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Increasing Cross Sections, Diffractive Excitation, and the Triple-Pomeron Coupling*

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Using model-independent sum rules, it is argued that the sharp peak near the kinematical boundary observed in some inclusive spectra and the increase of total cross sections are due to the same mechanism. Both phenomena are quantitatively described in proton-proton scattering, in terms of diffractive excitation into high-mass states with a triple-Pomeron coupling. This coupling is then incorporated in a two-component model. In this way we obtain a total inelastic proton-proton cross section which agrees with the data from $s = 30$ to 3000 GeV^2. The "break" in the elastic differential cross section at small $|t|$ is related to the increase with energy of the inelastic cross section. Predictions, based on the factorizability of the Pomeron, are given for the inclusive spectra, inelastic and elastic cross sections, at Serpukhov and National Accelerator Laboratory energies, for K^+p and other reactions. It is shown that a Pomeron with intercept slightly below unity can actually give rise to an increasing cross section and also that a perturbative approach to the Pomeron coupling with only the first few terms may be sufficient at and even far beyond CERN Intersecting Storage Rings energies, while the "true" asymptotic behavior appears only much later.

I. INTRODUCTION

Recent experiments at the CERN Intersecting Storage Rings (ISR) reveal that (1) there exists a sharp diffraction peak in inelastic proton inclusive cross sections,[1,2] which approximately scales[2]; (2) both the proton-proton (p-p) total and elastic cross sections have increased from accelerator energies[3]; and (3) certain inclusive cross sections have a nonscaling behavior.[4] These facts can be intimately related to one another and also to the sensitive subject of the triple-Pomeron coupling.[5]

These relationships are based on some model-independent sum rules, a phenomenological analysis of the inclusive data, which allows us to determine the triple-Pomeron coupling, and a two-component model.[6] The peak in the inclusive cross section near $x = 1$ is shown to produce an increase in the inelastic cross section, which is, in this way, related to the energy dependence of the diffractive component. By the optical theorem the increase in the inelastic cross section produces an increase in the optical point which may change the decreasing behavior of the elastic cross section, observed at low energies, into a flattening out and even a subsequent increase. Furthermore, the presence of a contribution to the optical point that increases with the energy may produce a "break" in the elastic differential cross section. This approach also provides a simple interpretation of the experimentally unclear situation concering the shrinkage of the elastic peak.

Since all these considerations are based on the triple-Pomeron coupling, they give unambiguous prediction for other reactions, especially for K^+p, which can be tested at Serpukhov and the National Accelerator Laboratory (NAL).

It is known that the triple-Pomeron coupling cannot be selfconsistent unless the intercept $\alpha_P(0)$ is below unity or the triple-Pomeron coupling vanishes in the forward direction. However, we are going to show that the behavior of the cross sections at ISR and even higher energies may be insenstive to this self-consistency question. In particular, even if $\alpha_P(0) < 1$, the cross sections can increase at those energies.

In Sec. II, we illustrate the relationships among the observed phenomena enumerated above in terms of some model-independent sum rules. In Sec. III, we discuss the effects of the diffraction peak on the inelastic cross section in terms of single-diffractive excitation and the triple-Pomeron coupling. These results are used in Sec. IV to compute the inelastic pp cross section in a two-component model, which includes nondiffractive, single-diffractive, and double-diffractive excitation processes, and compare with the data. In Sec. V, we discuss the justification of the two-component model and its generalization, namely, a perturbative approach of the Pomeron coupling, and its self consistency. Tests of these ideas in K^+p and other reactions are discussed in Sec. VI. In Sec. VII, we study the shape of the elastic cross section near the forward direction. Finally, we summarize the main results of this work in Sec. VIII.

II. MODEL-INDEPENDENT RELATIONS

We shall proceed in an increasing order of model dependence. To begin with, let us recall some simple sum rules, which have to be trivially satisfied by a complete set of data, but can provide useful constraints for models and for unknown pieces of physical information. Even when there is not enough data to impose strong constraints, these relationships can still provide certain indications as to what are the viable models that we shall consider. First we have the relationship among the average proton multiplicity $\bar{n}_p$, the total cross section σ_T, and the proton inclusive cross section

$$\bar{n}_p\sigma_T = \iint \frac{d^2\sigma_p}{dx dp_\perp^2}\, dx\, dp_\perp^2\,, \tag{1}$$

where $x = 2p_\parallel/\sqrt{s}$ is the Feynman scaling variable. $p_\parallel$ and $p_\perp$ are, respectively, the longitudinal and transverse momenta of the observed proton in the center-of-mass frame, $\sqrt{s}$ is the total invariant energy, and the invariant-inclusive cross section for a particle c will be denoted by

$$f_c(x, p_\perp, s) \equiv (p_\parallel^2 + p_\perp^2 + m_c^2)^{1/2}\, \frac{d^2\sigma}{dp_\parallel dp_\perp^2}$$

$$= \left(x^2 + 4\frac{p_\perp^2 + m_c^2}{s}\right)^{1/2} \frac{d^2\sigma}{dx dp_\perp^2}\,. \tag{2}$$

Since $\bar{n}_p$ (which is an independently measurable quantity) is observed to be constant or slightly decreasing over a wide energy range and approximately equal to 1.5 in pp reactions,[7] an increase of the right-hand side of Eq. (1) directly implies an increase of σ_T and vice versa.

If f_p is independent of s, i.e., scales near the kinematical limits $x = \pm 1$, we can easily see that the contribution to the integral in Eq. (1) from this region is energy-dependent. Indeed, the kinematical limits of the integration are restricted by

$$x^2 + 4\frac{p_\perp^2 + m_p^2}{s} \leq 1\,, \tag{3}$$

which is s-dependent. If f_p is sharply peaked near $x = \pm 1$, such a small increase of the region of integration can give rise to a substantial increase of the integral and therefore σ_T.

Another region of integration for Eq. (1) that may contribute to an increasing amount is for $x \simeq 0$. If $f_c(0, p_\perp, s)$ is independent of s, then this contribution increases logarithmically with s. For $c = \pi$, such an increase is usually associated with the increase in $\bar{n}_\pi$. But for $c = p$, $f(0, p_\perp, s)$ decreases with s, and $\bar{n}_p$ is observed to be constant[7]; we shall not concentrate our discussion in this region. In the next paragraph, we shall suggest another relation which is much less sensitive to this region.

The second simple relation is about the average inelasticity, $\bar{x}_p$:

$$\bar{x}_p\sigma_T = \tfrac{1}{2}\iint f_p(x, p_\perp, s)\, dx\, dp_\perp^2\,, \tag{4}$$

where $\bar{x}_p$ can also be measured independently. The integral in Eq. (4) is weighted more in the region $x \simeq \pm 1$ than the one in Eq. (1) and therefore is more sensitive to the peak. As indicated by the cosmic-ray and low-energy data, $\bar{x}_p$ is almost constant and equal to 0.5. If such a behavior could be verified at ISR, Eq. (4) would give an even stronger link between the peak and the rise in σ_T. Since the variation in σ_T is only about 10%, this link requires an accurate and independent measurement of $\bar{x}_p$ in the future. Furthermore, Eq. (4) together with the energy-conservation sum rule

$$\sigma_T = \sum_c \tfrac{1}{2}\iint f_c(x, p_\perp, s)\, dx\, dp_\perp^2\,, \tag{5}$$

where the summation is over all the secondary particles, would imply that the integrals over f_c for at least some particle $c \neq p$ must also increase with s. Since f_c for particles other than the "leading" proton are not peaked near $x = \pm 1$, such an increase must in turn imply some nonscaling behavior for f_c. Conversely, if f_c were to scale and σ_T increases, $\bar{x}_p$ must increase with s.

One more relationship is the optical theorem, by which an increasing σ_T implies an increasing elastic differential cross section $d\sigma_{el}/dt$ in the forward direction if the amplitude is predominantly imaginary.[8] As we shall see later, an increasing optical point together with the observed increase of the total elastic cross section[3] σ_{el}, has implications on the shape of $d\sigma_{el}/dt$ and can provide an explanation for the break observed[9] in pp reaction near the forward direction.

III. SINGLE DIFFRACTION AND THE TRIPLE-POMERON COUPLING

From the previous arguments, it seems natural to consider a process which gives a sharp scaling peak for f_p and contributes to an increasing σ_T. A plausible candidate is the single-diffractive excitation process (SDE) with a triple-Pomeron coupling (see Fig. 1). A phenomenological analysis by one of us[10] has shown that the observed proton inelastic inclusive cross section at ISR energies can indeed be described by a triple-Pomeron form, with the Pomeron parametrized as an "effective" pole:

$$f_p = \frac{d\sigma}{dt\, d(M^2/s)}$$

$$= G_P(t)\frac{1}{s}(s/M^2)^{2\alpha_P(t)}(M^2)^{\alpha_P(0)}, \tag{6}$$

with[11]

$$G_P(t) \simeq 2e^{4.65t}\,, \tag{7}$$

and $\alpha_P(t) \simeq 1$ for small values of both t and M^2/s, where t is the momentum transfer and M is the missing mass. These two variables are related to x and $p_\perp$ by $M^2/s \simeq 1 - x$ and $t \simeq m_p{}^2(2 - x - x^{-1}) - x^{-1}p_\perp{}^2$; the peaks near $x = \pm 1$ correspond to the diffraction peak for small t and M^2/s, and the energy-dependent kinematical limits are specified by $t \lesssim -m_p{}^2[(M^2 - m_p{}^2)/s]^2$.

Since the peaks at $x \simeq \pm 1$ are dominated by the SDE shown in Fig. 1, obviously the integral over the inclusive cross section under these two peaks gives the cross section which is an aggregate of all the SDE events. We thus have[12,13]

$$\sigma_{\text{SDE}} = \int_{\text{forward peak}} \frac{d\sigma}{dx dp_\perp{}^2}\, dx\, dp_\perp{}^2 + \int_{\text{backward peak}} \frac{d\sigma}{dx dp_\perp{}^2}\, dx\, dp_\perp{}^2 \,, \tag{8}$$

which is different from Eq. (1).[14] For p-p reactions, the right-hand side of Eq. (8) is just twice the integral over the forward peak. Since the data throughout the entire kinematical region of the peak, as required by Eq. (8), are not yet available, we take Eq. (6) as a representation of the data.[10] Then from Eqs. (6) and (8), we find that the peaks in f_p contribute to an increase of 2.3 mb in σ_T from $s = 200$ GeV^2 to $s = 3000$ GeV^2. Notice that we have not specified the lower limit of integration in Eq. (8). But for any two different limits, the resulting σ_{SDE} will only differ by a constant and thus have no effect on its s dependence.

Subtracting Eq. (8) from Eq. (1), we find that

$$\left(\iint - \iint_{\text{peaks}}\right) \frac{d^2\sigma}{dx dp_\perp{}^2}\, dx\, dp_\perp{}^2 = \bar{n}\sigma_T - \sigma_{\text{SDE}} \geqslant (\bar{n}_p - 1)\sigma_T \,, \tag{9}$$

which correlates f_p outside the peak and $\bar{n}_p$. If $\bar{n}_p$ is a constant, as experimentally observed, the right-hand side of Eq. (9) increases with energy and so must the left-hand side. Since in the left-hand side the contribution from the peak has been removed, such an increase must come from the contribution outside the peaks, corresponding to the proton emitted from some other processes.

It has been shown that if the Pomeron is a factorizable simple pole, then the triple-Pomeron form given by Eq. (6) cannot be self consistent unless $\alpha_P(0) < 1$ or $G_P(0) = 0$.[15] We shall take the point of view that

$$\alpha_P(t) \simeq 1 - \epsilon + \alpha_P{}'(0)t \tag{10}$$

for the sake of simplicity, while other methods of quenching the triple-Pomeron contribution can also be adopted without changing the results of our discussion. With the phenomenological value of $G_P(t)$, Eq. (7) and the self-consistency condition

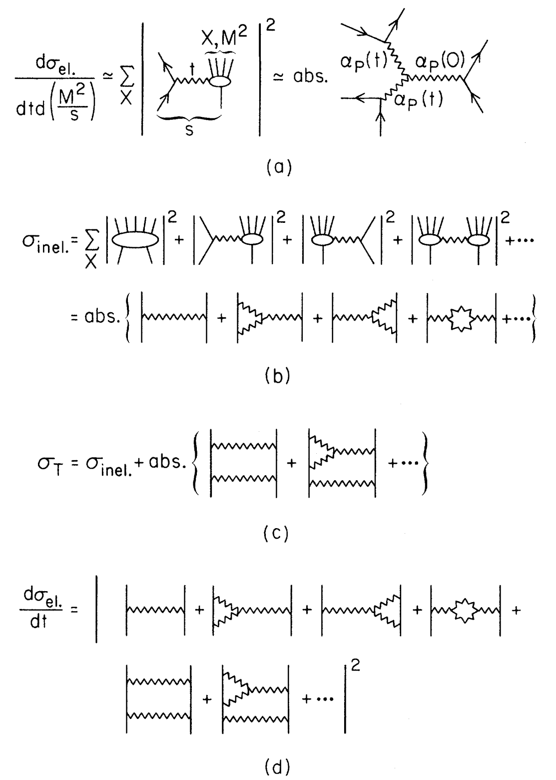

FIG. 1. (a) Graphical illustration of the dominant contribution to the inclusive cross section from single-diffractive excitation processes. The straight lines represent particles and the wavy lines represent the Pomeron. (b) Decomposition of the inelastic cross section according to the number of fireballs with Pomerons exchanged between each adjacent pair of fireballs with large rapidity gap in the rapidity space. Each diagram in the second line gives the aggregated Regge behavior for the processes shown by the corresponding diagram in the first line. (c) The total cross section as the sum of inelastic and elastic cross sections, where the total elastic cross section is graphically illustrated as absorptive part of Regge cuts. (d) Graphical illustration of the elastic differential cross section which satisfied perturbation unitarity.

of Ref. 15, one finds that[10] ϵ can be as small as 0.001 for $\alpha_P{}'(0) \simeq 0.5$ GeV^{-2}. This value can vary somewhat due to the uncertainties in $\alpha_P{}'(0)$. The diffraction peak then only has the approximate scaling property

$$\frac{d\sigma}{dt d(M^2/s)} = s^{-\epsilon} G_P(t) \left(\frac{s}{M^2}\right)^{1+2\alpha_P{}'(0)t-\epsilon} . \tag{11}$$

However, the break of scaling due to $s^{-\epsilon}$ is extremely small for small values of ϵ. It is true that the integral of Eq. (11) vanishes asymptotically and therefore the increasing cross section for SDE cannot persist. However, the s dependence of the integral at finite energies can be obtained straightforwardly from an analytical estimation and/or numerically. For $\epsilon \lesssim 0.01$, it can be shown that there is always a range of s and $\alpha_P(0)$ in which the result is well approximated by the logarithmic behavior in Ref. 15 (see Ref. 16). For $\epsilon = 0.01$, we find that the effect of the factor $s^{-\epsilon}$ is only 2% from $s = 200$ to $3000\,\mathrm{GeV}^2$. Therefore, this asymptotically vanishing cross section actually increases with s in the above energy range. Such a behavior persists even if $G_P(0) = 0$ but, if this is the case, the rate of increase would depend on how $G_p(t)$ approaches zero with t. At the present stage, our discussion is not very sensitive to the properties of the Pomeranchuk trajectory, namely, factorability, value of ϵ, etc. In Sec. V, we shall discuss the effects of these properties on the asymptotic behavior.

IV. A TWO-COMPONENT MODEL OF INELASTIC CROSS SECTION

To complete the picture for the total inelastic cross section, we shall adopt a two-component model, in which categories of events are characterized in terms of fireballs.[17] These categories are assumed to have aggregated energy dependence that corresponds to different powers of Pomeron coupling as shown in Fig. 1. This can be regarded as a perturbative approach of the Pomeron coupling, where the Pomerons being exchanged are the "bare" ones. However, our "bare" Pomeron is the physical Pomeron that governs the cross sections at intermediate energies, while we may regard the "renormalized" one as the one which governs the asymptotic behavior. In a self-consistent theory, one may require that these two are the same. Events with more than two fireballs are neglected at ISR energies in view of the smallness of the triple-Pomeron coupling.

In this model, the total inelastic cross section, σ_{inel}, is dominated by the nondiffractive (single fireball), SDE, and double-diffractive excitation (DDE) processes. The nondiffractive contribution is given by

$$\sigma_{\mathrm{ND}} = s^{-\epsilon} |\beta_{ppP}(0)|^2 , \tag{12}$$

where $\beta_{ppP}(0)$ is the proton-Pomeron residue at $t = 0$, which we shall estimate later.

The SDE processes have been already discussed in Sec. III. In all the calculations in this paper, the value of σ_{SDE} is obtained by computing numerically the integral in Eq. (8) with the integrand given by Eq. (11). Since this integrand is strongly peaked near $x = \pm 1$, we integrate it over the whole physical region — excluding the region near $x = 0$ where the integrand is kinematically enhanced. In this way, one avoids the ambiguity in the definition of the "peaks" in Eq. (8). As discussed in Sec. III, this ambiguity can lead to a constant (up to $s^{-\epsilon}$) difference. However, this difference behaves like σ_{ND}. Thus the sum $\sigma_{\mathrm{ND}} + \sigma_{\mathrm{SDE}}$ and the energy dependence of σ_{SDE} are not sensitive to this ambiguity. This integral can also be analytically evaluated provided that some simplifying assumptions are made. For ϵ in the range between 0 and 0.001, one obtains in this way the approximate form[16]

$$\sigma_{\mathrm{SDE}} \simeq 2\times(0.4\ \mathrm{mb}) \ln(s/m_p^2) . \tag{8a}$$

In computing the energy dependence of σ_{SDE} (which is related to the development of the peaks in f_p near $x = \pm 1$) from accelerator to ISR energies, there is an ambiguity due to the fact that at accelerator energies the peak (if present at all) is "covered" by resonances. The prescription used here to compute the increase with energy (by integrating the PPP contribution at two different energies and subtracting them) amounts to computing the surface under the peaks at ISR energies and subtracting the sum of the resonance cross sections at accelerator energies.[10]

The double-diffractive excitation cross section σ_{DDE} is much more difficult to analyze phenomenologically, since it is small and one also has to measure all the final states in order to select out all the two-fireball events. Only within the model, can it be obtained in terms of $\beta_{ppP}(0)$ and the triple-Pomeron coupling $g_P(t)$ from the assumption of factorization. We have[18]

$$\sigma_{\mathrm{DDE}} \simeq \frac{1}{32\pi} [\ln(s/m_p^2)]^2 \int_{-\infty}^{0} |\beta_{ppP}(0)|^2 g_P{}^2(t) dt , \tag{13}$$

where $g_P(t)$ is related to $G_P(t)$ in Eq. (7) by

$$g_P(t) = 16\pi |\beta_{ppP}(t)|^{-2} \beta_{ppP}(0)^{-1} G_P(t) , \tag{14}$$

and one can estimate $\beta_{ppP}(t)$ from the elastic differential cross section

$$|\beta_{ppP}(t)|^2 \simeq (16\pi)^{1/2} \left(s^{2\epsilon} \frac{d\sigma_{\mathrm{el}}}{dt} \right)^{1/2} e^{-\alpha_P'(0) t \ln(s/m_p^2)} . \tag{15}$$

Rigorously speaking, $d\sigma_{\mathrm{el}}/dt$ in this model must also have contributions from terms other than the single-Pomeron exchange and therefore $\beta_{ppP}(t)$ cannot be unambiguously determined, but these ambiguities in $\beta_{ppP}(t)$, which are characteristic of a perturbative approach, have little effect in our discussion. Using Eqs. (7) and (13)–(15), one gets for ϵ in the range 0–0.001 and $\alpha_P' = 0.33\ \mathrm{GeV}^{-2}$ (see Ref. 9 and Ref. 19) the approximate form

$$\sigma_{\mathrm{DDE}} \simeq (0.022\ \mathrm{mb}) [\ln(s/m_p^2)]^2 . \tag{13a}$$

The total inelastic cross section in the ISR energy range is then

$$\sigma_{\text{inel}} \simeq \sigma_{\text{ND}} + \sigma_{\text{SDE}} + \sigma_{\text{DDE}}, \tag{16}$$

where the right-hand side of Eq. (16) is given by Eqs. (9) and (11)–(13).

At this point, it is important to make a few comments regarding the justification of the perturbative approach, Eq. (16):

(1) One notices that the integral in Eq. (13) has the slope of $g_P(t)$ as the only cutoff in t, whereas the integral for σ_{SDE} contains the factor $|\beta_{ppP}(t)|^2$. Thus the ratio of the double-to single-diffractive excitation is very sensitive to the slope of $g_P(t)$. The phenomenological analysis in Ref. 10 shows that this slope is very small. This makes the evaluation of σ_{DDE} rather sensitive to uncertainties in the data and the value of $\alpha_P'(0)$.

(2) Obviously, Eq. (16) breaks down at energies much higher than the ISR when events with more than two fireballs may become important and also at low energies when non-Pomeron contributions are important

(3) There are corrections from the secondary Regge trajectories to the Pomeron contributions shown in Fig. 1. The corrections to the single-Pomeron exchange (ND) process are presumably small in exotic processes but rather important in nonexotic ones, especially at low energies. The ones to SDE and DDE processes may be important even at high energies, since each internal Pomeron in the latter processes corresponds to a subenergy which is much smaller than s. In particular, there is a 20% decrease of f_p at $x \approx 0.93$ from $s = 50$ to 500 GeV^2 (Ref. 20) and f_p only scales at ISR energies. Such a decrease can be attributed to the *PPR*, *PRR*, and *RRR* terms[21] which are not included in our approximation.

With the above comments in mind, we proceed to the phenomenological analysis in this model and shall defer some formal discussions to Sec. V. First of all, in order to determine our remaining unknown parameter $\beta_{ppP}(0)$, we should choose a value of s in Eq. (12) such that the uncertainties in the determination of $\beta_{ppP}(0)$ are minimized. We thus use Eq. (16) and the experimental value of σ_{inel} at $s = 100$ GeV^2 (Ref. 22) to obtain the value of $|\beta_{ppP}(0)|^2 = 27.4$ mb, where s is sufficiently large to justify Eq. (16), while σ_{DDE} is still small enough so that the uncertainties in this component do not affect the value of $\beta_{ppP}(0)$. With this value, we plot the s dependence of σ_{inel} for $\epsilon = 0$ and $\alpha_P'(0) = 0$ in Fig. 2 and compare with the data. As discussed before, the behavior of σ_{inel} is insensitive to the precise value of ϵ and $\alpha_P'(0)$ in the range $\epsilon < 0.001$ and $\alpha_P'(0) \leq 0.5$ GeV^{-2}. (See Ref. 16.)

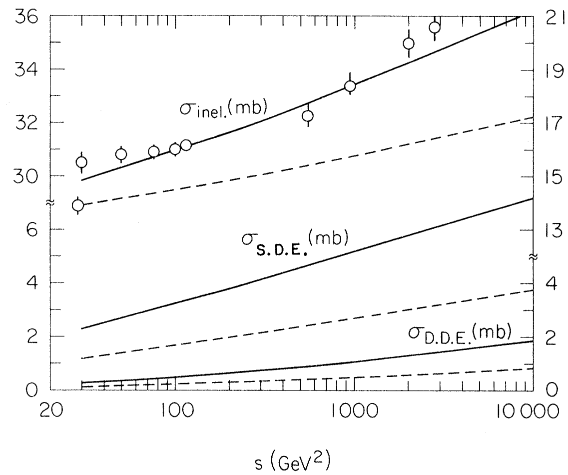

FIG. 2. The total inelastic, single-diffractive excitation, and double-diffractive excitation cross sections. The solid and dashed lines, respectively, represent these cross sections in pp and K^+p reactions calculated from the model and the data points represent the pp total inelastic cross section of Ref. 3 and the K^+p total inelastic cross section of Refs. 27 and 30. The scales on the left- and right-hand axes are for the pp and K^+p reactions, respectively.

As seen from Fig. 2, from $s = 550$ to 2800 GeV^2 we obtain an increase of 2 mb for σ_{inel} for $s = 500$ to 2800 GeV^2, as compared with the observed value of 3.3 ± 0.9 mb. These results indicate that the diffractive-excitation mechanism can be largely responsible for the observed increase in σ_{inel}. For s between 30 and 100 GeV^2 the curve in Fig. 2 seems to increase too fast and slightly underestimates σ_{inel}. This could be due to nonscaling contributions as discussed before. However, these contributions seem to have a rather small effect on σ_{inel}. On the other hand, at ISR and higher energies, there can be other contributions to the increasing σ_{inel}. For example, various electromagnetic processes always lead to a logarithmically increasing cross section[23] and such an increase may become observable at high energies.

To summarize, the sum of σ_{SDE} and σ_{DDE} in Fig. 2 represents the approximate amount of diffractive excitation expected from the model and its increase with energy. As discussed in Ref. 10, the diffractive components obtained in this way include the diffractive production of small mass fireballs, but the elastic process is not included.

The total single-diffractive cross section σ_{SD} is then the sum of σ_{SDE} and the elastic cross section. The magnitudes of σ_{ND}, σ_{SD}, and σ_{DDE} obtained from Fig. 2 are in agreement with their generally accepted values.

V. ASYMPTOTIC BEHAVIOR AND THE POMERANCHUK SINGULARITY

From Eq. (11), we can see that the triple-Pomeron term is relevant only when both s/M^2 and M^2 are large. A qualitative estimation immediately shows that if the single-Pomeron term becomes important only for s larger than a certain value N, then the triple-Pomeron terms start to be important for $s \geqslant N^2$ and so on. An alternative way of expressing this is simply that each internal Reggeon in Fig. 1(b) corresponds to a subenergy much less than s. The precise value of the threshold for s/M^2 then essentially sets a new energy scale for the various diffractive processes to become important.

Phenomenologically, the diffraction peak appears for $x > 0.95$, corresponding to $s/M^2 > 20$. The limit on M^2 is at least the highest resonance mass, which is above 5 GeV^2 in pp reactions. Thus we obtain an energy scale of about 100 GeV^2, which, however, may vary from one reaction to another. For example, in K^+p reaction, the kaon dissociation may start at a lower energy, since the highest resonant state for K^+ is only around 1 GeV.

The importance of this new energy scale is at least twofold: (1) It explains when new phenomena occur and, (2) it partially justifies the expansion in Eq. (16). At ISR energies, since the internal Pomerons correspond to an energy much less than s and below this energy scale of 100 GeV^2, we do not need to further iterate these internal Pomerons. Only at energies much higher than those of ISR, will such iterations become important. Together with the smallness of the triple-Pomeron coupling, this shows that it is reasonable to treat the triple-Pomeron in a perturbative approach as described in Sec. IV and to deal with only a few terms at finite energies, while the question of how to consistently include all the iterations is only relevant for the discussion of the asymptotic behavior as $s \to \infty$. Although different in the detailed mechanism, this physical idea is very similar to that of Chew, Rogers, and Snider.[24]

For $\epsilon > 0$, each term vanishes asymptotically in this perturbation series to any finite order. However, different terms start to become important at different energies, in analogy with the perturbation expansion in quantum electrodynamics, where higher-order terms are suppressed by factors of α but accompanied by factors of $\ln s$.[25] Each energy, where a higher-order term becomes important, effectively represents a new threshold. From this point of view, what we see at ISR or even higher energies in the foreseeable future may very well be the effect of the onset of these thresholds instead of the "true" asymptotic behavior.

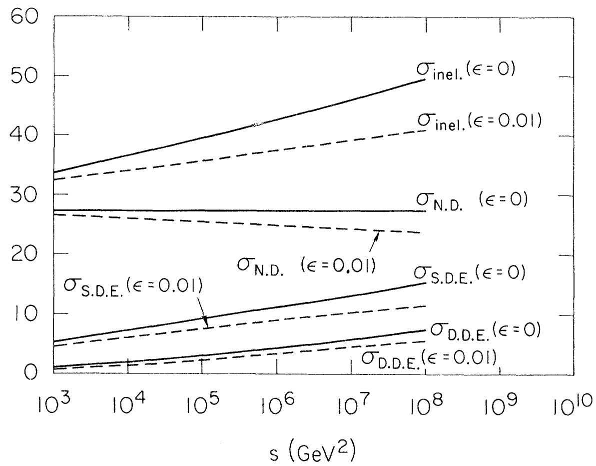

FIG. 3. Two-component model predictions of σ_{inel}, σ_{NP}, σ_{SDE} and σ_{DDE} in pp reaction. The solid and dashed curves, respectively, represent the predictions with $\epsilon = 0$ and $\epsilon = 0.01$.

From a more quantitative point of view, we have already seen that, at low energies, the cross section is dominated by simple Pomeron exchange and is almost constant in s. At higher energies, the SDE and then the DDE processes start to become important. From the knowledge of these processes, it is possible to estimate the relevant high-energy behavior. In Fig. 3, we plot σ_{inel}, σ_{ND}, σ_{SDE}, and σ_{DDE} of Eq. (16) from NAL to energies beyond laboratory experiments in the foreseeable future. For $\epsilon = 0$, the increase of σ_{inel} is practically linear in $\ln s$. For $\epsilon = 0.001$, we obtain almost the same rate of increase even at $s = 10^8$ GeV^2. For $\epsilon = 0.01$, the increase of σ_{SDE} is compensated by the decrease in σ_{ND} at $s \simeq 10^8$ GeV^2. However, σ_{inel} is still increasing at this value of s, due to the increasing σ_{DDE} and also to the possible onset of multiple diffraction processes which are not included in Eq. (16). From the fact that the diffractive cross sections remain smaller than σ_{ND}, such a perturbation expansion is still justified at such high energies. Therefore, we see that it is possible to start with a Pomeron with $\alpha_P(0) = 1 - \epsilon$ and consistently iterate it to the same singularity, i.e., to the asymptotic behavior $\sigma_{\text{inel}} \propto s^{-\epsilon}$, and still have a cross section increasing with s at finite but high energies. Even at the highest energies in Fig. 3, the relevant quantities are only the first few terms in the perturbation expansion. Thus the questions of whether there is an asymptotic behavior and/or whether $\alpha_P(0)$ is exactly unity seem to be academic ones unless future experimental data on f_p at ISR for very small t reveal a sufficiently strong turnover of $g_P(t)$ such that our estimation of σ_{SDE} and σ_{DDE} should be significantly reduced. In this case, $\sigma_{\rm inel}$ might exhibit an "early" asymptotic behavior at Isabelle or even at lower energies.

We further discuss the factorizability of the Pomeron. The relationship between $\sigma_{\rm SDE}$ and the increasing σ_T only depends upon the observation that f_p near $x=\pm 1$ can be parametrized in terms of an "effective pole" for the Pomeron. The estimation of $\sigma_{\rm DDE}$, however, depends on the factorizability of the Pomeron and may behave differently if the assumption is not true. Even if the input Pomeron is indeed a simple pole, some nonfactorizability of the cross sections can still arise as follows. In this approach, the input Pomeron is a factorizable simple pole. At low energies, where the single-Pomeron exchange is dominant, $\sigma_{\rm inel} \simeq \sigma_{\rm ND}$, the cross sections are approximately factorizable[26] apart from corrections due to secondary trajectories. At higher energies, where $\sigma_{\rm SDE}$ and $\sigma_{\rm DDE}$ become significant, the cross section is not single-Pomeron dominated and obviously no longer factorizable. Furthermore, since we cannot reach asymptotic behavior at any reasonable energies, the question of whether one can iteratively obtain an output Pomeron as a factorizable simple pole may also be academic.

VI. TEST OF THE MODEL IN K^+p SCATTERING AT NAL ENERGIES, AND PREDICTIONS FOR OTHER REACTIONS

In the previous sections the existence of the peak near $x=1$ in f_p has been correlated to an increase in $\sigma_{\rm inel}^{pp}$. This increase, due to an increase in the diffractive components, cannot be compensated by a decrease in the nondiffractive one in an exotic process, since the latter has presumably very little energy dependence due to the exchange degeneracy of secondary trajectories. The same arguments should apply to K^+p scattering, where exchange degeneracy is expected to be at least as good as in pp scattering. In the model we are discussing, the magnitude and shape of the peaks in K^+p inclusive reactions near $x=\pm 1$, and therefore the associated increase in $\sigma_{\rm inel}^{K^+p}$, can be predicted using the triple-Pomeron coupling $g_P(t)$ and factorization of the Pomeron. For $K^+p \to p+X$, we get

$$\frac{d\sigma^{K^+p}}{dt\, d(M^2/s)} \approx \frac{\beta_{KKP}(0)}{\beta_{ppP}(0)} \frac{d\sigma^{pp}}{dt\, d(M^2/s)} \approx \frac{\sigma_{\rm inel}^{K^+p}}{\sigma_{\rm inel}^{pp}} \frac{d\sigma^{pp}}{dt\, d(M^2/s)}, \qquad (17)$$

where the pp inclusive cross section is given by Eq. (11). Similarly, for $p+K^+ \to K^+ + X$, we have

$$\frac{d\sigma^{K^+p}}{dtd(M^2/s)} \approx \frac{|\beta_{KKP}(t)|^2}{|\beta_{ppP}(t)|^2} \frac{d\sigma^{pp}}{dtd(M^2/s)} \approx \frac{d\sigma_{\rm el}^{K^+p}/dt}{d\sigma_{\rm el}^{pp}/dt} \frac{d\sigma^{pp}}{dtd(M^2/s)}. \qquad (18)$$

The single-diffractive excitation cross section $\sigma_{\rm SDE}^{K^+p}$ can be obtained simply by integrating Eqs. (17) and (18) and adding these two contributions together. With the approximations $G_P(t) \propto e^{at}$ [see Eq. (2)],

$$|\beta_{ppP}(t)|^2 \simeq e^{b_p t} |\beta_{ppP}(0)|^2,$$

$$|\beta_{KKP}(t)|^2 \simeq e^{b_{K^+} t} |\beta_{KKP}(0)|^2,$$

and the same simplifying assumption in obtaining Eq. (8a), $\sigma_{\rm SDE}^{K^+p}$ can be approximated by

$$\sigma_{\rm SDE}^{K^+p} \simeq \frac{1}{2} \frac{\sigma_{\rm inel}^{K^+p}}{\sigma_{\rm inel}^{pp}} \left[1 + \frac{\sigma_{\rm inel}^{K^+p}}{\sigma_{\rm inel}^{pp}} \times \frac{a}{a+b_{K^+}-b_p}\right] \sigma_{\rm SDE}^{pp}, \qquad (19)$$

where $\sigma_{\rm SDE}^{pp}$ is given in Fig. 2 and has the approximate expression (8a).

For the DDE we have

$$\sigma_{\rm DDE}^{K^+p} \simeq \frac{\beta_{KKP}(0)}{\beta_{ppP}(0)} \sigma_{\rm DDE}^{pp} \simeq \frac{\sigma_{\rm inel}^{K^+p}}{\sigma_{\rm inel}^{pp}} \sigma_{\rm DDE}^{pp}, \qquad (20)$$

where $\sigma_{\rm DDE}^{pp}$ is given by Eq. (13a).

Equations (17)–(20) are only approximate in the sense that $\sigma_{\rm inel}$ should actually be $\sigma_{\rm ND}$ and also $d\sigma_{\rm el}/dt$ should be actually its single-Pomeron contribution (see Sec. VII for the discussion of $d\sigma_{\rm el}/dt$). Thus we should take $\sigma_{\rm inel}$ at a fixed energy, where the total effects due to the secondary trajectory and the diffractive components can be minimized. Furthermore, as discussed in pp scattering, these equations are expected to be more accurate at the highest NAL energies, due to the possible existence of nonscaling contribution in K^+p inclusive spectra at lower energies.

With $\sigma_{\rm ND}$ practically constant [see Eq. (12)], we can compute $\sigma_{\rm inel}^{K^+p}$ from Eqs. (16), (19), and (20) at high energies. For $a = 4.65$ GeV^{-2} and $b_p - b_{K^+} = 3$, we obtain the result shown by the dashed curve in Fig. 2. This curve has been normalized at the highest value of s ($=27.5$ GeV^2) available[27] by evaluating $\sigma_{\rm ND}^{K^+p}$ and the ratio $\sigma_{\rm inel}^{pp}/\sigma_{\rm inel}^{K^+p} = 30.4$ mb/13.88 mb at this energy. As seen from Fig. 2, the difference between this ratio and $\sigma_{\rm ND}^{pp}/\sigma_{\rm ND}^{K^+p}$ is very small. However, as discussed in Sec. IV, the normalization, and therefore $\sigma_{\rm ND}^{K^+p}$ as well as the ratio, should be better determined at $s \gtrsim 100$ GeV^2. A larger uncertainty can arise from that in $b_p - b_{K^+}$, which, in turn, is due to the uncertainties in the data and also to the effect of the real part of the elastic amplitudes at low energies. We estimate that there may be as much as 20% uncertainty in the increase of $\sigma_{\rm inel}^{K^+p}$ shown in Fig. 2, but this figure still gives an important estimation of the prediction of the model.

Similarly, the diffraction peaks in K^-p, $\pi^\pm p$, and $\bar{p}p$, as well as the diffractive components and their increase with the energy can be obtained in our model by simply replacing the K^+p cross sections in Eqs. (17)–(20) by the relevant cross sections. These predictions are, of course, subject to the same uncertainties mentioned above. Furthermore, since these processes are nonexotic, one expects that the increase in the diffractive components will be accompanied by a decrease in the nondiffractive component due to secondary Regge contributions. Only at high enough energies, where these contributions become sufficiently small, will the increase in the diffractive components result in a net increase in $\sigma_{\rm inel}$. Below those energies the increase of the diffractive components should be compared to that in the difference between $\sigma_{\rm inel}$ and its decreasing part (determined from a fit of the data). For instance in π^-p scattering $\sigma_{\rm inel} = 21.04$ mb at $p_{\rm lab} = 40$ GeV/c (see Ref. 28). From 40 to 205 GeV/c the increase in the diffractive component is about 0.9 mb. On the other hand, from a fit of the data of the form $a + bp_{\rm lab}{}^{-1/2}$ for $p_{\rm lab} < 65$ GeV/c, one gets a decreasing part of about 1.1 mb in the same range of $p_{\rm lab}$ between 40 and 205 GeV/c. Thus the value of $\sigma_{\rm inel}$ at 205 GeV/c should be approximately the same as at 40 GeV/c in agreement with preliminary results at NAL.[29] From 200 to 400 GeV/c the decrease is 0.2–0.3 mb, while the increase in the diffractive components is 0.4 mb and therefore no substantial increase of $\sigma_{\rm inel}$ is expected even at the highest NAL energies. (However, the increase in $\sigma_{\rm inel}^{\pi^-p}$ is estimated by Eqs. (19) and (20) with $|\beta_{\pi\pi P}(t)|^2 \sim |\beta_{\pi\pi P}(0)|^2 \times |\beta_{ppP}(t)|^2/|\beta_{ppP}(0)|^2$. The uncertainties in this residue may again make some difference. For instance, if its t dependence is much smaller than this, it may result in a larger increase in $\sigma_{\rm inel}^{\pi^-p}$.)

To conclude, we should make a few remarks concerning the hierarchy of these tests of our model. The most important of all is observing whether $\sigma_{\rm inel}^{K^+p}$ indeed increases and, if so, directly comparing with the inclusive cross sections for $K^+p \to pX$ and $K^+ + p \to pX$ near the kinematical limits to test our basic idea that an increasing $\sigma_{\rm inel}$ and the peaks in the inclusive cross sections are due to the same mechanism. The next one is to test Eq. (17), which is based on the factorizability of the Pomeron at $t=0$. The predictions of Eqs. (18) and (19) are based on the more subtle question of the factorizability for $t \neq 0$. Furthermore, these involve the difficult task of extracting out $\beta(t)$ from the elastic cross section. For the above reasons, those predictions are subjected to uncertainties. However, a direct comparison of $\sigma_{\rm inel}$ with Eqs. (16), (19), (20), and Fig. 2 gives a simple and very important test of our model.

VII. ELASTIC SCATTERING

Since the pp total cross section increases at ISR energies and the elastic amplitude at $t=0$ is largely imaginary at lower energies, $d\sigma_{\rm el}/dt$ at $t=0$ must increase by the optical theorem.

A large percentage of this increase, i.e., the part corresponding to the contribution of $\sigma_{\rm inel}$ to the optical point, can be obtained from our model. At low energies, where the increasing contribution is small, $\sigma_{\rm el}$ is expected to exhibit a decrease due to the shrinking of the elastic peak and/or the contribution of secondary trajectories. At higher energies when the processes involving the triple-Pomeron coupling become important, this decreasing behavior may be overtaken by the increasing contribution of those processes, and in this case $\sigma_{\rm el}$ will be flat for a while and then increase with the energy. However, whether or not this change of behavior occurs and at what value of s depends crucially on the slope parameters of the increasing components. We shall see that, with reasonable values of these slope parameters, $\sigma_{\rm el}$ can increase at ISR energies. In any case, it is very reasonable to conclude that the possible onset of flat and subsequent increasing $\sigma_{\rm el}$ must not occur earlier than that of $\sigma_{\rm inel}$, in agreement with the pp data. The apparent constancy of σ_T for $s \simeq 100$ GeV^2 is possibly a result of the compensation between an increasing $\sigma_{\rm inel}$ and a decreasing component (mainly $\sigma_{\rm el}$). On the other hand, an early increasing of σ_T, as observed in K^+p reaction[30] must imply an increasing $\sigma_{\rm inel}$. Since the increase in this quantity predicted by the model in the Serpukhov energy range (see Fig. 2) is comparable to the observed increase in σ_T, one expects $\sigma_{\rm el}$ to be constant or slightly increasing. Therefore a measurement of $\sigma_{\rm el}^{K^+p}$ at Serpukhov and also a measurement of $\sigma_T^{K^+p}$ and $\sigma_{\rm el}^{K^+p}$ at NAL are very important to test our ideas.

The existence of two types of contributions to the optical point, one almost independent of the energy and the other increasing with s, can produce a break in $d\sigma_{\rm el}/dt$.[31] The position of this break depends on s and on the slope parameters of the two (types of) contributions. We are going to see that in pp scattering, by supplementing our model with the experimental information on the integrated elastic cross section, one can obtain the shape of $d\sigma_{\rm el}/dt$ in reasonably good agreement with experiment.

The contribution to $d\sigma_{\rm el}/dt$ at $t=0$ from the diagrams of the first line of Fig. 1(d) is

$$\frac{1}{16\pi}(\sigma_{\rm ND}+\sigma_{\rm SDE}+\sigma_{\rm DDE})^2\,. \qquad (21)$$

This is the contribution of $\sigma_{\rm inel}$ to the optical point and represents approximately 80% of the total contribution. The missing part is given by the diagrams of the last line of Fig. 1(d). Notice that the diagrams in the first line of Fig. 1(d) which contain the triple-Pomeron coupling $g_P(t_1, t_2, t_3)$ cannot be computed at $t\neq 0$ since this coupling is only known for $t_1=t_2$ and $t_3=0$; therefore the iterated diagram in the second line of Fig. 1(d) cannot be computed either — not even at $t=0$. However, from the experimental data we know that the ratio $\sigma_{\rm el}/\sigma_{\rm inel}$ is roughly constant in the ISR range $s=500-2800$ GeV^2. This means that, in this energy range, the contribution of σ_T to the optical point is simply the contribution of $\sigma_{\rm inel}$, given by Eq. (21), multiplied by the constant factor $\sigma_T/\sigma_{\rm inel}$.

Let us now turn to the t dependence. The main contribution to the energy-independent part at $t=0$ is of course $\sigma_{\rm ND}$. Let us call $b(s)$ the slope parameter of this component and assume that this quantity represents an effective slope parameter for the entire energy-independent part of the elastic amplitude. Similarly the main contribution to the energy-dependent part of the amplitude is $\sigma_{\rm SDE}$. Let us denote by $b'(s)$ the slope parameter of this component, which again is assumed to represent the effective slope parameter of this energy-dependent part of the amplitude. One is led in this way to the following parametrization[32]:

$$\frac{d\sigma_{\rm el}}{dt}=\frac{1}{16\pi}\left(\frac{\sigma_T}{\sigma_{\rm inel}}\right)^2\left[\sigma_{\rm ND}\,e^{b(s)t/2}+(\sigma_{\rm SDE}+\sigma_{\rm DDE})e^{b'(s)t/2}\right]^2. \qquad (22)$$

It is clear that if $b(s)\neq b'(s)$, a break will appear in $d\sigma_{\rm el}/dt$.

The experimental situation concerning the s dependence of the elastic peak and thus the slope of the Pomeron is rather unclear.[9] In our model the two following situations seem to be possible *a priori:*

(i) $b'(s)>b(s)$. In this case the increasing component will be important mainly at very small $|t|$, and thus the slope of the Pomeron has to be linked to the small shrinkage observed at large $|t|$, the large shrinkage observed at small $|t|$ being produced by the increase of the optical point.

(ii) $b'(s)<b(s)$. Here the effect of the increasing contribution is more important at large $|t|$ and thus the slope of the Pomeron has to be linked to the shrinkage observed at small $|t|$.

In fact only the second possibility is truly consistent with our model. Indeed at conventional accelerator energies the increasing component is very small and therefore the slope of the Pomeron has to be linked to the shrinkage at these energies. This shrinkage is consistent with the one observed at the ISR at small values of $|t|$ as shown by the fit of the data on the slope parameter at $|t|<0.12$ GeV^2 given in Ref. 9:

$$b(s)=9.3+2\alpha_P'(0)[\ln(s/10\ {\rm GeV}^2)]\,, \qquad (23)$$

with $\alpha_P'(0)=0.33$ GeV^{-2} (see Ref. 33). That possibility (i) is indeed favored by our model can be shown in the following way: Using the values of $b(s)$ from Eq. (23) in Eq. (22), one can compute $b'(s)$ by demanding that the integrated elastic cross section obtained from Eq. (22) agree with the experimental ISR values. In this way, one obtains $b'\sim 8$ GeV^{-2} at $s=2800$ GeV^2. Furthermore with this value of b', one gets an increase of $\sigma_{\rm el}$ in the ISR energy range in reasonably good agreement with experiment. This does not prove that $b'(s)$ is s-independent, but shows that the model is consistent with a small s dependence of this quantity, as expected from the "cut" nature of the corresponding diagrams in Fig. 1(d) and in agreement with the rather small shrinkage observed of the ISR at large $|t|$. As discussed before, we cannot compute the value of $b'(s)$ unless some assumption is made on the triple-Pomeron coupling $g_P(t_1, t_2, t_3)$. For instance, if this quantity is symmetric in its three variables, we can take it to be a constant as a first approximation, since $g_P(t)\equiv g_P(t_1=t, t_2=t, t_3=0)$ depends very little on t. Then the second and third diagrams in Fig. 1(d) have a slope $b'(s)\simeq\frac{3}{4}b(s)$. We have also estimated that a more formal calculation of the diagrams in Fig. 1(d) in terms of certain parameters can indeed agree with the present more phenomenological approach.

Using the value $b'=8$ GeV^{-2}, one can compute the shape of $d\sigma_{\rm el}/dt$. The result for $s=2800$ GeV^2 is shown in Fig. 4 and compared with the data. As expected, one observes a break or rather a smooth change from an exponential e^{bt} at small $|t|$ to another exponential with smaller slope at larger values of $|t|$. It is clear that in our model this break disappears at low energies since the increasing components become negligible. However, a detailed description of its energy dependence is beyond the scope of the rather crude analysis presented here.

VIII. SUMMARY AND DISCUSSIONS

We summarize our main results as follows:

(1) The scaling peak in f_p near the kinematical limits implies that there is a mechanism responsible for this peak whose contribution to σ_T increases with s at least over a finite energy range.

(2) Both the peak in f_p and the increase in $\sigma_{\rm inel}$ at ISR energies can be explained in terms of dif-

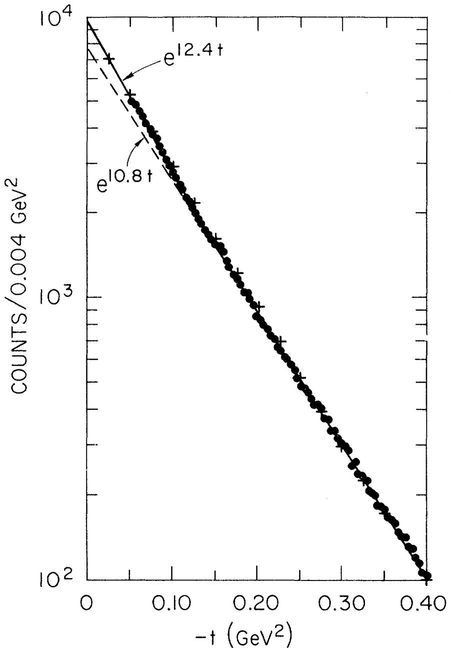

FIG. 4. Comparison between the data of Ref. 9 on the pp elastic cross section at $s = 2800$ GeV^2 and the model. The circles are the data points, the crosses are the results calculated from the model, and both the solid and dashed lines are simple exponentials given by Ref. 9 to illustrate the t dependence of the cross section for two different regions of t. Although difficult to see from the figure, the model gives a smoother transition of the cross section between these two lines than the data.

fractive excitation into high-mass states in the framework of a triple-Pomeron model. Therefore diffractive excitation provides for the mechanism alluded to in (1).

(3) The relationship between increasing σ_{inel} and diffractive processes can be tested in the K^+p reaction at Serpukhov and NAL and, very likely, also in other reactions.

(4) σ_{el} can also increase but this increase cannot start earlier than the increase of σ_T. In particular, $\sigma_{\text{el}}^{K^+p}$ at Serpukhov energies should be constant or slightly increasing.

(5) The shape of $d\sigma_{\text{el}}/dt$ and, in particular, the break at small $|t|$ is connected, via the optical theorem, to the existence of two types of components in σ_T, one of which is almost energy-independent and essentially nondiffractive, and the other which increases with s and is essentially diffractive.

(6) From energy conservation, an increasing σ_T implies that at least some of the inclusive cross sections, f_c, and/or the leading particle average inelasticity have to be s-dependent. In particular, f_c must increase with s if $\bar{x}_p$ is constant or decreases with s.

(7) If the peak in f_p is dominated by single diffraction and scales, and $\bar{n}_p$ stays constant, then f_p in some other region must also contribute to an increasing σ_T.

(8) In the two-component model, each diffractive-excitation process becomes important at certain energies and therefore effectively creates a new threshold for σ_T.

(9) Below all these thresholds, we have $\sigma_{\text{inel}} \simeq \sigma_{\text{ND}}$ and thus approximate factorization for the cross section, but this factorization property does not persist at high energies.

(10) The energy dependence of the cross sections at and even beyond ISR energies is most likely insensitive to the question of whether $\alpha_P(0)$ is exactly equal to unity or not.

To conclude, we have presented a model that correlates in a natural way such seemingly independent phenomena as inclusive peaks and rising cross sections, and describes them in terms of the simple physical concept of diffractive excitation. Furthermore, since diffractive-excitation processes are dominated by Pomeron exchange, the model relates in an unambiguous way the results in different reactions via factorization. Although the comparison with pp data is very encouraging, it is clear that the version of the model we have presented is oversimplified and corrections to it have to be expected. It is therefore important to obtain more detailed data in pp scattering and also data on the inclusive peaks and total and elastic cross sections in other processes, especially K^+p at Serpukhov and NAL energies in order to test the model and determine the importance of possible corrections to it. With respect to the model, experiments in the near future, although very important for a better understanding of the dynamics of strong interactions, may not provide us with the "true" asymptotic behavior.

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[1]M. G. Albrow *et al.*, Nucl. Phys. B51, 388 (1973).

[2]M. G. Albrow *et al.*, CERN report, 1972 (unpublished).

[3]U. Amaldi *et al.*, CERN report, 1973 (unpublished).

[4]See, for example, L. G. Ratner *et al.*, Phys. Rev. Lett. 27, 86 (1971).

[5]L. Caneschi and A. Pignotti, Phys. Rev. Lett. 22, 1219 (1969); C. E. DeTar *et al.*, *ibid.* 26, 675 (1971).

[6]The relationship between the increasing total cross section and the inclusive spectrum was first discussed in a paper by one of us. See A. Capella, this issue, Phys. Rev. D 8, 2047 (1973), and some of the qualitative discussion in Sec. II was presented in a subsequent publication by A. Capella, M.-S. Chen, M. Kugler, and R. D. Peccei, Phys. Rev. Lett. 31, 497 (1973).

[7]M. Antinucci *et al.*, Nuovo Cimento Lett. 6, 121 (1973).

[8]See, for example, Bellettini *et al.*, Phys. Lett. 19, 705 (1966).

[9]G. Barbiellini *et al.*, Phys. Lett. 39B, 663 (1972).

[10]A. Capella, this issue, Phys. Rev. D 8, 2040 (1973).

[11]The form of $G_P(t)$ given by Eq. (7) has the same integral as $G_P(t)$ in Ref. 10 and is a good enough approximation for the purposes of this paper.

[12]G. F. Chew, Phys. Rev. D 7, 3525 (1973).

[13]There may be a small amount of double counting in Eq. (8). See Ref. 12 for a discussion of this point.

[14]One can also derive the same result using an average multiplicity sum rule of the type given by Eq. (1) for σ_{SDE}. But, in this way, one has to include the contribution from the protons, associated with the diffractively excited states, which is not in the peaks.

[15]H. D. I. Abarbanel, G. F. Chew, M. L. Goldberger, and L. M. Saunders, Phys. Rev. Lett. 26, 937 (1971).

[16]The increase is proportional to $\ln s$ for $\alpha_P'(0)=0$ and to $\ln(\ln s)$ for $\alpha_P'(0)>0$, but these two behaviors are numerically very close to each other, and the effects of the slope of the Pomeron on the integral of Eq. (11) are negligible for $\alpha_P'(0)<0.5$ GeV^{-2}. In the following, we take $\alpha_P'(0)=0$ for the evaluation of σ_{SDE}.

[17]Our discussion is very similar to that of Ref. 12, where Chew originally studied the energy dependence of these categories of events in suggesting an explanation for the apparent constancy of σ_T even though some of these events are s-dependent. A perturbative description of the Pomeron was also studied by M. Bishari and J. Koplik [Phys. Lett. 44B, 175 (1973)].

[18]As discussed before and also in Ref. 10, both the diffractively excited low-mass states (predominantly the diffractive resonances) and high-mass states are included in the triple-Pomeron contribution to σ_{SDE}. By factorization, we thus expect that both of these states are also included in σ_{DDE}. If one takes an alternative point of view that the triple-Pomeron term only includes the high-mass states, then the resonance contributions have to be estimated separately. For example, the total resonance production contribution to σ_{SDE} should behave like σ_{el}, which also increases logarithmically with s at high energies and shall be discussed in Sec. VII. The cross section double diffractive excitation into one high- and one low-mass state should then behave like σ_{SDE} and be approximately equal to $R\sigma_{SDE}$, where R is the ratio of the total resonance production cross section to σ_{el} and is about 10%. There is also some double counting in Eq. (13), as discussed in Ref. 12.

[19]For reasons to be discussed below, σ_{DDE} is more sensitive to $\alpha_P'(0)$ than σ_{SDE}. For instance, with $\alpha_P'(0)=0$ we get in Eq. (13a) 0.03 instead of 0.022 mb. The choice of $\alpha_P'(0)=0.33$ will become clear in Sec. VII in connection with the discussion of the elastic cross section.

[20]F. Sannes *et al.*, results reported at the Proceedings of the International Conference on High Energy Particle Interactions, Vanderbilt University, Nashville, Tennesse, NAL report, 1973 (unpublished).

[21]All these terms are nonscaling and peaked near $x=1$—the last one only for small $|t|$. Although only the first one is properly diffractive, all of them contribute to the single excitation of the proton into high-mass objects and their contribution should be included in σ_{inel}.

[22]G. G. Beznogikh *et al.*, Phys. Lett. 43B, 85 (1973); S. P. Denisov *et al.*, *ibid.* 36B, 415 (1971).

[23]See, for example, H. Cheng and T. T. Wu, Phys. Rev. Lett. 23, 1311 (1969); Phys. Rev. D 1, 2775 (1970), and references therein.

[24]See Sec. IV of G. F. Chew, T. Rogers, and D. R. Snider, Phys. Rev. D 2, 765 (1970).

[25]Except that in our case these logarithmic factors will eventually disappear at very high energies if $\alpha_P(0)<1$.

[26]See, for example, M.-S. Chen *et al.*, Phys. Rev. Lett. 26, 1585 (1967); Chan Hong-Mo, M. L. Miettinen, and R. G. Roberts, Rutherford report, 1972 (unpublished).

[27]K. J. Foley *et al.*, Phys. Rev. Lett. 11, 503 (1963).

[28]For $\sigma_{el}^{\pi^- p}$, see Yu. M. Antipov *et al.*, CERN report, 1973 (unpublished); for $\sigma_T^{\pi^- p}$ see, for example, E. Bracci *et al.*, CERN Report No. CERN/HERA 72-1 (unpublished).

[29]D. Bogert *et al.*, NAL-Conf. Report No. 73/30-EXP, 1973 (unpublished).

[30]S. P. Denisov *et al.*, Phys. Lett. 36B, 415 (1971).

[31]A description of this break in the framework of Gribov's Reggeon calculus has been given by T. N. Ng and V. P. Sukhatme, University of Washington report, 1973 (unpublished).

[32]We have neglected a possible real part of the elastic amplitude at ISR energies. However, the effect of this real part is presumably small since the main contribution to the elastic amplitude, i.e., the nondiffractive component is essentially imaginary at small $|t|$ and thus there is very little interference between this contribution and a possible real part. We thank V. Baluni for a discussion concerning the real part.

[33]The same value of $\alpha_P'(0)$ has been obtained from a fit of the pp elastic data at conventional accelerator energies using an eikonal Regge model. It has also been shown that even at these energies the secondary Regge trajectories cannot reproduce the observed shrinkage of the elastic peak. See A. Capella, J. Kaplan, A. Krzywicki and D. Schiff, Nuovo Cimento 63 A, 141 (1969), and especially J. Kaplan and D. Schiff, Nuovo Cimento Lett. 3, 19 (1970).