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[9]S. D. Ellis and A. I. Sanda, Phys. Lett. 41B, 87 (1972).

[10]A. I. Sanda, Phys. Rev. D 6, 280 (1972); M. B. Einhorn, J. Ellis, and J. Finkelstein, *ibid.* 5, 2063 (1972).

[11]H. Harari, Phys. Rev. Lett. 20, 1395 (1968); P. G. O. Freund, Phys. Rev. Lett. 20, 235 (1968).

[12]For other parametrizations, such as

$$\nu^{-3/2} \underset{M^2 \to 0}{\sim} M^{-3},$$

the local duality as described in Sec. II B does not appear to work. Why the parametrizations (12a) and (12b) are better than others is a mystery—the necessity of using the variable ν in the FESR for two-body processes is rather mysterious—and the question of whether a parametrization works better than the others in all inclusive processes is an open question.

[13]J. H. Schwarz, Phys. Rev. 159, 1269 (1967).

[14]No attempt has been made to check whether $R_r(t)$ in Eq. (16) is linked to a singularity at $\alpha(t) \sim 1$ for all values of t. In order to interpret it as a wrong-signature fixed pole, one has to assume that all the other singularities give rise to the asymptotic Regge behavior used in the FMSR.

[15]A. B. Kaidalov, Yad. Fiz. 13, 401 (1971) [Sov. J. Nucl. Phys. 13, 226 (1971)].

[16]We have verified that the narrow-width approximation tends to overestimate $g(t)$ by 10–20%. Therefore we take $g^R_{PP}(t) = 0.84 g_1(t)$.

[17]An alternative parametrization which gives an equally good fit of the data is the following: For the second term in Eq. (22), one takes the result of the fit in the first paper of Ref. 1, i.e.:

$$c_1 = 5.7,\ c_2 = 0,\ \beta(t) = 24.6 e^{5.37t + 2.11t^2};$$

the first term in Eq. (22) is replaced by

$$\frac{1}{s}(58 e^{5.16t} + 369 e^{21t})(s/M^2)^{1.04} \nu^{0.41},$$

and the third term is left unchanged.

[18]In the region $M > 2.5$ GeV, the contribution of the *PPR* term is so small that one can safely use the analytic expression (20) in the whole range of t.

[19]M. G. Albrow *et al.*, Nucl. Phys. B51, 388 (1973); J. C. Sens, in *Proceedings of the XVI International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 1, p. 255.

[20]Note that the low- and high-energy data we are comparing correspond roughly to the same value of the momentum transfer.

[21]In the region $x \sim 1$, the points in Figs. 4 and 5 correspond to rather different values of t, and therefore the theoretical curve in Fig. 4 at $x \sim 1$ could be modified by slightly changing the t dependence in Eq. (24). Notice that the spectrum in Fig. 4 is a mixture of elastic and inelastic protons, whereas the data in Fig. 5 are inelastic. (We thank Dr. J. C. Sens for a correspondence on these data.)

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Evidence for a Dual Triple-Pomeron Coupling from Inclusive Intersecting-Storage-Rings Data*

A. Capella†

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

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The proton inclusive spectrum in the diffractive region is described by a triple-Pomeron term with no free parameters. Our input is the cross sections for the production of N^*'s at CERN accelerator energies, which, according to duality rules for Pomeron-particle reactions, are related to the triple-Pomeron coupling via finite-mass sum rules. The extrapolated value of this coupling to $t = 0$ induces rather weak constraints on the parameters of the Pomeron, and *no* sharp turnover of the proton spectrum near $t = 0$ is expected.

In a recent phenomenological analysis of $p + p \rightarrow p + X$ inclusive reactions it has been shown[1] that most of the CERN Intersecting Storage Rings (ISR) proton spectrum in the diffractive region has to be due to a triple-Pomeron term—in contrast with previous works that attempted to describe it with a Pomeron-Pomeron-Reggeon (*PPR*) term. This conclusion is supported by a recent experiment at $s = 929.5$ GeV^2.[2] On the one hand, from the comparison with the results obtained at $s = 1995$ GeV^2,[3] " one observes quantitative agreement between the two spectra all the way out to $x \equiv 2p_L/\sqrt{s} = 1$."[2] On the other hand, the new results, which are very detailed in the diffractive region, $0.95 \lesssim x \lesssim 1$, show the following features:

(i) An e^{at} dependence at fixed missing mass, M, for $|t| \lesssim 0.5$ GeV^2, with the slope parameter a independent of M.

(ii) An approximate M^{-2} dependence at fixed t, for $10 \lesssim M^2 \lesssim 50$ GeV^2.

Property (1) is in agreement with the small slope of the Pomeron. With $\alpha_P'(0) \sim 0$ and $\alpha_P(0) \sim 1$, property (ii) favors a triple-Pomeron term, which behaves like M^{-2}, versus a PPR, which behaves like M^{-3} with $\alpha_R(0) \sim \frac{1}{2}$.

In this note we describe the above results in terms of a PPP term with no free parameters. At the same time we check the duality rules for Pomeron-particle amplitudes based on perturbative dual models. These rules state[4] that the resonances in the s channel are dual to the Pomeron in the t channel and that the PPR terms are small since one cannot draw dual diagrams for such terms—which thus can only appear as a nonleading contribution. With these duality rules, one can deduce the triple-Pomeron coupling from the production cross sections $pp \to pN^*$. The latter will be our only input, and we shall use for their values the results of Ref. 5 at 24 GeV/c.

With $\alpha_P(t) \sim 1$, the finite-mass sum rules (FMSR) in the narrow-width approximation lead to the relation[6]

$$\sum_i \nu_i \frac{d\sigma_i}{dt} = (\bar{\nu} - \nu_0) G_P(t), \tag{1}$$

where $\nu \equiv M^2 - M_P^2 - t$, $d\sigma_i/dt$ is the cross section for the production of N_i^*, and $\bar{\nu}$ is the cut in the FMSR. The quantity $G_P(t)$ is related to the triple-Pomeron coupling, $g_P(t)$,[7] by

$$g_P(t) = \frac{(16\pi)^{1/2} G_P(t)}{(\sigma_T)^{1/2} [(d\sigma/dt)_{\rm el}]^{1/2}}, \tag{2}$$

where σ_T and $d\sigma/dt$ are the asymptotic values of the total and elastic differential pp cross sections, and the equality is strictly true only for $\alpha_P(0) = 1$.

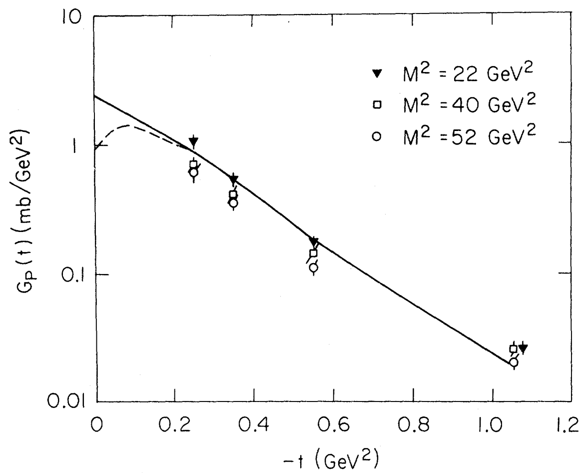

FIG. 1. The values of $G_P(t)$ in Table I plotted against t and compared with the t dependence of the data of Ref. 2 at several values of M^2. The dashed curve is obtained when the 1400 enhancement is not included. In this figure the normalization of the data has been arbitrarily chosen.

The normalization of $G_P(t)$ is

$$E \frac{d\sigma_D}{d^3p} = \frac{1}{\pi} G_P(t) \frac{s}{M^2}. \tag{3}$$

(The symbol D stands for diffractive contribution.) In the sum in Eq. (1) we include, besides the nucleon itself, the 1400 enhancement, the $N^*(1520)$, the $N^*(1690)$, and the $N^*(2190)$, and the cut is taken at $M = 2.4$ GeV after the last broad bump at $M = 2.2$ GeV. The FMSR is saturated in an essentially local way.[8] The values of $G_P(t)$ obtained from Eq. (1) are given in Table I. The results, when the 1400 enhancement is not included in Eq. (1), are also given. For $|t| \gtrsim 0.25$ GeV^2, where ISR data exist, the results are the same in both cases. The values of the triple-Pomeron coupling are also given.

The values of $G_P(t)$ in Table I are plotted in Fig. 1 and compared with the t-dependence of the data

TABLE I. The values of $G_P(t)$ and the PPP coupling, $g_P(t)$, computed from Eqs. (1) and (3).

$-t$ (GeV^2)	0.005 [a]	0.10	0.25	0.35	0.55	0.80	1.05
$G_P(t)$ (mb/GeV^2)	2.3 (0.9) [b]	1.65 (1.4) [b]	0.9	0.55	0.19	0.058	0.019
$g_P(t)$ (GeV^{-1}) [c]	0.3 (0.1) [b]	0.36 (0.31) [b]	0.36	0.33	0.26	0.23	0.22

[a] For this value of t we have used the data of Bellettini *et al.* at 19.3 GeV/c as given in Table VI of Ref. 5.

[b] Values obtained when the 1400 enhancement is not included.

[c] We have used $\sigma_T = 40$ mb and the values of $(d\sigma/dt)_{\rm el}$ of Ref. 5.

of Ref. 2.

The values of $G_P(t)$ are subject to uncertainties due to error bars in $d\sigma_i/dt$. These are random errors of the order of 6–10% and normalization errors of 10–15%. Besides, another 10–15% normalization error due to arbitrariness in the choice of $\bar{\nu}$ is possible. To facilitate the comparison with experiment, and in view of the uncertainty in normalization, from now on for both $G_P(t)$ and $g_P(t)$ we take the values in Table I multiplied by a factor 0.75.

Using these values of $G_P(t)$ in Eq. (3) we obtain the dashed curves shown in Fig. 2; the t-dependence of the data is well reproduced. Notice also that with a small $\alpha_P'(0) \neq 0$, the M^2 dependence would be slightly flatter, especially at the largest values of $|t|$, resulting in an even better agreement with the data.

We turn now to a discussion of the triple-Pomeron coupling. One can see from Table I that it has some turnover near $t=0$ when the 1400 enhancement is not included. This turnover is due to the vanishing of the nucleon contribution at $t=0$ in Eq. (1) ($\nu_i = -t$ for the nucleon). However, its value at $t=0$ is not zero unless all other $d\sigma_i/dt$ vanish at $t=0$, which does not appear to be the case. When the 1400 enhancement is included the turnover almost disappears, and one gets a value for the triple-Pomeron coupling which approximately coincides with the one obtained by extrapolating exponentially to $t=0$ the ISR data in the diffractive region—under the assumption that these data are essentially due to a *PPP* term (see Fig. 1). The value of the triple-Pomeron coupling at $t=0$ is related to the parameters of the Pomeron trajectory as follows[7]:

$$\eta_P(0) \equiv \frac{1}{16\pi}\,\frac{1}{2\alpha_P'(0)}\,g_P^2(0) \lesssim 1-\alpha_P(0). \qquad (4)$$

Our value $g_P(0) \sim 0.3\times0.75\ \text{GeV}^{-1}$ gives a rather weak constraint on $\alpha_P'(0)$ and $\alpha_P(0)$:

$$1-\alpha_P(0) \gtrsim \frac{10^{-3}}{2\alpha_P'(0)}\,.$$

With $0.05 \lesssim \alpha_P'(0) \lesssim 0.5\ \text{GeV}^{-2}$, one can have $0.990 \lesssim \alpha_P(0) \lesssim 0.999$. If the contribution of the 1400 enhancement is not included, the constraint is even weaker. These values of $\alpha_P(0)$ are so close to unity that no zero or sharp turnover of the triple-Pomeron coupling at $t=0$ appears to be required by Eq. (4). Therefore, a sharp turnover near $t=0$ of the proton inclusive spectra in the diffractive region is not required either. In fact, with our dual triple-Pomeron coupling such a sharp turnover does not occur—one might observe at most the flattening out or slight turnover of the curves in Fig. 1.

A very recent theoretical value of the integrated triple-Pomeron coupling, with a maximum estimated error of a factor 2, is given in Ref. 9:

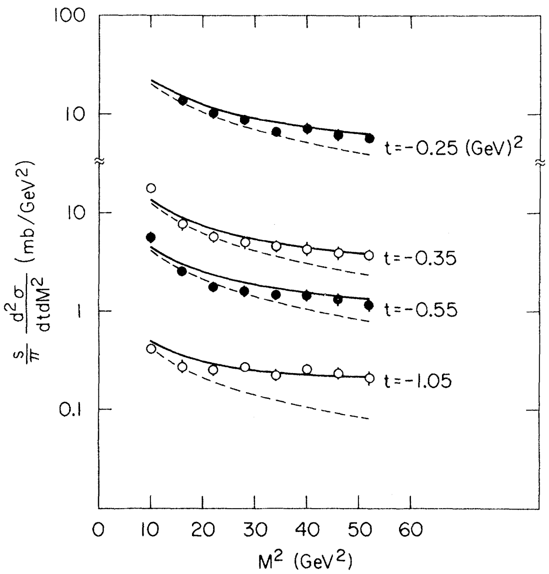

FIG. 2. Missing-mass spectra at $s = 929.5\ \text{GeV}^2$ from Ref. 2. The dashed line is obtained from the triple-Pomeron term, Eq. (3). The full line is obtained by adding to this *PPP* term the nondiffractive contribution of Ref. 1 [first two terms in Eq. (22)].

$$\int_{-\infty}^{0} dt\, e^{bt/2}\, g_P(t) \approx \frac{2\alpha_P'}{b^2}(\sigma_T)^{1/2}\,, \qquad (5)$$

where b is the slope parameter of the pp elastic differential cross section. With our values of $g_P(t)$ (see Ref. 10) one gets $\alpha_P' \sim 0.3\ \text{GeV}^{-2}$ (see Ref. 11).

As for the possible presence of a *PPR* term, it is clear from the uncertainties in absolute normalization, together with the factor 0.75 discussed above, that one cannot exclude the existence of such a term with a residue of order 10–50% of the triple-Pomeron term. This would, however, alter very little both our general scheme and the quantitative values discussed above. As far as the comparison with the ISR results is concerned, such a *PPR* term, due to its extra M^{-1} power as compared to a *PPP* one, would affect our figures by 3–17% at $M^2 = 10\ \text{GeV}^2$ ($x \sim 0.99$) and 1.5–7% at $M^2 = 50\ \text{GeV}^2$ ($x \sim 0.95$).

Our results, relating experimental results at ISR and CERN accelerator energies without any free parameter, provide in our opinion a striking confirmation of a (dominating) triple-Pomeron term in the diffractive region. This term appears to be dual to the diffractively produced resonances in the s channel, in agreement with duality rules for inclusive reactions. The model leads to a nonvanishing triple-Pomeron coupling at $t=0$, and predicts no sharp turnover near $t=0$ of the proton inclusive spectrum in the diffractive region.

Another interesting point is that the integral over M^2 and t of our PPP term increases by about 2 mb from $s=50$ to $s=3000$ GeV^2. This gives rise to a contribution to the proton-proton total cross section which increases by about 4 mb in the same range of energy.[12] The value of this contribution at $s=50$ GeV^2 is about 3 mb. From the dual construction of the PPP term one might think that this contribution includes most of the diffractive component—including the elastic part. However, looking at the zero-moment (wrong-signature) FMSR, one finds that its left-hand side is larger than the right-hand side by an amount almost equal to the contribution of the proton.[13] Therefore, the elastic cross section is not included in the 3 mb.[14]

As a final remark, it is instructive to add to our PPP term, Eq. (3), a phenomenological parametrization of the proton spectrum outside the diffractive peak, obtained from a fit of data at CERN accelerator energies [see Ref. 1, first two terms in Eq. (22)]. One obtains in this way a very good description of the proton spectrum at CERN energies in a large range of the variable x. At $s=930$ GeV^2, one gets the full curves in Figs. 2 and 3; one can see from Fig. 2 that the new terms are very small for x near 1, so that our previous results are almost unchanged, and Fig. 3 shows that they provide a reasonably good parametrization of the data at smaller values of x. This parametrization also describes[1] the ISR data at $s=1995$ GeV^2,[3] since they scale with the ones in Fig. 3, and the model has scaling built in—except for an RRR term, which is very small at $s=930$ GeV^2.

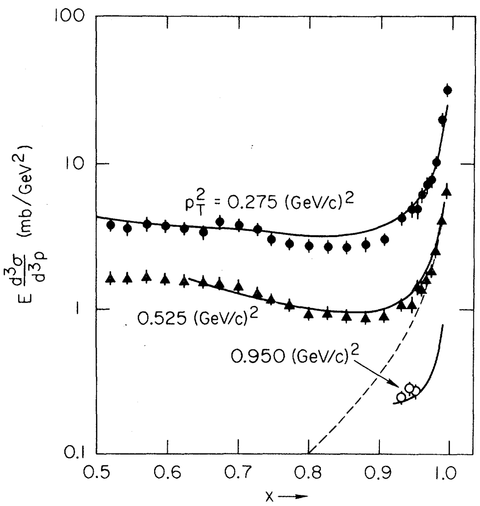

FIG. 3. Inelastic proton spectra at $s=929.5$ GeV^2. The full curves are obtained by adding the diffractive and nondiffractive contributions, given by Eq. (3) and Ref. 1 [first two terms in Eq. (22)], respectively. The dashed curve is the contribution to the triple-Pomeron term, Eq. (3), at $p_T{}^2 = 0.525\,(\text{GeV}/c)^2$.

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†On leave of absence from the Laboratoire de Physique Théorique et Particules Elementaires, Orsay, France.

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[2]M. G. Albrow *et al.*, CERN report, 1972 (unpublished).

[3]M. G. Albrow *et al.*, Nucl. Phys. B51, 388 (1973).

[4]See, for instance, M. B. Einhorn, M. B. Green, and M. A. Virasoro, Phys. Lett. 37B, 292 (1971); Phys. Rev. D 7, 102 (1973). A phenomenological analysis supporting these duality rules can be found in Chan Hong-Mo, M. I. Miettinen, and R. G. Roberts, Rutherford report, 1972 (unpublished).

[5]J. V. Allaby *et al.*, contribution to the Fourth International Conference on High Energy Collisions, Oxford, 1972; and CERN report, 1972 (unpublished).

[6]The details of its derivation in a slightly different case can be found in Ref. 1.

[7]H. D. I. Abarbanel, G. F. Chew, M. L. Goldberger, and L. M. Saunders, Phys. Rev. Lett. 26, 937 (1971).

[8]Local duality works essentially as in Ref. 1, where a different sum rule was considered.

[9]G. F. Chew, Phys. Rev. D 7, 3525 (1973).

[10]For $|t| < 0.8$ GeV^2, our values of $g_P(t)$ are smaller than the upper limit for this quantity obtained by Rajaraman under very different assumptions. However, our $g_P(t)$ is much flatter and the two curves cross over at $t \sim -0.8$ GeV^2. See R. Rajaraman, Phys. Rev. Lett. 27, 693 (1971).

[11]If some preliminary results on the total proton-proton cross section at the ISR were confirmed, Eq. (5) should be replaced by an inequality with the left-hand side larger than the right-hand side (G. F. Chew, private communication). The value $\alpha_P'(0) \sim 0.3$ GeV^{-2} would then give an upper limit for the Pomeron slope.

[12]This remark arose during a conversation with M.-S. Chen and M. Kugler. It will be discussed further in a forthcoming publication by M.-S. Chen and the author.

[13]See R. Dolen, D. Horn, and C. Schmid, Phys. Rev. 166, 1768 (1968), for a similar situation in pion-nucleon charge exchange.

[14]I thank V. Rittenberg for a useful conversation on this point.