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Coupling-Constant Sum Rules for Broken SU(3)

E. Golowich and V. Kapila
Department of Physics and Astronomy, University of Massachusetts, Amherst, Massachusetts 01002
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Both theoretical and phenomenological aspects of broken-SU(3) coupling-constant sum rules are considered. Assumptions underlying the conventional derivation of these sum rules are discussed. A coupling-constant sum rule is derived in a model involving broken scale invariance. A phenomenological analysis of tensor-meson decays is carried out. Four coupling-constant sum rules are analyzed, and the ambiguity problem in defining empirical coupling constants is studied.

I. INTRODUCTION

It is clear from the occurrence of mass degeneracies in both the ground state and also low-lying excitations of mesons and baryons that SU(3) must play a central role in our description of hadrons. This is even more evident when one considers the striking manner in which the symmetry is broken (octet dominance). However, the physical basis of the symmetry is still not understood. Because of this, we are unable to answer several fundamental questions about the behavior of hadronic matter. For example, does the symmetry breaking decrease and eventually vanish for highly excited hadronic states (and if so, at what energy scale), or is it comparable to that of the ground state? Presumably, progress can be made in answering this question empirically when the appropriate data become available. Unfortunately, we currently lack the information on resonant states needed to resolve the question. As an illustration of this, we list in Table I for certain of the baryon multiplets the average multiplet mass,[1]

$$\bar{M}=\frac{1}{N}\sum_{i=1}^{N}M_i\,, \tag{1}$$

the average mass per baryon due to symmetry breaking,

$$\langle\Delta M\rangle=\frac{1}{N}\sum_{i=1}^{N}|M_i-\bar{M}|, \tag{2}$$

and finally $\langle\Delta M\rangle/\bar{M}$. In (1) and (2), the summation goes over the members of a given multiplet. We see in Table I a decrease of $\langle\Delta M\rangle/\bar{M}$ with energy, i.e., the mass contribution due to symmetry breaking is not keeping pace with the unitary singlet mean energy. Of more interest is the apparent slight decrease in the absolute size of $\langle\Delta M\rangle$ with increasing energy. However, given the uncertainty in high-spin masses, this cannot justifiably be distinguished from constant behavior. Further experimental work is needed to clarify this point. Were the value of $\langle\Delta M\rangle\cong 100$ MeV to remain constant, it would constitute a basic parameter for theorists to calculate. Incidentally, some indirect evidence on this subject comes from particle production data. There, it appears that SU(3) does not improve as energy increases. For example, at the highest accelerator energies, the production of kaons is still not comparable to that of pions.[2]

Our purpose in the discussion so far is simply to point out how deficient even our qualitative understanding of SU(3) is. In view of this, we feel that the phenomenological aspect of this subject deserves continued study. The area on which we concentrate in this paper is the relation between SU(3) and hadronic couplings. There are several ways in which one can define hadronic couplings. These exhibit varying degrees of SU(3) invariance. For example, according to the Ademollo-Gatto theorem,[3] matrix elements of the vector current between single-particle states of the same SU(3) multiplet evaluated at zero momentum transfer, are SU(3) symmetric to a good approximation. Another example of the possibility of SU(3)-invariant hadronic couplings, is the Gell-Mann, Oakes, Renner solution[4] for the pseudoscalar-meson decay constants, for which it is predicted that $F_\pi\cong F_K\cong F_\eta$. Our primary concern in this paper is with hadronic coupling constants. These describe the strength of the transition $\alpha+\beta\to\gamma$, where α,β,γ are on-shell hadrons. The easiest way to experimentally investigate these couplings is for one or more of the particles to be a resonance. By measuring the decay width and dividing out some appropriate measure of phase space, the decay coupling constant can be extracted from the data. When this is done, it is generally found that effects of SU(3) breaking are still present. As an example of this, we present in Table II numerical values associated with certain of the transitions $\underline{10}(\frac{3}{2}^+)\to\underline{8}(\frac{1}{2}^+)\otimes\underline{8}(0^-)$ among baryon states. For this case, a suitable relation between coupling constant and decay width is

$$g^2 = \frac{3m_\pi^2 M_R \Gamma}{k^3(E+M)}, \tag{3}$$

where M_R is the resonance mass and k, E, M are the momentum, energy, and mass, respectively, of the decay baryon evaluated in the parent rest frame. To estimate the degree of SU(3) breaking in these couplings, we can perform a least-squares analysis by minimizing the non-negative function,

$$F(g) = [g(\Delta N\pi) - g/\sqrt{2}\,]^2 + [g(Y_1^*\Lambda\pi) - g/2]^2 + [g(Y_1^*\Sigma\pi) - g/\sqrt{6}\,]^2 + [g(\Xi^*\Xi\pi) - g/2]^2. \tag{4}$$

The second term in each square bracket is the SU(3)-invariant coupling, with over-all strength g. Upon performing the minimization $\partial F/\partial g = 0$, we obtain $g = 0.73$. The average degree of symmetry breaking (in percent) can then be calculated for each of the coupling constants, and is given in Table II. The root-mean-square average is seen to be about 15%.

There exists a sum rule which relates the coupling constants which appear in Eq. (4), viz.,

$$\sqrt{2}\,g(\Delta N\pi) + 2g(\Xi^*\Xi\pi) = 3g(Y_1^*\Lambda\pi) + \tfrac{1}{2}\sqrt{6}\,g(Y_1^*\Sigma\pi). \tag{5}$$

This sum rule is compared with the current data[1] in Table II. It is obeyed to an accuracy of about 4%. The existence of such coupling sum rules can have the following practical significance: As increasingly accurate data become available, coupling-constant sum rules, if valid, can provide rather sensitive consistency checks on the experimental numbers. The remainder of this paper contains theoretical and phenomenological studies of this subject. In Sec. II, we discuss theoretical means by which the sum rule (5), and others like it, are conventionally derived. Part of the discussion in Sec. II involves a dynamical model incorporating the concept of scale invariance. Then, in Sec. III, we perform a phenomenological analysis of tensor-meson decays into two pseudoscalar mesons. The data are used as input to several coupling sum rules. Section IV contains our conclusions. There are also two Appendixes. The first is on the relation between Gell-Mann—Okubo mass sum rules and the concept of scalar dominance, whereas in the second, the decay of tensor mesons into vector and pseudoscalar mesons is examined.

TABLE I. Baryon multiplet masses. The average multiplet mass is denoted by $\overline{M}$, and $\langle \Delta M \rangle$ is the average mass per baryon associated with symmetry breaking as defined in Eq. (2) of the text.

	$\overline{M}$	$\langle \Delta M \rangle$	$\langle \Delta M \rangle / \overline{M}$
$\frac{1}{2}^+$	1151	115	0.1
$\frac{3}{2}^+$	1383	118	0.085
$\frac{5}{2}^-$	1798	88	0.049
$\frac{5}{2}^+$	1865	101	0.054

TABLE II. Coupling-constant sum rule for $\frac{3}{2}^+ \to \frac{1}{2}^+$ baryon transitions. The couplings shown in the first column are calculated from Eq. (3) of the text. The second column gives the percent deviation from the SU(3) fit of each empirical coupling constant. The final two columns display the content of sum rule (5) in the text.

	Isospin-invariant coupling constants		Contribution to sum rule	
	$g(\frac{3}{2}^+ \to \frac{1}{2}^+ 0^-)$	$\left\lvert \frac{\Delta g}{g} \right\rvert$ (%)	SU(3)	Empirical
$\Delta N\pi$	0.60	16.3	0.717	0.85
$\Xi^*\Xi\pi$	0.29	20.2	0.717	0.58
$Y_1^*\Sigma\pi$	0.26	12.3	0.358	0.32
$Y_1^*\Lambda\pi$	0.35	4.1	1.076	1.05

II. THEORETICAL ASPECTS OF COUPLING-CONSTANT SUM RULES

We begin this section by reviewing a method for deriving coupling-constant sum rules that has generally been the basis of previous studies.[6] Suppose SU(3) is broken by one or more terms in the Hamiltonian which transform as the isoscalar member of an octet. Then the coupling $g(\alpha+\beta\to\gamma)$, where the states α, β, γ belong to SU(3) representations $\underline{A}$, $\underline{B}$, $\underline{C}$, respectively, is no longer given by its SU(3)-invariant value. Additional parameters are needed to describe it. We may generate expressions for these by analyzing T_8, the symmetry-breaking part of the transition operator, taken between the states α, β, γ. If, in the matrix element $\langle \underline{C}(\gamma) | T_8 | \underline{A}(\alpha)\underline{B}(\beta)\rangle$, we assume that the states α, β, γ are correctly given by their original SU(3) assignments, even though the symmetry is broken, then application of the Wigner-Eckart theorem implies the following formula:

$$g(\alpha+\beta\to\gamma) = g_{SU(3)}(\alpha+\beta\to\gamma) + \sum_N X_N \begin{pmatrix} C & 8 & N \\ \gamma & 0 & \gamma \end{pmatrix} \begin{pmatrix} A & B & N \\ \alpha & \beta & \gamma \end{pmatrix}. \tag{6}$$

The X_N are the additional parameters, mentioned above, needed to describe $g(\alpha+\beta\to\gamma)$ in the presence of symmetry breaking, and the quantities in parentheses are SU(3) isoscalar factors. The index N runs over all SU(3) representations contained in the Clebsch-Gordan series of both $\underline{A}\otimes\underline{B}$ and $\underline{C}\otimes\underline{8}$. Indices representing the possibility of F, D type couplings have been suppressed. If we are considering a set of m couplings, and n is the number of independent parameters needed in (6) to describe them, then there are $m-n$ sum rules. A useful sum rule involving pion emission from $\frac{3}{2}^+$ baryon resonances has been given in Eq.(5). Here is another, describing the $\frac{3}{2}^- \to \frac{3}{2}^+$ baryon transition with pion emission,

$$2g(\Xi'\,\Xi^*\pi)+g(N'\,\Delta\pi) - \frac{3}{\sqrt{6}}g(Y_1'\,Y_1^*\pi) - \sqrt{3}\,g(Y_0'\,Y_1^*\pi)=0. \quad (7)$$

The primed baryons have spin-parity $\frac{3}{2}^-$. One must be careful to allow for the presence of singlet-octet mixing in the $\frac{3}{2}^-$ isoscalar states. There is probably good reason to believe in the sum rule (7). We discuss this point further in Section IV.

Sum rules (5) and (7) may be shown to follow from the decomposition of $g(\alpha+\beta\to\gamma)$ given in Eq. (6). The main assumption underlying the derivation of Eq. (6) is that each particle state transforms as a member of a particular SU(3) multiplet even in the presence of symmetry breaking. Although certainly popular, this kind of assumption is not above criticism. Sometimes it can lead to difficulties. In the following, we discuss two examples of this.

Despite the model calculation of Ref. 4, it appears from experiment that the pseudoscalar decay constants are not equal[7]; evidently F_K and F_π differ appreciably. These decay constants appear in the matrix elements

$$\langle 0|A_a^\mu(0)|P_b(\vec{k})\rangle = i\,F_a\,\delta_{ab}\,k^\mu, \quad (8)$$

where A_a^μ is an axial-vector current, P_b is a pseudoscalar meson, and $a, b=1,\ldots,8$. If we assume that the vacuum is SU(3)-invariant,[8] that the states $|P_b(\vec{k})\rangle$ transform as members of an octet, and use current algebra to describe how the A_a^μ transform, then SU(3) rotations may be used to show $F_\pi=F_K$ F_η. This unwanted result is produced directly by the assumption that symmetry breaking does not affect the SU(3) description of $|0\rangle$ and $|P_b(\vec{k})\rangle$.

A more interesting, although somewhat more involved, example of the pitfalls which can occur when SU(3) properties of hadron states are assumed, is shown by analyzing matrix elements of the energy-momentum trace operator, θ. The outlook that hadron physics might be elegantly described in terms of broken SU(3)×SU(3) symmetry has attracted much attention recently. It is popular to suppose[9] that the symmetry is broken by operators u_0, u_8 belonging to the $(3,\overline{3})+(\overline{3},3)$ representation of SU(3)×SU(3). Thus, one can write for the energy density θ^{00},

$$\theta^{00}=\overline{\theta}^{00}+u_0+c\,u_8+\delta\,, \quad (9)$$

and for the energy-momentum trace,

$$\theta=(4-d)(u_0+cu_8)+4\delta\,. \quad (10)$$

In Eq. (9), $\overline{\theta}^{00}$ is an SU(3)×SU(3) singlet, δ is a c-number contribution to the energy density, and the constant c determines the amount of SU(3) breaking present. In Eq. (10), the constant d is the dimension of the symmetry-breaking operators u_0, u_8. Our point is to show that by using the naive SU(3) states freely in certain meson and baryon matrix elements, it is possible to obtain a contradiction in the evaluation of the dimension. First, consider the rigorous energy density and trace equalities for baryons

$$\langle\vec{P}, a|\,\theta^{00}|\vec{P}, a\rangle_{\vec{P}=0}=\langle\vec{P}, a|\theta|\vec{P}, a\rangle = M_a\,,\quad a=1,\ldots,8 \quad (11a)$$

and for mesons,

$$\langle\vec{p}, a|\theta^{00}|\vec{p}, a\rangle_{\vec{p}=0}=\langle\vec{p}, a|\theta|\vec{p}, a\rangle = 2\,m_a^{\;2},\quad a=1,\ldots,8\,. \quad (11b)$$

By applying the Wigner-Eckart theorem (without justification) to matrix elements of both θ^{00} and θ as in (11a) and (11b), one can deduce that $d=3$. Alternatively, let us employ the Wigner-Eckart theorem (again, without justification) in a soft-meson analysis of 0^- meson matrix elements of the trace

$$\langle\vec{p}\,';\,b|\theta|\vec{p}, a\rangle=\delta_{ab}E_0(t)+d_{ab8}\,E_8(t)\,. \quad (12)$$

The trace constraint relation of (11b) implies for (12) that

$$E_8(0)=\frac{4}{\sqrt{3}}(m_\pi^{\;2}-m_K^{\;2})\,,\qquad E_0(0)=\tfrac{2}{3}(m_\pi^{\;2}+2\,m_K^{\;2})\,. \quad (13)$$

However, we may use a soft-meson analysis of Eq. (12) to show that

$$E_8(0)=-\frac{(4-d)c}{F_a}\langle 0|\,v_a(0)|\,\vec{p}, a\rangle\,,\qquad E_0(0)=-\frac{\sqrt{2}}{\sqrt{3}}\frac{(4-d)}{F_a}\langle 0|\,v_a(0)|\,\vec{p}, a\rangle\,, \quad (14)$$

where F_a is the decay constant of meson a, and the v_a may be related[4] to the current divergences $\partial_\mu A_a^\mu$,

$$\partial_\lambda A_a^\lambda=-\frac{\sqrt{2}+c}{\sqrt{3}}\,v_a\,,\quad a=1,2,3. \quad (15)$$

By substituting Eq. (13) into Eq. (14), we obtain the familiar relation $c=2\sqrt{2}\,(m_\pi^{\;2}-m_K^{\;2})/(m_\pi^{\;2}+2m_K^{\;2})$. Then by substituting Eq. (15) into (14), we finally get $d=2$. In this model, the dimension should have a unique value, so there is an inconsistency between the two analyses just performed. It is probably attributable to the free use of the Wigner-Eckart theorem throughout the calculation.

At the very least, these examples indicate that we must be careful in using the SU(3) wave functions. Although the SU(3) assumption might be valid in some cases, it is likely to cause problems in others. Thus, derivations, such as the one which led to Eq. (6), are not rigorous unless supported by additional dynamical knowledge.

For the rest of this section, we shall examine a simple model,[10] based on the concepts of scale and chiral symmetries, in which a coupling sum rule emerges. The model is of interest because assumptions regarding the transformation properties of states do not directly appear. Furthermore, the idea that symmetry breaking in coupling constants is likely to accompany that which occurs in masses is readily made.

We begin by defining an off-shell vertex function $T_a(q^2, p^2; t)$, which describes the coupling of pseudoscalar mesons to the energy-momentum trace,

$$T_a(q^2, p^2; t) = i^2 \frac{(m_a^2 - q^2)(m_a^2 - p^2)}{F_a^2 m_a^4} \int d^4x d^4y\, e^{iq\cdot x - ip\cdot y} \langle 0 | T \partial_\lambda A_a^\lambda(x) \partial_\sigma A_a^\sigma(y)\, \theta(0) | 0 \rangle . \tag{16}$$

In (16), $a=1, \ldots, 8$ and is not summed over, $t = (p-q)^2$, m_a is the mass of the 0^- meson carrying index a, and F_a is defined in (8). We adopt the following representation for $T_a(q^2, p^2; t)$:

$$T_a(q^2, p^2; t) = \frac{X + Y(q^2 + p^2) + Zt}{m_\epsilon^2 - t}, \tag{17}$$

where X, Y, Z are constants to be determined by dynamical considerations, and m_ϵ is the mass of an $I = J = 0$ meson, the ϵ. Our use of the pole dominated formula (17) is motivated by the assumption that the scale-invariant symmetry limit, in which all on-shell matrix elements of θ vanish, is accompanied by the presence of a zero-mass scalar meson, ϵ. If the symmetry is not too badly broken, then Eq. (17) is thought to be a reasonable approximation to $T_a(q^2, p^2; t)$ for moderate values of momentum transfer, say, $|t| \lesssim m_\epsilon^2$. We can use soft-meson theorems to gather the following constraints:

$$T_a(m_a^2, m_a^2; 0) = 2m_a^2, \tag{18a}$$

$$T_a(0, m_a^2; m_a^2) = (4-d)m_a^2, \tag{18b}$$

$$T_a(0,0;0) = (4-2d)m_a^2. \tag{18c}$$

In deriving Eqs. (18a)–(18c), we have used Eqs. (9) and (10). The dimension d appearing in (18b) and (18c) is common to the operators u_0, u_8, and $\partial_\lambda A_a^\lambda$, $a = 1, \ldots 8$. Inserting the pole representation (17) into Eqs. (18a)–(18c), we get

$$T_a(q^2, p^2; t) = \frac{(4-2d)m_a^2 m_\epsilon^2 + (q^2+p^2)m_\epsilon^2(d-1) + t[m_\epsilon^2 + (d-4)m_a^2]}{m_\epsilon^2 - t}. \tag{19}$$

The physical matrix element of the trace operator taken between single 0^- meson states with index a is obtained by passing to the on-shell limit $q^2 = p^2 = m_a^2$. The form factor so defined contains an ϵ-pole, with residue

$$\text{Res } T_a(m_a^2, m_a^2; m_\epsilon^2) = m_\epsilon^2[m_\epsilon^2 + (d-2)m_a^2]. \tag{20}$$

This residue contains information on the coupling between the 0^+ meson ϵ and the 0^- meson with index a. Let us define coupling-constants $g(\epsilon\pi\pi)$, $g(\epsilon\bar{K}K)$, $g(\epsilon\eta\eta)$ with the Lagrangian

$$\mathcal{L} = g(\epsilon\pi\pi)\epsilon\, \vec{\pi}\cdot\vec{\pi} + g(\epsilon\eta\eta)\epsilon\eta^2 + g(\epsilon\bar{K}K)\epsilon\bar{K}K. \tag{21}$$

We may express the residue of $T_a(m_a^2, m_a^2; t)$ in terms of these couplings,

$$\text{Res } T_a(m_a^2, m_a^2; m_\epsilon^2) = m_\epsilon^2 F_\epsilon\, \alpha_a\, g(\epsilon aa), \tag{22}$$

where $\alpha_a = 2$ for $a = 1, 2, 3, 8$ and $\alpha_a = 1$ otherwise, and F_ϵ is defined by

$$\langle 0 | \theta(0) | \epsilon \rangle = m_\epsilon^2 F_\epsilon . \tag{23}$$

Equating Eqs. (20) and (22) for each value of a, we find

$$\begin{aligned} g(\epsilon\pi\pi) &= (m_\epsilon^2 + (d-2)\, m_\pi^2)/2F_\epsilon , \\ g(\epsilon\bar{K}K) &= (m_\epsilon^2 + (d-2)\, m_K^2)/F_\epsilon , \\ g(\epsilon\eta\eta) &= (m_\epsilon^2 + (d-2)\, m_\eta^2)/2F_\epsilon . \end{aligned} \tag{24}$$

It is also of interest to exhibit the values of these couplings in the SU(3) limit. We have

$$\begin{aligned} \mathcal{L} &= g_0 \epsilon \vec{\mathrm{M}}\cdot\vec{\mathrm{M}} \\ &= g_0 \epsilon(\vec{\pi}\cdot\vec{\pi} + \eta^2 + 2\bar{K}K) . \end{aligned} \tag{25}$$

Thus, the SU(3)-symmetric couplings are $g(\epsilon\pi\pi) = g(\epsilon\eta\eta) = g(\epsilon\bar{K}K)/2 = g_0$.

The first thing to note about the relations in (24) is that if the masses m_π, m_K, m_η are nondegenerate, then so are the couplings $g(\epsilon\pi\pi)$, $g(\epsilon\eta\eta)$, and $g(\epsilon\bar{K}K)/2$. Mass and coupling-constant breaking appear simultaneously in this model. Moreover, the masses are squared, so that if we have a Gell-Mann–Okubo mass formula,

$$3m_\eta^2 = 4m_K^2 - m_\pi^2 , \tag{26}$$

then we obtain the coupling sum rule

$$3g(\epsilon\eta\eta) - 2g(\epsilon K\overline{K}) + g(\epsilon\pi\pi) = 0. \tag{27}$$

This sum rule can also be derived from Eq. (6). Evidently, the dynamical assumptions involved in the model just discussed (particularly single-particle dominance) are equivalent to those underlying (6).

In Sec. III, we shall perform a phenomenological analysis of several coupling sum rules, employing as input empirical tensor-meson coupling constants. The sum rules to be used are obtained from Eq. (6), even though, in this section, we have questioned the rigor underlying this approach.

We conclude this section with an observation pertaining to the concept of broken scale invariance. It has been pointed out[11] that Gell-Mann–Okubo mass relations can be generated in a model containing ϵ-pole dominance *and* SU(3)-invariant couplings of the ϵ meson. This latter feature turns out to be unnecessary. A discussion of this point is presented in Appendix A.

III. PHENOMENOLOGICAL ANALYSIS OF TENSOR-MESON DECAYS

Gathering the data necessary for evaluating coupling sum rules is not easy. As mentioned earlier, the most forthright method for obtaining couplings is from the decays of resonances. If the multiplet to which the resonance belongs is too light in mass, then some of the particles involved in the sum rules occur as bound states in certain of the "decay" channels. Unfortunately, only a very few couplings which occur as residues of bound state poles have been determined from S-matrix theory. At the other extreme, if the multiplet of resonances has too large a mass, accurate experimental data are invariably lacking. This can be blamed partly on the sharp increase with energy in the hadronic density of states. As the energy approaches 2 GeV, sorting out individual resonances becomes a significant problem.

The tensor mesons with positive parity are one of the few systems likely to be amenable to analysis. There are nine of them—$f(1269)$, $f'(1514)$, $A_2(1310)$, and $K_T(1421)$—[12] forming an octet and a singlet in the SU(3) limit. The physical f and f' states exhibit singlet-octet mixing. For want of a better approach,[13] we adopt the following mixing parametrization:

$$\begin{aligned} f &= T_0 \cos\theta_T + T_8 \sin\theta_T , \\ f' &= -T_0 \sin\theta_T + T_8 \cos\theta_T , \end{aligned} \tag{28}$$

where T_0, T_8 belong to the singlet and octet multiplets, respectively. A fit to the Gell-Mann–Okubo mass formula gives $\theta_T = 30.6°$. This value is not far from the "canonical" mixing angle $\theta_c = \sin^{-1}(1/\sqrt{3}) = 35.3°$, which in a simple quark model, signifies the decoupling of $f'(1514)$ from nonstrange quarks. This is consistent with current data,[1] for which only an upper bound for the decay width $\Gamma(f'\pi\pi)$ exists. In this paper, we shall take $\Gamma(f'\pi\pi) \cong 0$.

The two most predominant decay modes of the tensor mesons consist of two pseudoscalar ($2^+ \to 0^-0^-$) and one vector–one pseudoscalar ($2^+ \to 1^-0^-$) meson composites. Most of the analysis to follow will be concerned with the first of these. We identify the various $2^+ \to 0^-0^-$ couplings by means of the Lagrangian

$$\begin{aligned} \mathcal{L} = {} & g(f\pi\pi) f\vec{\pi}\cdot\vec{\pi} + g(f'\pi\pi) f'\vec{\pi}\cdot\vec{\pi} + g(f\overline{K}K) f\overline{K}K \\ & + g(f'\overline{K}K) f'\overline{K}K + g(f\eta\eta) f\eta^2 + g(f'\eta\eta) f'\eta^2 \\ & + g(A_2\pi\eta)\vec{A}_2\cdot\vec{\pi}\eta + g(A_2\overline{K}K)\vec{A}_2\cdot\overline{K}\vec{\tau}K \\ & + g(K_T K\eta)(\overline{K}_T K\eta + \text{H.c.}) \\ & + g(K_T K\pi)(\overline{K}_T \vec{\tau} K\cdot\vec{\pi} + \text{H.c.}) . \end{aligned} \tag{29}$$

We have suppressed the spin and momentum structure of the $2^+ \to 0^-0^-$ couplings. A particular choice for this is given in Eqs. (30) and (31) which also define the SU(3)-invariant couplings for the octet and singlet tensor-meson states,

$$\mathcal{L}_1(\underline{T}\,\underline{P}\,\underline{P}) = \sqrt{3}\, g_8 d_{ijk} T_i^{\mu\nu} P_j \partial_\mu \partial_\nu P_k \tag{30}$$

and

$$\mathcal{L}_1(T_0\,\underline{P}\,\underline{P}) = g_0 T_0^{\mu\nu} P_j \partial_\mu \partial_\nu P_j , \tag{31}$$

where all repeated indices are summed over and $i, j, k = 1, \ldots, 8$. Notice that as defined in (30) and (31), all coupling constants have the dimension of inverse energy. There is no rationale underlying this except that the choice of kinematics in (30) and (31) seems a "natural" one. Other, equally plausible, definitions will be considered later in this section. The relation between decay width Γ and coupling constant g_1 which follows from (30) and (31) is

$$\Gamma = C \frac{g_1^2}{4\pi} \frac{q^5}{M^2} , \tag{32}$$

where q is the decay momentum in the parent rest frame, M is the mass of the decaying particle, and C is an isospin-dependent numerical factor. For the transition $f \to \pi\pi$, we find $C = 2/5$.

Of the ten couplings in (29), only the $\eta\eta$ decay mode of $f(1269)$ and $f'(1514)$ is not sufficiently well known to be of any use. Let us see how well the data are fit by the SU(3)-invariant couplings. Using (30), (31) and the mixing formula (28), we can perform a least-squares minimization relative to the variables g_0 and g_8. Doing so, we find[14] $g_0=1.82$ and $g_8=1.16$. The root-mean-square percent deviation of the empirical couplings in Eq. (29) from SU(3),

$$\Delta g=\left[\frac{\sum_i (g_i - g_i^{\mathrm{SU}(3)})^2}{\sum_i g_i^{\,2}}\right]^{1/2}\times 100\,, \tag{33}$$

is numerically about 8%. The agreement between the empirical $2^+\to 0^-0^-$ couplings and their SU(3) counterparts is rather good. This lends hope that existing data are of sufficient quality to provide a fair test of the coupling sum rules. Recall that the $\frac{3}{2}^+\to\frac{1}{2}^+0^-$ decays, for which a sum rule is valid, exhibit a mean square deviation from SU(3) of about 15%.

Using the parametrization implied by Eq. (6), it is possible to derive the following four sum rules for $2^+\to 0^-0^-$ decays[6]:

$$\frac{2}{\sqrt{3}}g(A_2\bar{K}K)+\frac{1}{\sqrt{3}}g(K_TK\pi)=-g(K_TK\eta)+g(A_2\pi\eta)\,, \tag{34}$$

$$\frac{4}{\sqrt{3}}g(K_TK\pi)=-g(T_8\bar{K}K)+2\,g(T_8\pi\pi)+\frac{1}{\sqrt{3}}g(A_2\bar{K}K), \tag{35}$$

$$2g(K_TK\eta)+g(T_8\bar{K}K)+\frac{1}{3\sqrt{3}}g(A_2\bar{K}K)+\frac{2}{3\sqrt{3}}g(K_TK\pi)-2g(T_8\eta\eta)=0\,, \tag{36}$$

$$3g(T_0\eta\eta)+g(T_0\pi\pi)-2g(T_0\bar{K}K)=0\,. \tag{37}$$

As a check against errors in Eqs. (34)–(38), note that these relations are consistent with the SU(3)-invariant couplings of Eqs. (30) and (31). Unfortunately, until the $\eta\eta$ decay modes are measured, the sum rules (36) and (37) cannot be tested. Therefore, we are left with the two sum rules (34) and (35). If desired, a linear combination of (34) and (35) can be used to eliminate either of the $A_2\bar{K}K$ or $K_TK\pi$ couplings. Thus, we can also write

$$\frac{9}{\sqrt{3}}g(K_TK\pi)=4g(T_8\,\pi\pi)-2g(T_8\bar{K}K)+g(A_2\pi\eta)-g(K_TK\eta) \tag{38}$$

or

$$\frac{9}{\sqrt{3}}g(A_2\bar{K}K)-g(T_8\bar{K}K)+2g(T_8\pi\pi)=-4g(K_TK\eta)+4g(A_2\pi\eta)\,. \tag{39}$$

The results of testing the sum rules (34), (35), (38), (39) by inserting numerical values of the empirical coupling constants is given in Table III. From the second row of Table IIIa, we see that the percent differences between the left- and right-hand sides of the four sum rules are 18.7%, 17.1%, 8.3%, and 24.8%.

A necessary criterion for the sum rules to be judged successful is that they be obeyed to an accuracy at least better than that of the SU(3)-invariant fit. Clearly, the couplings defined in Eq. (32) are not. There are several ways to interpret this negative result. The most obvious of these is that the data are not yet of sufficient accuracy. Another is that the coupling constants being tested are not being extracted from the measured decay widths in the "correct" manner. We shall investigate this latter point in the rest of this section, deferring a discussion of the former until the Conclusion. That is, we shall study the arbitrariness involved in defining empirical coupling constants by considering the effect upon the coupling sum rules of using different definitions. The key point to remember in choosing other definitions is the requirement from special relativity that the lifetime of a decaying resonance transform as the fourth component of a four-vector. Thus, the class of allowed redefinitions can involve only masses of the participating particles. Despite the apparently bewildering freedom of choice, there are certain of these redefinitions which appear more natural than others. Two definitions which come to mind are defined by the Lagrangians

$$\mathcal{L}_2=g_2MT^{\mu\nu}P\partial_\mu\partial_\nu P \tag{40}$$

and

$$\mathcal{L}_3=\frac{g_3}{M}T^{\mu\nu}P\partial_\mu\partial_\nu P\,, \tag{41}$$

where we have suppressed the internal-symmetry indices. These definitions are of interest because the first gives a phase space containing only (aside from numerical factors) the q^5 momentum dependence,

$$\Gamma=C\frac{g_2^{\,2}}{4\pi}q^5\,, \tag{42}$$

and the second involves a dimensionless coupling constant,

$$\Gamma=C\frac{g_3^{\,2}}{4\pi}\frac{q^5}{M^4}\,. \tag{43}$$

From Table IIIa, we see that each definition leads again to poor results. Although the root-mean-square amount of symmetry breaking is not large, 10.2% for (40) and 10.6% for (41), the coupling sum rules are not obeyed to a reasonable degree of accuracy.

It has recently been suggested[15] that for pseudoscalar mesons, the effects of SU(3) symmetry breaking appear to be more naturally described by the Duffin-Kemmer-Petiau (DKP) equation rather than by the Klein-Gordon (KG) equation. The former is a first-order differential equation, analogous in this respect to the Dirac equation, whereas the latter is of second order. One can write interactions similar to (30), (31), (40), and (41) in terms of DKP field operators, instead of the more conventional KG operators. This procedure introduces certain mass factors into the coupling-constant–decay-width relation. Naturally, the value consequently attained by the coupling constant is thus affected. In principle, this approach is afflicted with the same ambiguity as the KG formalism; one is free to redefine coupling constants with factors of the appropriate masses. However, we have performed a numerical analysis of two coupling definitions which seem reasonable candidates to us. The interactions are defined by

$$\mathcal{L}_4 = g_4 T^{\mu\nu} \bar{\psi}(\beta_\mu \beta_\nu + \beta_\nu \beta_\mu)\psi \tag{44}$$

and

$$\mathcal{L}_5 = g_5 T^{\mu\nu} \bar{\psi} \partial_\mu \partial_\nu \psi , \tag{45}$$

where we again suppress internal-symmetry indices. The respective decay widths are given by

$$\Gamma(\alpha+\beta\to\gamma) = C\frac{g_4^2}{4\pi}\frac{q^5}{M_\gamma^2 M_\alpha M_\beta} \tag{46}$$

and

$$\Gamma(\alpha+\beta\to\gamma) = C\frac{g_5^2}{4\pi}\frac{q^5[M_\gamma^2-(M_\alpha+M_\beta)^2]^2}{M_\gamma^2 M_\alpha M_\beta}. \tag{47}$$

In (44) and (45), we have described the decaying tensor meson in the usual (KG) manner. The other symbols are characteristic of the DKP formalism and are discussed in Ref. 15. Note that g_4 is dimensionless, whereas g_5 has the dimension of inverse mass squared. The last two rows of Table IIIa give the result of inserting the DKP couplings of (44) and (45) into the four coupling-constant sum rules. Before commenting on them, let us first point out that the root-mean-square deviation of the SU(3) fit is 32% and 46% for these couplings, respectively. This is substantially larger than the symmetry breaking associated with the KG couplings. The numbers in Table IIIa show reasonably positive results for the DKP couplings of Eq. (44), but not for those defined in Eq. (45).

TABLE III. Coupling-constant sum rules for $2^+ \to 0^- 0^-$ meson transitions. The column headings I–IV represent the sum rules (34), (35), (38), and (39) of the text, respectively. The row headings define the kind of constant being analyzed; $\mathcal{L}_1, \ldots, \mathcal{L}_5$ refer to the interactions given by Eqs. (30), (40), (41), (44), and (45) of the text. The Table entries give the magnitude, in percent, of the difference between the left- and right-hand sides of a given sum rule. The symbol R, which distinguishes the results in (a) from those in (b), refers to hadronic radius which appears in the barrier penetration factor of Eq. (48).

	I	II	III	IV
(a) R = 0.0 fm				
SU(3)	0.0	0.0	0.0	0.0
$\mathcal{L}_1$	18.7	17.1	8.3	24.8
$\mathcal{L}_2$	19.1	16.7	7.5	24.7
$\mathcal{L}_3$	18.3	16.9	8.6	24.7
$\mathcal{L}_4$	5.7	2.0	4.3	5.2
$\mathcal{L}_5$	17.6	12.3	27.7	16.1
(b) R = 1.0 fm				
SU(3)	0.0	0.0	0.0	0.0
$\mathcal{L}_1$	7.8	26.1	18.4	13.6
$\mathcal{L}_2$	6.8	29.2	21.4	12.8
$\mathcal{L}_3$	8.8	22.8	15.4	14.2
$\mathcal{L}_4$	10.2	11.9	19.0	8.5
$\mathcal{L}_5$	37.8	119.0	273.0	32.5

In particular, despite the relatively large deviation of the empirical coupling constants of Eq. (44) from the SU(3) fit, the sum rules are obeyed to an accuracy appreciably better than the other definitions. We shall comment further on this in the Conclusion.

So far, we have considered only those redefinitions of coupling-constant sum rules which follow from various effective Lagrangians, calculated to lowest order. A rather different type of redefinition involves use of a factor which explicitly takes into account the extended spatial structure of hadrons. For example, suppose hadrons are characterized by some uniform size R. Then, if q is the decay momentum of a two-particle composite from some resonant state, damping of the decay rate for those momenta obeying $qR \gtrsim 1$ might be expected to occur. This idea is a well-known one in nuclear physics and is discussed in Refs. 1 and 5. We have incorporated this approach into our phenomenological analysis with the following substitution in Eqs. (32), (42), (43), (46), and (47):

$$\Gamma \to \Gamma[1+(qR)^2]^{-2}. \tag{48}$$

The effect of a variety of sizes R has been examined. Representative of these are the sum-rule values given in Table IIIb with $R = 1$ fm. Table IIIa corresponds, of course, to $R=0$. Generally, the pure SU(3) fits with $R=1$ fm are worse than those with $R=0$. From Table IIIb, it is clear that the success of the broken SU(3) sum rules is not impressive. Incidentally, the effect of these considerations on the $\frac{3}{2}^+ \to \frac{1}{2}^+ 0^-$ sum rule (5) for $R=1$ fm is to worsen its agreement from about 4% to 7%.

Up to this point, we have not considered the decay of a tensor meson into a vector and a pseudoscalar pair. This is done in Appendix B, where it is shown that, because of kinematical considerations, the coupling sum rules are not likely to be testable.

IV. CONCLUSION

Our phenomenological analysis of tensor-meson decays has yielded a rather diverse set of empirical coupling constants. The various classes of decay kinematics which we employed gave SU(3) fits ranging from a root-mean-square deviation of about 8% to 56%. None of the fits to the sum rules can be considered outstanding, although one [see Eq. (44) and Table IIIa] appears to be better than the rest. There are clearly three factors which can be blamed for any of the negative results in Table III: invalidity of the sum rules themselves, inappropriate definition of the empirical coupling constants, or inaccurate data. Let us discuss each of these in turn.

First, there is the problem concerning the overall validity of the coupling sum rules. This is actually part of a larger question about the nature of SU(3) and its breaking. The problem of understanding which internal symmetries are perturbative and which are not is actively being studied in various model calculations.[16] Even the Gell-Mann–Okubo mass sum rules continue to be a fertile area for theoretical investigation; the question of linear versus quadratic sum rules for mesons has not yet been satisfactorily resolved.[15] Although we expressed reservations in Sec. II regarding the derivation of coupling sum rules given there, we feel that there is no compelling experimental evidence against them at this time. The model calculation involving broken scale invariance was of interest in this regard. Further work on the theoretical foundations of this subject is currently underway.[17]

Concerning the means by which coupling constants should be extracted from decay widths, we have taken the attitude in this paper that effective Lagrangians such as Eqs. (30), (40), (41), (44), and (45) are a kind of sophisticated bookkeeping which generates the correct momentum and spin dependence of the decay amplitude. Unless one is going to completely solve the field theory, there seems to be no *a priori* way of choosing between various couplings. Of course, when comparing these couplings with the symmetry prediction, the various definitions lead to very different results. The search for a particular choice which works better than the others is worthwhile because it may lead to a better understanding of the underlying dynamical processes. For example, there have been claims[7,15] that the Duffin-Kemmer-Petiau description of mesons displays a systematic superiority over the usual Klein-Gordon approach in taking symmetry-breaking effects into account. Of course, one could employ the somewhat extreme approach of constructing more or less arbitrary functions of the relevant masses and letting a computer search decide which choice of decay kinematics fits the sum rules best. Unfortunately, this viewpoint does not leave one with much physical understanding of the final answer; surely *some* theoretical foundation to the phenomenology should be present.

Interestingly enough, there are situations in which the ambiguity problem in defining coupling constants is greatly alleviated, if not entirely absent. Recall that the $\frac{3}{2}^+ \to \frac{1}{2}^+ 0^-$ sum rule contains only pions as the decay mesons. There, the problem of mesonic kinematic corrections is not present. Unfortunately, it is not possible to do the same for the tensor-meson sum rules because terms containing π, K, and η are unavoidably intertwined. The problem associated with kinematic corrections is correspondingly a more difficult one. For this reason, we mentioned in Sec. II that the sum rule (7) (describing $\frac{3}{2}^- \to \frac{3}{2}^+ 0^-$ transitions) should be of special interest—aside from Eq. (5), it is experimentally the most accessible sum rule which contains only pions. Before it can be tested, there must be improved data for each of the decay modes in Eq. (7), particularly the $\Xi^*(1820) \to \Xi^*(1530)\pi$ and $Y_0^*(1690) \to Y_1^*(1385)\pi$ decays. When this is done, the sum rule (7) could be a useful indicator of the accuracy of the data.

To conclude our discussion of the broken SU(3) sum rules and tensor-meson decays, we consider the possibility that the data have not yet reached the desired state of accuracy and stability. Perhaps, there is a lesson to be learned from the $\frac{3}{2}^+ \to \frac{1}{2}^+ 0^-$ sum rule (5). Over the past several years, coupling constants defined as in Eq. (3) have fitted the sum rule (5) to better than 1%. Changes in the most recent data[1] decreased this excellent result to about 4%. Since the $\frac{3}{2}^+$ baryons are probably the best studied resonance system in the hadron spectrum, one should clearly be wary of over-interpreting analyses of particles such as the tensor mesons. Thus, although the numbers in Table IIIa show that the best fit to current data is with the DKP formula (44), it might be premature for the

DKP advocates to claim this as a success for their method. In particular, a mystifying feature of the preferred definition (44) is that its over-all fit to SU(3) is substantially poorer than each of the KG definitions (30), (40), and (41). Numerically, this is due mainly to the large $f\pi\pi$ contribution in Eq. (33). Our choice of f-f' mixing could be responsible, but this is by no means clear.[18] A final caveat regarding the tensor-meson decay data concerns the failure of the barrier penetration factor of Eq. (48) to improve the $R=0$ fits to the sum rules. The idea that damping of large momenta should accompany intrinsic hadronic structure is an attractive one; the negative nature of the $R\neq 0$ values in Table IIIb is unexpected. It will be of interest to follow this aspect of our results as more data become available.

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APPENDIX A

The scalar-dominance hypothesis[11,19] is simply the conjecture that matrix elements of the trace operator can be approximated by scalar-meson pole contributions for moderate values of momentum transfer. For example, the elastic matrix element of baryon a is written

$$\langle \vec{q}, \lambda, a \mid \theta \mid \vec{p}, \lambda', a\rangle = \left(\frac{g_{\epsilon aa}\, m_\epsilon{}^2 F_\epsilon}{m_\epsilon{}^2 - t} + \frac{g_{\epsilon' aa}\, m_{\epsilon'}{}^2 F_{\epsilon'}}{m_{\epsilon'}{}^2 - t}\right)\bar{u}(\vec{q}, \lambda) u(\vec{p}, \lambda')\,. \tag{A1}$$

We have considered two $I=J=0$ mesons, $\epsilon(700)$ and $\epsilon'(1060)$, in (A1), in order to relate to previous calculations on the subject. At zero momentum transfer, we deduce the following mass relations for the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryon multiplets:

$$\begin{aligned}
\tfrac{1}{2}^+:\ & N = g_{\epsilon NN}F_\epsilon + g_{\epsilon' NN}F_{\epsilon'}\,, & \tfrac{3}{2}^+:\ & \Delta = g_{\epsilon\Delta\Delta}F_\epsilon + g_{\epsilon'\Delta\Delta}F_{\epsilon'}\,,\\
& \Lambda = g_{\epsilon\Lambda\Lambda}F_\epsilon + g_{\epsilon'\Lambda\Lambda}F_{\epsilon'}\,, & & Y_1 = g_{\epsilon Y_1 Y_1}F_\epsilon + g_{\epsilon' Y_1 Y_1}F_{\epsilon'}\,,\\
& \Sigma = g_{\epsilon\Sigma\Sigma}F_\epsilon + g_{\epsilon'\Sigma\Sigma}F_{\epsilon'}\,, & & \Xi^* = g_{\epsilon\Xi^*\Xi^*}F_\epsilon + g_{\epsilon'\Xi^*\Xi^*}F_{\epsilon'}\,,\\
& \Xi = g_{\epsilon\Xi\Xi}F_\epsilon + g_{\epsilon'\Xi\Xi}F_{\epsilon'}\,, & & \Omega = g_{\epsilon\Omega\Omega}F_\epsilon + g_{\epsilon'\Omega\Omega}F_{\epsilon'}\,,
\end{aligned} \tag{A2}$$

where the particle symbol represents the mass of that particle (e.g., $N=m_N$). If the physical states ϵ,ϵ' correspond to states σ_0,σ_8 in the SU(3) limit, we can write

$$\epsilon = \sigma_0\cos\theta - \sigma_8\sin\theta\,, \quad \epsilon' = \sigma_8\cos\theta + \sigma_0\sin\theta\,. \tag{A3}$$

Substituting (A3) into (A2), we find (using obvious notation)

$$B = g_{0BB}F_0 + g_{8BB}F_8\,, \tag{A4}$$

where the symbol B ranges over each of the baryons in Eq. (A2). The quantities F_0, F_8 are related to $F_\epsilon, F_{\epsilon'}$ as σ_0,σ_8 are to ϵ,ϵ' in Eq. (A3).

Let us observe the consequence of assuming SU(3)-invariant coupling constants in Eq. (A4). We denote the coupling of σ_0,σ_8 to the $\frac{1}{2}^+$ baryons as $g_0{}^{(0)}, g_8{}^{(0)}$, and the coupling of σ_0,σ_8 to the $\frac{3}{2}^+$ baryons as $h_0{}^{(0)}$, $h_8{}^{(0)}$. Then,

$$\begin{aligned}
\tfrac{1}{2}^+:\ & N = F_0 g_0^{(0)} + [(3-4\alpha)/\sqrt{3}\,]F_8 g_8^{(0)}, & \tfrac{3}{2}^+:\ & \Delta = F_0 h_0^{(0)} + F_8 h_8^{(0)}\,,\\
& \Lambda = F_0 g_0^{(0)} - (2\alpha/\sqrt{3})F_8 g_8^{(0)}, & & Y_1 = F_0 h_0^{(0)},\\
& \Sigma = F_0 g_0^{(0)} + (2\alpha/\sqrt{3})F_8 g_8^{(0)}, & & \Xi^* = F_0 h_0^{(0)} - F_8 h_8^{(0)},\\
& \Xi = F_0 g_0^{(0)} - [(3-2\alpha)/\sqrt{3}\,]F_8 g_8^{(0)}, & & \Omega = F_0 h_0^{(0)} - 2F_8 h_8^{(0)}\,.
\end{aligned} \tag{A5}$$

In (A5), α is the F/D parameter occurring in the coupling of σ_8 to the $\frac{1}{2}^+$ baryons. To repeat, Eq. (A5) gives mass formulas for the $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decuplet under the assumption that the *only* source of SU(3) breaking is in the mixing of the σ_0,σ_8 states. The usual Gell-Mann–Okubo mass formulas are seen to follow,

$$\begin{aligned}
N + \Xi &= \frac{3\Lambda + \Sigma}{2}\\
\Delta - Y_1 &= Y_1 - \Xi^*\\
&= \Xi^* - \Omega\,.
\end{aligned} \tag{A6}$$

Thus, by incorporating the assumption of SU(3)-

invariant coupling constants into a scalar-dominance model, one is able to generate the usual baryon-mass sum rules.

What we wish to point out here is that, if the restrictive assumption of SU(3)-invariant coupling constants in dropped, but the broken couplings employed obey sum rules of the type discussed in this paper, then to a good approximation the Gell-Mann–Okubo mass relations survive. To be specific, we study the effect of broken-SU(3) coupling constants upon the $\frac{3}{2}^+$ system. We employ for a given coupling $h_{\sigma DD}$ the notation

$$h_{\sigma DD}=h^{(0)}_{\sigma DD}+h^{(1)}_{\sigma DD}\,, \tag{A7}$$

where $h^{(1)}_{\sigma DD}$ is the symmetry-breaking contribution. The mass formula (A4) for the Δ baryon becomes

$$\Delta=h^{(0)}_{0\Delta\Delta}F_0+[h^{(1)}_{0\Delta\Delta}F_0+h^{(0)}_{8\Delta\Delta}F_8]+h^{(1)}_{8\Delta\Delta}F_8\,. \tag{A8}$$

We have lumped the contributions in (A8) according to the size we expect each to have (zeroth order, first order, and second order). The first-order contribution comes from two separate terms. The other members of the $\frac{3}{2}^+$ system have similar type mass relations. The coupling sum rules appropriate to this discussion are

$$\begin{aligned} &h^{(1)}_{0\Delta\Delta}+h^{(1)}_{0\Xi\Xi}-2h^{(1)}_{0YY}=0\,,\\ &h^{(1)}_{0YY}+h^{(1)}_{0\Omega\Omega}-2h^{(1)}_{\Xi\Xi}=0\,. \end{aligned} \tag{A9}$$

The couplings $h^{(1)}_{8\Delta\Delta}, \ldots$ etc, do not obey analogous sum rules. In view of Eqs. (A4), (A8), and (A9), the zeroth- and first-order terms give rise to exact Gell-Mann–Okubo mass formulas. The second-order terms do not. We can estimate the magnitude of the zeroth- and first-order terms, and thus by inference, also the second order. The zeroth order gives rise to the average mass of the $\frac{3}{2}^+$ decuplet, $\overline{M}=1385$ MeV, whereas the first order governs the magnitude of the almost equally spaced splittings, $\Delta M\cong 147$ MeV. From the smallness of the ratio $\Delta M/\overline{M}\cong 0.11$, we expect the second-order contributions to be of the order of a percent of the SU(3)-invariant contribution.

In summary, even if the scalar-meson coupling constants are not SU(3)-invariant, if they exhibit symmetry breaking of the expected type, then the mass sum rules are maintained to a good degree of approximation. Although we do not show it here, the same result holds for broken baryon and meson octets as well.

APPENDIX B

At first sight, it might appear that the decays, $2^+\to 1^-0^-$, constitute an appropriate system for studying coupling sum rules. The SU(3)-invariant Lagrangians are

$$\mathcal{L}(\underline{T}\,\underline{V}\underline{P})=igf_{abc}\,\epsilon_{\alpha\beta\gamma\delta}\,\partial^\alpha T^{\gamma\nu}P\,\partial_\nu\partial^\delta V^\beta\,, \tag{B1}$$

$$\mathcal{L}(T_0\underline{V}\underline{P})=\mathcal{L}(\underline{T}\,V_0\underline{P})=0\,. \tag{B2}$$

The latter two equalities follow from charge-conjugation invariance. There are seven nonzero couplings contained in (B1). When one allows for symmetry breaking, these seven couplings may be described in terms of four independent parameters, so there are three sum rules:

$$3g(K_T\rho K)-2\sqrt{3}\,g(T_8K_V\overline{K})-\sqrt{3}\,g(K_TV_8K)+2g(K_TK_V\pi)-g(A_2\rho\pi)=0\,, \tag{B3}$$

$$g(A_2\rho\pi)-2g(A_2K_V\overline{K})-2g(K_TK_V\pi)-g(K_T\rho K)-\sqrt{3}\,g(K_TV_8K)=0\,, \tag{B4}$$

$$g(K_T\rho K)-g(K_TK_V\pi)-\sqrt{3}\,g(K_TK_V\eta)-\sqrt{3}\,g(K_TV_8K)=0\,. \tag{B5}$$

We have denoted the strangeness carrying vector mesons by K_V, and the octet isoscalar vector meson by V_8. The second sum rule (B4) is clearly not useful because of the presence of $g(A_2K_V\overline{K})$; the A_2 meson is a bound state in the $K_V\overline{K}$ channel. To analyze the first and third sum rules further, we must determine whether the couplings $g(T_8K_V\overline{K})$, and $g(K_TV_8K)$ are observable (the remaining couplings in these sum rules have been measured). The physical states to which V_0 and V_8 correspond are $\omega(783)$ and $\phi(1020)$. We shall not have to discuss just how one should describe the singlet-octet mixing of vector mesons. However, having chosen a mixing scheme, we would, in general, need the values of $g(fK_V\overline{K})$ and $g(f'K_V\overline{K})$ in order to determine $g(T_8K_V\overline{K})$. Unfortunately, there is no way of measuring $g(fK_V\overline{K})$, so the evaluation of $g(T_8K_V\overline{K})$ appears doomed. A possible way to save the situation might appear to lie in the equation (B2). The coupling $g(T_0K_V\overline{K})$ vanishes in the SU(3) symmetry limit. If it identically vanished even with the symmetry broken, then a measurement of $g(f'K_V\overline{K})$ would suffice to give the information required to evaluate $g(T_8K_V\overline{K})$. However, when Eq. (6) is used to analyze the broken symmetry structure of $g(T_0K_V\overline{K})$, we find that it is generally nonzero. Thus, the sum rule (B3) contains a nonmeasurable coupling. The same argument applies for the coupling $g(K_TV_8K)$. Thus, although there is an impressive accumulation of data on the $2^+\to 1^-0^-$ mesonic transitions, the sum rules (B3), (B4), and (B5) turn out to contain nonmeasurable couplings.

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[14]For convenience in our numerical analysis, we have chosen an over-all normalization such that $g^2(f\pi\pi) = 4.0$ for all the effective Lagrangians which we consider.

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Unified Theory of Nonleptonic Hyperon Decays

Michael D. Scadron*
Department of Theoretical Physics, Imperial College, London, SW7, England

Lawrence R. Thebaud
Department of Physics, University of Arizona, Tucson, Arizona 85721
(Received 20 June 1972; revised manuscript received 5 March 1973)

Inclusion of baryon resonances in the current-algebra analysis of $B \to B'\pi$ and $B \to B'\gamma$ decays explains both the mismatch of the s- and p-wave pion amplitudes and the nonvanishing negative asymmetry parameter for $\Sigma^+ \to p\gamma$.

I. INTRODUCTION

Despite certain successes of the current-algebra approach to the nonleptonic hyperon decays,[1-3] two important problems remain unsolved. First, the $B \to B'\pi$ s-wave amplitudes are mismatched relative to the p-wave amplitudes, due to the s-wave Suzuki-Sugawara[1] current commutator and the p-wave baryon octet poles.[2,3] Second, the parity-violating amplitude is predicted to vanish for the weak radiative decay $\Sigma^+ \to p\gamma$,[4] in contradiction with the large measured asymmetry parameter.[5]

It is the purpose of this paper to solve *both* of these problems simultaneously by inclusion of decuplet intermediate states along with the baryon octet poles. Decuplet poles dominate the pion photoproduction background amplitudes at low energy and therefore account for most of the anomalous magnetic moment of the nucleon as expressed by the current-algebra low-energy theorem of Fubini, Furlan, and Rossetti (FFR).[6,7] The Adler-Weisberger[8] (AW) background amplitudes in low-energy pion-nucleon scattering are also dominated by decuplet states.[8,9] It is therefore not surprising that decuplet poles play an important role in weak hyperon decays. However, in contrast with AW, FFR, and the weak radiative decays $B \to B'\gamma$, the decuplet poles in $B \to B'\pi$ dominate the axial-vector current algebra background amplitudes.