and $B \to B'\gamma$ provide a very significant positive test for the $H_w \sim J \cdot J \sim \lambda_6$ and $J = V - A$ assumptions along with the current-algebra equal-time commutator

$$[F_5, H_w^{\mathrm{PC,PV}}] = -[F, H_w^{\mathrm{PV,PC}}]$$

needed in the derivation of (3). We have adopted a pragmatic attitude toward the isotopic properties of H_w in that we have not derived octet dominance, but instead use it along with a 2% empirical contamination by a 27-plet in H_w.

A further probe into the current-current structure of H_w is to dominate the complete set of states between the two currents by baryon octet and decuplet states according to the procedure of Chiu, Schechter, and Ueda,[40] but with the most recent fits to baryon form-factor data. This not only demonstrates the octet-dominance property of H_w, but also verifies our fitted value for the octet weak-spurion ratio $d_w/f_w \sim -0.9$.[41] Thus, the s-wave value $d_w/f_w = -0.3$ (Ref. 13) is inconsistent with the p-wave amplitudes and with the baryon saturation of the current-current H_w.[41] Further, any attempt to relate $d_w/f_w = -0.3$ to the K^* vector-dominance model is inconsistent with SU_3.[16] We therefore reject the notion that d_w/f_w is somehow related to $(d/f)_{\text{medium strong}} \sim -0.3$.

As for the experimental consequences of our theory, we would hope that future experiments be directed toward the various $B \to B'\gamma$ decay modes. The error on $\alpha(\Sigma^+ \to p\gamma)$ must be lowered if we are to know by just how much the Hara theorem[4] is broken. Measurements of the branching ratios and asymmetry parameters of $\Lambda \to n\gamma$, $\Xi \to \Lambda\gamma$, and $\Xi \to \Sigma\gamma$ should be considered, in spite of their difficulty. The small predicted ratio $\Gamma(\Lambda \to n\gamma)/\text{all} \sim 10^{-3}$ will be hard to measure, but an $\alpha(\Lambda \to n\gamma)$ of ~ -1 should be easier to detect because the competing pion decay has $\alpha(\Lambda \to n\pi^0) \sim 0.66$. The larger relative Ξ^0 branching ratios ($\sim 10^{-2}$ due to the Y_0^*) might make these measurements feasible in spite of the difficulty in producing Ξ^0's.

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APPENDIX

We display all contributions to the various amplitudes A, B, C, and D. We denote the masses as N for m_N, etc.

$$A_{\mathrm{cc}}(\Lambda_-^0) = -\sqrt{2}A_{\mathrm{cc}}(\Lambda_0^0) = -\frac{1}{\sqrt{2}f_\pi}H_{n\Lambda},$$

$$A_{\mathrm{cc}}(\Sigma_0^+) = \frac{1}{2f_\pi}H_{p\Sigma^+},$$

$$A_{\mathrm{cc}}(\Sigma_+^+) = -\frac{1}{f_\pi}\left(H_{n\Sigma^0} + \frac{1}{\sqrt{2}}H_{p\Sigma^+}\right),$$

$$A_{\mathrm{cc}}(\Sigma_-^-) = \frac{1}{f_\pi}H_{n\Sigma^0},$$

$$A_{\mathrm{cc}}(\Xi_-^-) = -\sqrt{2}A_{\mathrm{cc}}(\Xi_0^0) = -\frac{1}{\sqrt{2}f_\pi}H_{\Lambda\Xi^0},$$

$$B_8(\Lambda_-^0) = -\frac{(\Lambda+N)}{f_\pi}\left[\frac{g_{pn\pi^+}H_{n\Lambda}}{N(\Lambda-N)} - \frac{2g_{\Sigma^+\Lambda\pi^+}H_{p\Sigma^+}}{(\Sigma+\Lambda)(\Sigma-N)}\right],$$

$$B_8(\Lambda_0^0) = -\frac{(\Lambda+N)}{f_\pi}\left[\frac{g_{nn\pi^0}H_{n\Lambda}}{N(\Lambda-N)} - \frac{2g_{\Sigma^0\Lambda\pi^0}H_{n\Sigma^0}}{(\Sigma+\Lambda)(\Sigma-N)}\right],$$

$$B_8(\Sigma_0^+) = -\frac{(\Sigma+N)}{f_\pi}\left[\frac{g_{pp\pi^0}H_{p\Sigma^+}}{N(\Sigma-N)} - \frac{g_{\Sigma^+\Sigma^+\pi^0}H_{p\Sigma^+}}{\Sigma(\Sigma-N)}\right],$$

$$B_8(\Sigma_+^+) = -\frac{(\Sigma+N)}{f_\pi}\left[\frac{g_{np\pi^-}H_{p\Sigma^+}}{N(\Sigma-N)} - \frac{g_{\Sigma^0\Sigma^+\pi^-}H_{n\Sigma^0}}{\Sigma(\Sigma-N)} - \frac{2g_{\Lambda\Sigma^+\pi^-}H_{n\Lambda}}{(\Sigma+\Lambda)(\Lambda-N)}\right],$$

$$B_8(\Sigma_-^-) = \frac{(\Sigma+N)}{f_\pi}\left[\frac{g_{\Sigma^0\Sigma^-\pi^+}H_{n\Sigma^0}}{\Sigma(\Sigma-N)} + \frac{2g_{\Lambda\Sigma^-\pi^+}H_{n\Lambda}}{(\Sigma+\Lambda)(\Lambda-N)}\right],$$

$$B_8(\Xi_-^-) = -\frac{(\Xi+\Lambda)}{f_\pi}\left[\frac{2g_{\Lambda\Sigma^-\pi^+}H_{\Sigma^-\Xi^-}}{(\Sigma+\Lambda)(\Xi-\Sigma)} - \frac{g_{\Xi^0\Xi^-\pi^+}H_{\Lambda\Xi^0}}{\Xi(\Xi-\Lambda)}\right],$$

$$B_8(\Xi_0^0) = -\frac{(\Xi+\Lambda)}{f_\pi}\left[\frac{2g_{\Lambda\Sigma^0\pi^0}H_{\Sigma^0\Xi^0}}{(\Sigma+\Lambda)(\Xi-\Sigma)} - \frac{g_{\Xi^0\Xi^0\pi^0}H_{\Lambda\Xi^0}}{\Xi(\Xi-\Lambda)}\right],$$

$$C_8(\Sigma^+ \to p\gamma) = -\frac{\kappa_p}{2\Sigma N}H_{p\Sigma^+},$$

$$C_8(\Lambda \to n\gamma) = -\frac{\kappa_n}{2\Lambda N}\left(\frac{\Lambda - \frac{1}{2}N}{\Lambda - N}\right)H_{n\Lambda} + \frac{\kappa_{\Sigma\Lambda}H_{n\Sigma^0}}{(\Sigma+\Lambda)(\Sigma-N)},$$

$$C_8(\Xi^0\to\Lambda\gamma)=\frac{\kappa_n}{2\Xi\Lambda}\left(\frac{\Lambda-\frac{1}{2}\Xi}{\Xi-\Lambda}\right)H_{\Lambda\Xi^0}-\frac{\kappa_{\Sigma\Lambda}H_{\Sigma^0\Xi^0}}{(\Sigma+\Lambda)(\Xi-\Sigma)},$$

$$C_8(\Xi^0\to\Sigma^0\gamma)=\frac{\kappa_n}{2\Xi\Sigma}\left(\frac{\Sigma+\frac{1}{2}\Xi}{\Xi-\Sigma}\right)H_{\Sigma^0\Xi^0}-\frac{\kappa_{\Sigma\Lambda}H_{\Lambda\Xi^0}}{(\Sigma+\Lambda)(\Xi-\Lambda)},$$

$$C_8(\Xi^-\to\Sigma^-\gamma)=\frac{(\kappa_p+\kappa_n)}{2\Xi\Sigma}H_{\Sigma^-\Xi^-},$$

where $H_{fi}=\langle B_f|H_w^{\rm PC}|B_i\rangle$, with

$$H_{n\Lambda}=-\tfrac{1}{2}h_1(\tfrac{3}{2})^{1/2}(f_w+\tfrac{1}{3}d_w)+\tfrac{1}{2}(\tfrac{3}{2})^{1/2}h_{27},$$

$$H_{\Lambda\Xi^0}=\tfrac{1}{2}h_1(\tfrac{3}{2})^{1/2}(f_w-\tfrac{1}{3}d_w)+\tfrac{1}{2}(\tfrac{3}{2})^{1/2}h_{27},$$

$$H_{p\Sigma^+}=-\tfrac{1}{2}h_1(f_w-d_w)+h_{27},$$

$$H_{n\Sigma^0}=\tfrac{1}{2}(\tfrac{1}{2})^{1/2}h_1(f_w-d_w)+\tfrac{3}{2}(\tfrac{1}{2})^{1/2}h_{27},$$

$$H_{\Sigma^-\Xi^-}=\tfrac{1}{2}h_1+h_{27},$$

$$H_{\Sigma^0\Xi^0}=-\tfrac{1}{2}(\tfrac{1}{2})^{1/2}h_1+\tfrac{3}{2}(\tfrac{1}{2})^{1/2}h_{27},$$

and

$$g_{pp\pi^0}=-g_{nn\pi^0}=g,$$

$$g_{pn\pi^+}=g_{np\pi^-}=\sqrt{2}\,g,$$

$$g_{\Sigma^+\Lambda\pi^+}=g_{\Sigma^0\Lambda\pi^0}=g_{\Lambda\Sigma^+\pi^-}=\frac{2}{\sqrt{3}}d_Pg,$$

$$g_{\Sigma^+\Sigma^+\pi^0}=-g_{\Sigma^0\Sigma^+\pi^-}=g_{\Sigma^0\Sigma^-\pi^+}=2f_Pg,$$

$$g_{\Xi^0\Xi^0\pi^0}=-(\tfrac{1}{2})^{1/2}g_{\Xi^0\Xi^-\pi^+}=(f_P-d_P)g,$$

and

$$\kappa_p=\kappa_{\Sigma^+}=1.79,$$

$$\kappa_n=\kappa_{\Xi^0}=2\kappa_\Lambda=-2\kappa_{\Sigma^0}=-(2/\sqrt{3})\kappa_{\Sigma\Lambda}=-1.91,$$

$$\kappa_{\Sigma^-}=\kappa_{\Xi^-}=-(\kappa_p+\kappa_n).$$

The decuplet contributions are (we assume $H_w\sim\lambda_6$ here)

$$A_{10}(\Lambda^0_-)=-\sqrt{2}A_{10}(\Lambda^0_0)$$
$$=-\frac{h_2g_2}{3\sqrt{6}}(\Lambda-N)\left(\frac{\Sigma^*+N}{\Sigma^{*2}}\right),$$

$$A_{10}(\Sigma^+_0)=\tfrac{1}{9}\sqrt{2}h_2g_2(\Sigma-N)\left(\frac{\Delta+\Sigma}{\Delta^2}+\frac{1}{2}\frac{\Sigma^*+N}{\Sigma^{*2}}\right),$$

$$A_{10}(\Sigma^+_+)=-\tfrac{1}{9}h_2g_2(\Sigma-N)\left(\frac{\Delta+\Sigma}{\Delta^2}-\frac{1}{2}\frac{\Sigma^*+N}{\Sigma^{*2}}\right),$$

$$A_{10}(\Sigma^-_-)=-\tfrac{1}{3}h_2g_2(\Sigma-N)\left(\frac{\Delta+\Sigma}{\Delta^2}+\frac{1}{6}\frac{\Sigma^*+N}{\Sigma^{*2}}\right),$$

$$A_{10}(\Xi^-_-)=-\sqrt{2}A_{10}(\Xi^0_0)$$
$$=\frac{h_2g_2}{3\sqrt{6}}(\Xi-\Lambda)\left(\frac{\Sigma^*+\Xi}{\Sigma^{*2}}+\frac{\Xi^*+\Lambda}{\Xi^{*2}}\right),$$

$$B_{10}(\Lambda^0_-)=-\sqrt{2}B_{10}(\Lambda^0_0)$$
$$=-\frac{h_3g_2}{3\sqrt{6}}(\Lambda+N)\left(\frac{\Sigma^*-N}{\Sigma^{*2}}\right),$$

$$B_{10}(\Sigma^+_0)=\tfrac{1}{9}\sqrt{2}h_3g_2(\Sigma+N)\left(\frac{\Delta-\Sigma}{\Delta^2}+\frac{1}{2}\frac{\Sigma^*-N}{\Sigma^{*2}}\right),$$

$$B_{10}(\Sigma^+_+)=-\tfrac{1}{9}h_3g_2(\Sigma+N)\left(\frac{\Delta-\Sigma}{\Delta^2}-\frac{1}{2}\frac{\Sigma^*-N}{\Sigma^{*2}}\right),$$

$$B_{10}(\Sigma^-_-)=-\tfrac{1}{3}h_3g_2(\Sigma+N)\left(\frac{\Delta-\Sigma}{\Delta^2}+\frac{1}{6}\frac{\Sigma^*-N}{\Sigma^{*2}}\right),$$

$$B_{10}(\Xi^-_-)=-\sqrt{2}B_{10}(\Xi^0_0)$$
$$=\frac{h_3g_2}{3\sqrt{6}}(\Xi+\Lambda)\left(\frac{\Sigma^*-\Xi}{\Sigma^{*2}}+\frac{\Xi^*-\Lambda}{\Xi^{*2}}\right),$$

$$C_{10}(\Sigma^+\to p\gamma)=-\frac{h_3\lambda_{p\Delta^+}}{6\sqrt{3}}\left[\frac{\Delta-\Sigma}{\Delta^2}-\left(\frac{\lambda_{\Sigma^{*+}\Sigma^+}}{\lambda_{p\Delta^+}}\right)\frac{\Sigma^*-N}{\Sigma^{*2}}\right],$$

$$C_{10}(\Lambda\to n\gamma)=-\frac{h_3\lambda_{\Sigma^{*0}\Lambda}}{6\sqrt{6}}\left(\frac{\Sigma^*-N}{\Sigma^{*2}}\right),$$

$$C_{10}(\Xi^0\to\Lambda\gamma)=-\frac{h_3\lambda_{\Lambda\Sigma^{*0}}}{6\sqrt{6}}\left[\frac{\Sigma^*-\Xi}{\Sigma^{*2}}+\sqrt{3}\left(\frac{\lambda_{\Xi^{*0}\Xi^0}}{\lambda_{\Lambda\Sigma^{*0}}}\right)\frac{\Xi^*-\Lambda}{\Xi^{*2}}\right],$$

$$C_{10}(\Xi^0\to\Sigma^0\gamma)=-\frac{h_3\lambda_{\Sigma^0\Sigma^{*0}}}{6\sqrt{6}}\left[\frac{\Sigma^*-\Xi}{\Sigma^{*2}}+\left(\frac{\lambda_{\Xi^{*0}\Xi^0}}{\lambda_{\Sigma^0\Sigma^{*0}}}\right)\frac{\Xi^*-\Sigma}{\Xi^{*2}}\right],$$

$$C_{10}(\Xi^-\to\Sigma^-\gamma)=0,$$

$$D_{10}(\Sigma^+\to p\gamma)=-\frac{h_2\lambda_{p\Delta^+}}{6\sqrt{3}}\left[\frac{\Delta+\Sigma}{\Delta^2}+\left(\frac{\lambda_{\Sigma^{*+}\Sigma^+}}{\lambda_{p\Delta^+}}\right)\frac{\Sigma^*+N}{\Sigma^{*2}}\right],$$

$$D_{10}(\Lambda\to n\gamma)=-\frac{h_2\lambda_{\Sigma^{*0}\Lambda}}{6\sqrt{6}}\left(\frac{\Sigma^*+N}{\Sigma^{*2}}\right),$$

$$D_{10}(\Xi^0\to\Lambda\gamma)=-\frac{h_2\lambda_{\Lambda\Sigma^{*0}}}{6\sqrt{6}}\left[\frac{\Sigma^*+\Xi}{\Sigma^{*2}}-\sqrt{3}\left(\frac{\lambda_{\Xi^{*0}\Xi^0}}{\lambda_{\Lambda\Sigma^{*0}}}\right)\frac{\Xi^*+\Lambda}{\Xi^{*2}}\right],$$

$$D_{10}(\Xi^0\to\Sigma^0\gamma)=-\frac{h_2\lambda_{\Sigma^0\Sigma^{*0}}}{6\sqrt{6}}\left[\frac{\Sigma^*+\Xi}{\Sigma^{*2}}-\left(\frac{\lambda_{\Xi^{*0}\Xi^0}}{\lambda_{\Sigma^0\Sigma^{*0}}}\right)\frac{\Xi^*+\Sigma}{\Xi^{*2}}\right],$$

$$D_{10}(\Xi^-\to\Sigma^-\gamma)=0,$$

with

$$\lambda_{\Delta^+p}=\lambda_{\Delta^0n}=-\lambda_{\Sigma^{*+}\Sigma^+}=2\lambda_{\Sigma^{*0}\Sigma^0}$$
$$=-(2/\sqrt{3})\lambda_{\Sigma^{*0}\Lambda}=-\lambda_{\Xi^{*0}\Xi^0}\approx2.34.$$

Finally

$$A_{\Lambda'}(\Sigma^+_+)=A_{\Lambda'}(\Sigma^-_-)$$
$$=\frac{(\Sigma-N)g_{\pi\Lambda'\Sigma}h^{\rm PV}_{\Lambda'}}{(\Lambda'-\Sigma)(\Lambda'-N)},$$

and all other $A_{\Lambda'}$ are 0;

$$B_{\Lambda'}(\Sigma^+_+)=B_{\Lambda'}(\Sigma^-_-)$$
$$=\frac{(\Sigma+N)g_{\pi\Lambda'\Sigma}h^{\rm PC}_{\Lambda'}}{(\Lambda'-\Sigma)(\Lambda'-N)},$$

and all other $B_{\Lambda'}$ are 0;

$$C_{\Lambda'}(\Lambda\to n\gamma)=\frac{bh^{\rm PC}_{\Lambda'}}{\Lambda'+N},$$

$$C_{\Lambda'}(\Xi^0\to\Lambda\gamma)=\frac{1}{\sqrt{3}}C_{\Lambda'}(\Xi^0\to\Sigma^0\gamma)=\frac{bh^{\rm PC}_{\Lambda'}}{\Lambda'+\Xi},$$

$$C_{\Lambda'}(\Sigma^+\to p\gamma)=C_{\Lambda'}(\Xi^-\to\Sigma^-\gamma)=0,$$

$$D_{\Lambda'}(\Lambda\to n\gamma)=-\frac{bh^{\rm PV}_{\Lambda'}}{\Lambda'-N},$$

$$D_{\Lambda'}(\Xi^0\to n\gamma)=\frac{1}{\sqrt{3}}D_{\Lambda'}(\Xi^0\to\Sigma^0\gamma)=\frac{bh^{\rm PV}_{\Lambda'}}{\Lambda'-\Xi},$$

$$D_{\Lambda'}(\Sigma^+\to p\gamma)=D_{\Lambda'}(\Xi^-\to\Sigma^-\gamma)=0.$$

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[20] We use $g^{fiP}=2g(d_P d_{fiP}-f_P i f_{fiP})$ and $g_A^{fiP}=g_A(d_A d_{fiP}-f_P i f_{fiP})$, with $g=13.6$ and $g_A=1.24$ (Refs. 11 and 21). The Goldberger-Treiman value of f_π is then $f_\pi=mg_A/g=85.6$ MeV, which compares favorably with the "true" value extracted from π decay, $f_\pi\sim 91$ MeV. We shall use the latter value in our analysis.

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[25] It should be noted that by using the hard-pion technique of Okubo (Ref. 10) we obtain all our results without explicit reference to g_A.

[26] Accordingly, we take $\kappa_p=\kappa_{\Sigma^+}=1.79$, $\kappa_n=\kappa_{\Xi^0}=2\kappa_\Lambda=-2\kappa_{\Sigma^0}=-(2/\sqrt{3})\kappa_{\Sigma\Lambda}=-1.91$, and $\kappa_{\Sigma^-}=\kappa_{\Xi^-}=-(\kappa_p+\kappa_n)$. The predicted Λ and Σ^+ moments are then within 2 standard deviations of experiment (c.f. Ref. 12); whereas if $\kappa/2m$ were the SU_3 quantities the predicted Λ moment would be almost 6 standard deviations from experiment.

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[30] The choice of relative sign $gg_2>0$ ($g=13.6$, $g_2=15.7$) is consistent with higher symmetry groups such as U(6,6). See, e.g., R. Delburgo, A. Salam, and J. Strathdee, in *Proceedings of the Seminar on High-Energy Physics and Elementary Particles, Trieste, Italy, 1965* (International Atomic Energy Agency, Vienna, Austria, 1965).

[31] If H_w transformed like λ_7, then $h_2\to -h_2$ in (32).

[32] R. Peccei (Ref. 9) has found a unique (but not necessar-

ily correct) way to resolve this formal field theory ambiguity. However, we have applied his method to this problem and find that it gives results for A_{10} and D_{10} which are twice as large as those given by dispersion theory. What is worse, it does not generate the PC spurion vertex $l=2$ suppression factor $(m_D - m_B)$ in B_{10} and C_{10}.

[33]This gives a sensitive test of our choice of dispersion covariants for the spurion process. Schnitzer (Ref. 9) and Chan (Ref. 10) choose axial vertex covariants $g_{\mu\nu}$ and three others which are orthogonal to q_ν. This choice contains kinematic zeros which eventually destroy the needed $m_D - m_B$ suppression at the $l=2$ PC spurion vertex.

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[37]Reference 7 finds that corrections to the narrow-width approximation (38) reduces (40) to 22% below the experimental value for κ_p.

[38]The $U=1$ state Σ_3 will rule out possible Y_0^* resonant contributions.

[39]It should be noted that the various measurements of the $\Sigma^+ \to p\gamma$ branching ratio are somewhat inconsistent, but that the experiment with the smallest errors gives $|C|^2 + |D|^2 = (2.7 \pm 0.4) \times 10^{-20}$ MeV^{-2}, closer to our theoretical prediction.

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From a Hard-Pion Model to a Phenomenological Lagrangian

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A hard-pion model is presented for a general symmetry. In the case of strong partially conserved currents the proper vertices are assumed to be analytical. Ward-like identities are obtained for the contact terms. For the soft-pion process in the symmetry limit of SU(2) × SU(2) a phenomenological Lagrangian is built which reproduces the results of the hard-pion model. As an illustration we deal with the K_{l3} problem, and build an amplitude for the photoproduction process $\gamma N \to \pi N$.

I. INTRODUCTION

Hard pions (H.P.) and the phenomenological Lagrangians (P.L.) have been successfully used in describing various low-energy phenomena.[1,2] It has been pointed out that both methods give similar results. More systematic proof of the equivalence of the results given by P.L. with those obtainable from current algebra was given by Dashen and Weinstein.[3] The equivalence is clear, in particular for soft-pion emissions in the symmetry limit. Our aim is to present a general H.P. model which in some sense is a generalization of the Gell-Mann–Oakes–Renner (GMOR) model[4] for symmetry breaking and the work of Gerstein *et al.*[5] on the structure of 3-point functions.

The main advantage of the H.P. model presented is the possibility of extracting the scalar (pseudoscalar) contribution from the nonconserved currents. Having constructed the H.P. model, we shall discuss soft-pion processes in the symmetry limit. We shall show that the same results may be obtained using a P.L. in the tree approximation. As an illustration for the use of the H.P. model we shall deal with the K_{l3} problem and with the photoproduction process $\gamma N \to \pi N$ in the $P_\gamma < 1$ GeV/c region.

II. THE HARD-PION MODEL

The construction of the H.P. amplitudes will be achieved in four steps.

Step 1: The symmetry structure. The symmetry structure of the model is defined by the following commutation relations between the currents and their divergences[5]:

$$[J_a^0(x), J_b^\mu(y)]\,\delta(x^0 - y^0) = iC_{abc}J_c^\mu(x)\delta^4(x-y) + \text{S.T.}, \tag{1a}$$