ANL/HEP 7139, 1971 (unpublished).
[10]W. D. Shepard, J. T. Powers, N. N. Biswas, N. M. Cason, V. P. Kenney, R. R. Riley, D. W. Thomas, J. W. Elbert, and A. R. Erwin, Phys. Rev. Lett. 27, 1164 (1972). There are also data with somewhat lower statistics from D. J. Crennell *et al.*, Phys. Rev. Lett. 28, 643 (1972).
[11]S. Stone, T. Ferbel, P. Slattery, and B. Werner, Phys. Rev. D 5, 1621 (1972).
[12]Winston Ko and Richard L. Lander, Phys. Rev. Lett. 26, 1064 (1971).
[13]J. V. Allaby *et al.*, CERN report (unpublished); and A. N. Diddens, private communication.
[14]E. Flaminio, J. D. Hansen, D. R. O. Morrison, and N. Tovey, CERN Report No. CERN/HERA 70-3, 1970 (unpublished).
[15]E. L. Berger, B. Y. Oh, and G. A. Smith, Phys. Rev. Lett. 29, 675 (1972).
[16]K. Fialkowski, K. Rybicki, and R. Witt, Nucl. Phys. B44, 509 (1972).
[17]G. R. Charlton *et al.*, Phys. Rev. Lett. 29, 515 (1972).
[18]$\Gamma \simeq 0$, however, if one uses the asymptotic cross section $\sigma_{tot}(\infty) = 40$ mb, instead of the cross section at 3.5 GeV/c as a normalization.
[19]C. Quigg, Jiunn-Ming Wang, and Chen Ning Yang, Phys. Rev. Lett. 28, 1290 (1972).
[20]Winston Ko, Phys. Rev. Lett. 28, 935 (1972).
[21]J. V. Beaupre *et al.*, Phys. Lett. 40B, 510 (1972).
[22]M. Kugler and R. G. Roberts, Weizmann Inst. Report No. WIS 71/51, 1971 (unpublished).
[23]Note however that the peak vanishes with the normalization $\sigma_{tot}(\infty)$; see also Ref. 18.
[24]W. D. Shepard, J. T. Powers, N. N. Biswas, N. M. Cason, V. P. Kenney, and D. W. Thomas, Phys. Rev. Lett. 28, 703 (1972).
[25]T. F. Hoang, D. Rhines, and W. A. Cooper, Phys. Rev. Lett. 27, 1681 (1971).

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$\bar{p}p$ Elastic Scattering at 2.85 GeV/c

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$\bar{p}p$ elastic scattering at an incident beam momentum of 2.85 GeV/c is analyzed using 18 412 events. The simple exponential parametrization of the diffraction peak is found to be a poor representation of the data. Two other parametrizations are tried and the estimates of $d\sigma/dt$ at $t=0$ and of the slope of the diffraction peak are found to differ significantly between various parametrizations. It is found that two coherent interfering exponentials are able to represent the differential cross section over the range $0.04 \le |t| \le 1.8$ (GeV/c)^2 with a χ^2 probability of approximately 40%.

In elastic scattering the parametrization can be important in obtaining an estimate of the differential cross section at $t=0$. Above 1.5 GeV/c incident beam momentum, the parametrization which has normally been used in $\bar{p}p$ elastic scattering represents a limited region of the diffraction peak with a simple exponential:

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt}\right)_0 e^{bt} . \quad (1)$$

In one of the experiments, Parker *et al.*[1] concluded after studying this reaction from 1.5 to 2.9 GeV/c that " ...no significant consistent improvement in the fits is realized by allowing curvature in the very low-t region." In a more recent experiment, Ambats *et al.*[2] include a curvature term; that is, they use exp $(bt + ct^2)$ in analyzing their data from 3 to 6 GeV/c.

We have measured the differential cross section for $\bar{p}p$ elastic scattering at 2.85 GeV/c using the 31-in. bubble chamber and a separated beam at the Brookhaven AGS. The present analysis is based on 18 412 events which are in a limited fiducial volume and which satisfy the elastic scattering hypothesis with a χ^2 less than 12. The resulting differential cross sections are presented in Table I for the t range $0.04 \le |t| \le |t_{max}|$ in (GeV/c)^2, where $t_{max} \approx -4.0$ (GeV/c)^2. The data for the diffraction peak region, that is, $0.04 \le |t| \le 0.45$ (GeV/c)^2, have been plotted in Fig. 1. For $|t| < 0.240$ (GeV/c)^2 the bin size used is 0.0050 (GeV/c)^2, which is significantly larger than the resolution. At larger t the bin size has been increased to keep the statistical errors small.

An attempt to fit the limited t range $0.04 \le |t| \le 0.25$ (GeV/c)^2 to Eq. (1) was made. The resulting fit is relatively poor with a χ^2/ν of 1.39 or a χ^2 probability of approximately 5%. It should be noted, however, that the resulting parameters agree quite well with those given for this parametrization for data from other experiments at nearby energies. Another indication of the poor fit is the variation in the slope parameter, and the intercept as the t range of the data used in the determination

TABLE I. The values of $d\sigma/dt$ measured in this experiment. The errors listed are the point-to-point errors and do not include the uncertainty in normalization which is 2.2%.

$-t_{min}$ [(GeV/c)^2]	$-t_{max}$ [(GeV/c)^2]	$d\sigma/dt$ [mb/(GeV/c)^2]	Error	$-t_{min}$ [(GeV/c)^2]	$-t_{max}$ [(GeV/c)^2]	$d\sigma/dt$ [mb/(GeV/c)^2]	Error
0.040	0.045	171.8	6.2	0.280	0.290	6.82	0.88
0.045	0.050	167.2	6.2	0.290	0.300	2.95	0.58
0.050	0.055	144.0	5.7	0.300	0.310	4.66	0.73
0.055	0.060	140.4	5.6	0.310	0.320	3.07	0.59
0.060	0.065	138.6	5.6	0.320	0.330	2.95	0.58
0.065	0.070	126.8	5.4	0.330	0.340	2.61	0.54
0.070	0.075	126.3	5.4	0.340	0.350	1.48	0.41
0.075	0.080	116.3	5.1	0.350	0.375	1.14	0.23
0.080	0.085	112.2	5.1	0.375	0.400	0.55	0.16
0.085	0.090	101.6	4.8	0.400	0.450	0.75	0.13
0.090	0.095	97.9	4.7	0.450	0.500	0.68	0.12
0.095	0.100	87.7	4.5	0.500	0.550	0.82	0.14
0.100	0.105	89.7	4.5	0.550	0.600	0.82	0.14
0.105	0.110	82.5	4.3	0.600	0.650	1.07	0.16
0.110	0.115	82.3	4.3	0.650	0.700	1.64	0.19
0.115	0.120	71.1	4.0	0.700	0.750	1.20	0.17
0.120	0.125	66.1	3.9	0.750	0.800	1.52	0.19
0.125	0.130	61.8	3.7	0.800	0.850	1.32	0.17
0.130	0.135	60.4	3.7	0.850	0.900	1.18	0.16
0.135	0.140	53.2	3.5	0.900	0.950	0.95	0.15
0.140	0.145	49.1	3.3	0.950	1.000	0.70	0.13
0.145	0.150	50.7	3.4	1.000	1.050	0.82	0.14
0.150	0.155	42.7	3.1	1.05	1.10	0.52	0.11
0.155	0.160	42.9	3.1	1.10	1.15	0.66	0.12
0.160	0.165	37.3	2.9	1.15	1.20	0.39	0.09
0.165	0.170	35.9	2.9	1.20	1.30	0.364	0.064
0.170	0.175	31.4	2.7	1.30	1.40	0.352	0.063
0.175	0.180	29.8	2.6	1.40	1.50	0.239	0.052
0.180	0.185	26.4	2.4	1.50	1.60	0.216	0.050
0.185	0.190	30.7	2.6	1.60	1.70	0.148	0.041
0.190	0.195	25.7	2.4	1.70	1.80	0.114	0.036
0.195	0.200	25.5	2.4	1.80	1.90	0.181	0.043
0.200	0.205	20.2	2.1	1.90	2.00	0.161	0.040
0.205	0.210	18.9	2.1	2.00	2.10	0.121	0.035
0.210	0.215	16.8	2.0	2.10	2.20	0.141	0.038
0.215	0.220	18.9	2.1	2.20	2.40	0.051	0.016
0.220	0.225	14.8	1.8	2.40	2.60	0.081	0.020
0.225	0.230	14.3	1.8	2.60	2.80	0.071	0.019
0.230	0.235	11.8	1.6	2.80	3.00	0.055	0.017
0.235	0.240	13.0	1.7	3.00	3.20	0.020	0.010
0.240	0.250	9.66	1.05	3.20	3.40	0.010	0.007
0.250	0.260	9.66	1.05	3.40	3.60	0.010	0.007
0.260	0.270	8.52	0.98	3.60	3.80	0.015	0.009
0.270	0.280	6.93	0.89	3.80	4.00	0.005	0.005

is varied. The data from 0.04 $(\text{GeV}/c)^2 \leq |t| \leq |t'|$ have been fitted to determine the best value of b and $(d\sigma/dt)_0$ in Eq. (1); $|t'|$ was varied over the range $0.215 \leq |t'| \leq 0.35$ $(\text{GeV}/c)^2$. That value of $(d\sigma/dt)_0$ is plotted against t' in Fig. 2(a) and b is plotted against t' in Fig. 2(b). The reduced χ^2, χ^2/ν, is plotted versus t' in Fig. 2(c).

Since the variation of the parameters of the fit with the exact t range used could be taken as indicative of a poor parametrization, it was decided to attempt to fit the data with other parametrizations as well. The form

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt}\right)_0 \exp(bt + ct^2) \qquad (2)$$

was fitted to the data over the range $0.04 \leq |t| \leq |t'|$,

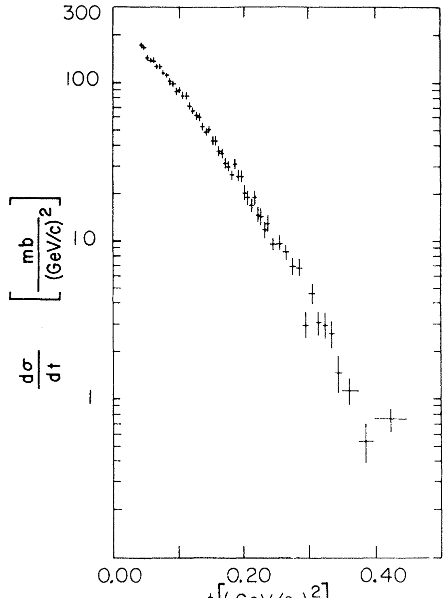

FIG. 1. The differental cross section in the range $|t| < 0.45$ (GeV/c)^2 for $\bar{p}p$ elastic scattering at an incident laboratory momentum of 2.85 GeV/c.

and $|t'|$ was varied from 0.215 to 0.35 (GeV/c)^2 as before. An excellent fit was achieved ($\chi^2/\nu \sim 0.6$ for between 35 and 51 data points) whose parameters were rather insensitive to the particular t range used. The behavior of the parameters and

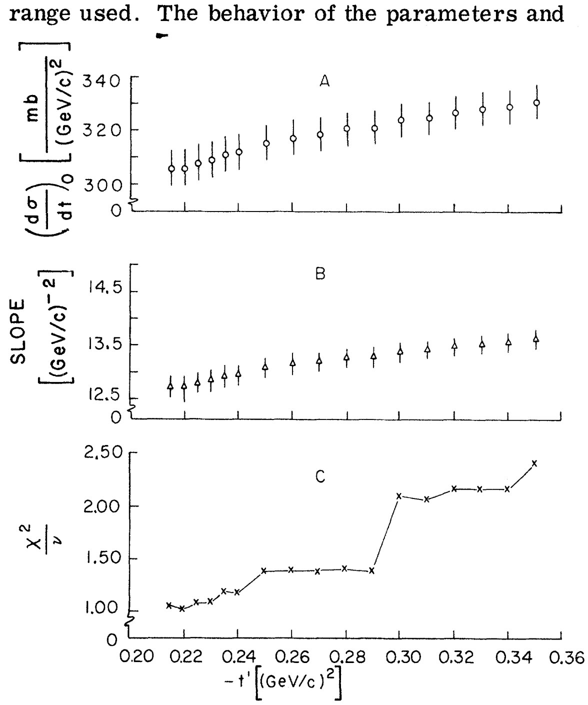

FIG. 2. The results of fitting Eq. (1) to the data in the range 0.04 (GeV/c)$^2 \leq |t| \leq |t'|$ versus the value of t'. (A) The value of the intercept $(d\sigma/dt)_0$. (B) The slope parameter b. (C) The reduced χ^2.

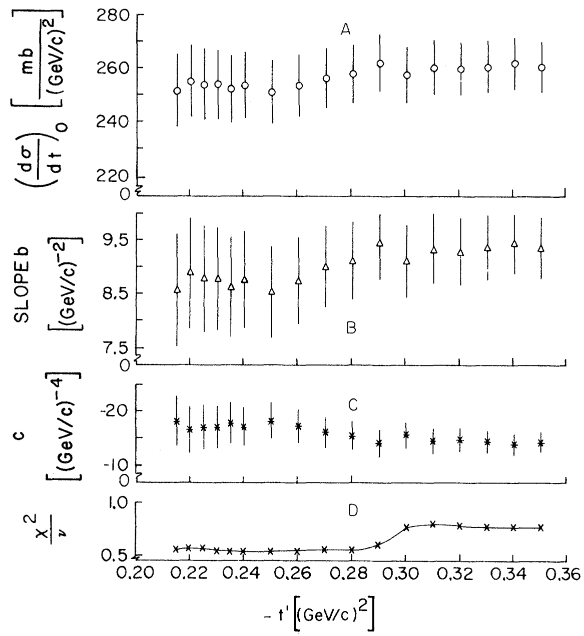

FIG. 3. The results of fitting Eq. (2) to the data in the range 0.04 (GeV/c)$^2 \leq |t| \leq |t'|$ versus the value of t'. (A) The value of the intercept $(d\sigma/dt)_0$. (B) The "slope" parameter b. (C) The "curvature" parameter c. (D) The reduced χ^2.

of χ^2/ν is plotted in Fig. 3. For $t' = -0.215$ (GeV/c)^2 the results of the fit are presented, along with the results from other parametrizations, in Table II. The points to note are that the estimated value of $(d\sigma/dt)_0$ changes significantly between parametrizations, and the slope b is markedly different.

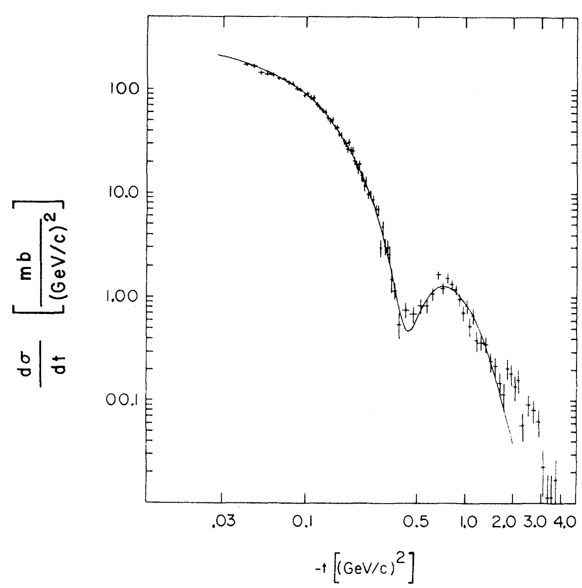

FIG. 4. The differential cross section plotted versus t. The solid line is the result of fitting Eq. (3) to the data.

TABLE II. Results of fits to $\bar{p}p$ elastic differential cross section at 2.85 GeV/c.

	Equation (1)	Equation (2)	Equation (3)
Parametrization	$\left(\frac{d\sigma}{dt}\right)_0 e^{bt}$	$\left(\frac{d\sigma}{dt}\right)_0 e^{bt+ct^2}$	$\left(\frac{d\sigma}{dt}\right)_0 \frac{\lvert e^{(b_1t)/2}+\lvert A\rvert e^{i\varphi}e^{(b_2t)/2}\rvert^2}{\lvert 1+\lvert A\rvert e^{i\varphi}\rvert^2}$
$\left(\frac{d\sigma}{dt}\right)_0$	305.9 ± 7.0	251.5 ± 13.8	292.7 ± 5.5 [a]
b or b_1	12.70 ± 0.21	8.5 ± 1.1	9.24 ± 0.38
c or b_2	···	−18.2 ± 4.6	3.83 ± 0.30
$\lvert A\rvert$	···	···	0.319 ± 0.045
φ (degrees)	···	···	168.0 ± 2.1
t range of fit	$0.04 \le \lvert t\rvert \le 0.215$	$0.04 \le \lvert t\rvert \le 0.215$	$0.04 \le \lvert t\rvert \le 1.8$
No. of data points	35	35	75
χ^2/ν	1.06	0.56	1.04

[a] This error does not include the 2.2% normalization uncertainty.

Two coherent, interfering exponentials which correspond to the equation

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt}\right)_0 \frac{\lvert e^{(b_1t)/2} + \lvert A\rvert e^{i\varphi}e^{(b_2t)/2}\rvert^2}{\lvert 1+\lvert A\rvert e^{i\varphi}\rvert^2} \qquad (3)$$

were also fitted to the data, but over the t range $0.04 \le \lvert t\rvert \le 1.8$ (GeV/c)^2. The form given in Eq. (3) is very similar to that used by Kalbfleisch *et al.*[3] from 1.0 to 1.5 GeV/c. Over the large t range used an acceptable fit was achieved, with a χ^2 of 72.6 for 75 data points and five parameters. This corresponds to a χ^2/ν of 1.04 or a χ^2 probability of approximately 40%. The results of this fit are shown in Fig. 4 along with the data used in the determination. There is apparently structure beyond $t = -1.8$ (GeV/c)^2, but no attempt to parametrize that region has been made. The log-log plot was chosen simply to compress the scale for presentation. For this parametrization $(d\sigma/dt)_0$ is more consistent with the value estimated from Eq. (1) than with the value estimated from Eq. (2). The positions of the minimum and secondary maximum corresponding to this fit are 0.443 ± 0.004 (GeV/c)^2 and 0.727 ± 0.008 (GeV/c)^2, respectively.

In summary, if the accuracy of the data is sufficiently high, the question of curvature becomes important. Estimates of $(d\sigma/dt)_0$, on which calculations of real to imaginary ratios of the amplitudes at $t = 0$ can be based, and the determination of the slope b are, at least in part, dependent on a reasonable solution of an acceptable parametrization to use.

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[1] D. L. Parker, B. Y. Oh, G. A. Smith, and R. J. Sprafka, Nucl. Phys. B32, 29 (1971).

[2] I. Ambats, D. S. Ayres, R. Diebold, A. F. Greene, S. L. Kramer, A. Lesnik, D. R. Rust, C. E. W. Ward, A. B. Wicklund, and D. D. Yovanovitch, Phys. Rev. Lett. 29, 1415 (1972).

[3] G. R. Kalbfleisch, R. C. Strand, and V. Vanderburg, Nucl. Phys. B30, 466 (1971).