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Threshold Enhancements in Hadronic Reactions*

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A resonance-dominance picture of reactions is shown to lead to threshold enhancements in angle-integrated cross sections under a random-sign or oscillating-sign condition for coupling constants. Formulas are derived for threshold-enhanced reaction rates. Application is made to several reactions and compared to measured cross sections.

I. INTRODUCTION

Threshold enhancements are seen in many hadronic reactions. A description of these enhancements in production processes has recently been given[1] in terms of an incoherent sum of a statistically dense set of resonances with partial widths which are inversely proportional to the density of resonances. From duality,[2] we have learned to expect that resonance-dominated amplitudes have high-energy tails, characteristic of Regge behavior, decreasing with increasing s as an inverse power of s. The threshold enhancements on the other hand are characterized by much more rapid decrease with increasing s, after the initial rise from threshold.

An example of a reaction showing Regge behavior is $\bar{p}p \to \bar{n}n$. The reaction $\bar{p}p \to \pi^+\pi^-$, on the other hand, falls exponentially with increasing s up to $s = 8$ GeV^2. This reaction is of course exothermic and so the rise from threshold is not seen. In this paper an explanation of the different behavior of these reactions will be given.

We examine here classes of reactions with a view to understanding under what conditions strong threshold enhancements are present or absent. Specifically, we consider reactions of the types

$$a + b \to c + M, \quad M \to d + e, \tag{A}$$

$$a + b \to M, \quad M \to c + d. \tag{B}$$

In the above a and b are stable particles and c, d, and e are stable particles or single resonances with well-defined quantum numbers. The reaction product M is a hadronic state of mass M which is in general a superposition of resonances with different quantum numbers. Diagrams corresponding to reactions (A) and (B) are shown in Fig. 1.

The reactions above include many cases where threshold enhancements of substantial proportions are seen. In fact, threshold enhancements are seen generally if a reaction fits into class (A). Examples treated in Ref. 1 are the strong enhancements in the region of the A_1, A_3, Q, and L structure and N^* enhancements. For reactions of class (B), however, strong enhancements may or may not exist. Reactions of type (B), we will see, will not show strong threshold enhancements in elastic reactions or reactions which are simply related to elastic scattering amplitudes, say, through symmetry considerations. For example, in πp scattering the charge exchange reaction $\pi^- p \to \pi^0 n$ has an amplitude simply related to $\pi^- p$ and $\pi^+ p$ elastic scattering by charge independence. Another reaction not showing strong enhancement at low energies is $\bar{p}p \to \bar{n}n$, which is simply related to $\bar{p}p$ and $\bar{n}n$ elastic scattering by charge independence. More generally, using SU(3) symmetry one can relate[3] a wide class of amplitudes to the nondiffractive component of elastic scattering of pions from nucleons. All these related amplitudes do not show strong threshold enhancement.

On the other hand, the reaction $\bar{p}p \to \pi^+\pi^-$ is not simply related to forward elastic scattering, and this reaction, as well as some related amplitudes, shows a strong threshold enhancement. We will relate this behavior to randomness of sign or rapid-sign oscillations of the ratio of the coupling constants at vertices 1 and 2 in Fig. 1.

There is of course no fundamental difference between reactions related to elastic scattering and others. The differences which exist are merely quantitative, not qualitative. Generally some enhancement near threshold exists.

In Sec. II we present a picture of reaction amplitudes in terms of resonance superposition. We show the relevance of coupling-constant oscillation to strong threshold enhancement. Section III contains formulas for threshold enhancements obtained from the considerations of Sec. II and the statistical bootstrap. Using these formulas, we compare with data for a number of reactions of type (A) and (B). Conclusions are given in Sec. IV.

II. RESONANCE-SUPERPOSITION PICTURE OF REACTION AMPLITUDES

We consider a class of reactions

$$a+b\to c+d \tag{1}$$

which proceed entirely through s-channel resonance formation. Then, neglecting spins, we write the amplitude for such a process as a sum of Breit-Wigner amplitudes:

$$f_{ab,cd}^{(E,\theta)}=-\frac{1}{2k}\sum_l(2l+1)P_l(\cos\theta)\sum_j\frac{\gamma_{ab}^{jl}\gamma_{cd}^{jl}}{E-E_{jl}+\frac{1}{2}i\Gamma^{jl}}, \tag{2}$$

where γ_{ab}^{jl} is the square root of the width for elastic scattering of a by b for the resonance of mass E_{jl} with angular momentum l. Γ^{jl} is the total width of the level E_{jl}; k and E are the incident c.m. momentum and energy, respectively. The square root of the partial widths, γ_{ab}^{jl}, may be either positive or negative, being coupling constants.

We have then, for the integrated cross section for reaction (1),

$$\sigma_{ab,cd}=\int|f_{ab,cd}|^2d\Omega$$
$$=\frac{\pi}{k^2}\sum_l(2l+1)\times\sum_{j,m}\frac{\gamma_{ab}^{jl}\gamma_{cd}^{jl}\gamma_{ab}^{ml}\gamma_{cd}^{ml}}{(E-E_{jl}+\frac{1}{2}i\Gamma^{jl})(E-E_{ml}-\frac{1}{2}i\Gamma^{ml})} \tag{3}$$

$$=\frac{\pi}{k^2}\sum_l(2l+1)\sum_j\frac{\Gamma_{ab}^{jl}\Gamma_{cd}^{jl}}{(E-E_{jl})^2+\frac{1}{4}(\Gamma^{jl})^2}+\text{terms with } j\neq m\,. \tag{3'}$$

In (3′), $\Gamma_{ab}^{jl}=(\gamma_{ab}^{jl})^2$ is the partial width for elastic scattering of a by b through the resonance level E_{jl}.

If reaction (1) corresponds to the resonant part of elastic scattering, i.e., if $\gamma_{ab}^{jl}=\gamma_{cd}^{jl}$, then

$$f_{ab,ab}^{(E,\theta)}=-\frac{1}{2k}\sum_l(2l+1)\sum_j\frac{\Gamma_{ab}^{jl}P_l(\cos\theta)}{E-E_{jl}+\frac{1}{2}i\Gamma^{jl}}\,. \tag{4}$$

Then in the forward direction we have an amplitude of the form

$$f_{ab,ab}^{(E,0)}=-\frac{1}{2k}\sum_l(2l+1)\sum_j\frac{\Gamma_{ab}^{jl}}{E-E_{jl}+\frac{1}{2}i\Gamma^{jl}} \tag{5}$$

$$\approx-\frac{1}{2k}\int\frac{\Gamma_{ab}(E_j)\rho(E_j)dE_j}{E-E_j+\frac{1}{2}i\Gamma(E_j)}, \tag{5'}$$

where we have assumed that the density of resonances is high enough that we can replace the sums in (5) by the integral in (5′). Here $\rho(E)$ is the density of resonances of all angular momenta of mass E. According to average duality principles[2] the amplitude $f(E,0)$ at high energy has Regge behavior, with

$$\mathrm{Im} f(E,0)\sim E^{2\alpha(0)}, \tag{6}$$

where $\alpha(0)$ is the intercept of the highest Regge trajectory that can be exchanged in the t channel.

The condition (6) is satisfied to within powers of E if

$$\Gamma_{ab}(E)\propto\frac{1}{\rho(E)}\,. \tag{7}$$

The exact power-law behavior of (5) is related to the energy dependence of Γ_{total}.

We now return to reaction (1) not including the elastic scattering case. We consider the sum corresponding to terms $j=m$ in formula (3′).

Again converting sums to an integral, we have

$$\sigma_{ab,cd}^{D}\equiv\frac{\pi}{k^2}\sum_l(2l+1)\sum_j\frac{\Gamma_{ab}^{jl}\Gamma_{cd}^{jl}}{(E-E_{jl})^2+\frac{1}{4}(\Gamma^{jl})^2}$$
$$\approx\frac{\pi}{k^2}\int\frac{\Gamma_{ab}(E_j)\Gamma_{cd}(E_j)\rho(E_j)dE_j}{(E-E_j)^2+\frac{1}{4}\Gamma^2(E_j)}, \tag{8}$$

and for Γ small enough

$$\sigma_{ab,cd}^{D}\approx\frac{\pi}{k^2}\frac{2\pi}{\Gamma(E)}\Gamma_{ab}(E)\Gamma_{cd}(E)\rho(E)\,. \tag{8'}$$

We make the approximation in (8′) that the integrand is peaked at the peak in the Breit-Wigner component. This is exact in the narrow-width approximation. It is expected to be a good approximation, as the width of a threshold enhancement[1] is typically 600 MeV whereas Γ is typically $\sim m_\pi$.

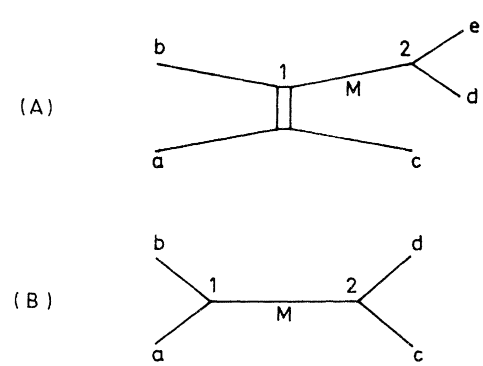

FIG. 1. Two types of processes in which threshold enhancements are seen in the distributions in the energy variable M. In type (A) resonances of mass M are produced by Pomeranchukon or Regge exchange and subsequently decay into a two-body final state. Type-(B) processes occur when resonances are formed and decay into the $d+e$ channel.

In formula (8′) the $\Gamma(E)$ are average values of the $\Gamma(E_i)$ at $E_i = E$. To the extent that widths and spacings fluctuate from their mean values, there will be, in addition, a fluctuating component to (8′). Using (7) and assuming that the total width is a slowly varying function of E, we find that the contribution of terms with $j = m$ to $\sigma_{ab,cd}$ is approximately

$$\sigma^D_{ab,cd} \propto \frac{1}{\rho(E)} . \tag{9}$$

Provided the terms with $j \neq m$ are not important, we thus find that reaction (1) will rise from threshold and then fall as $\rho^{-1}(E)$. According to the statistical bootstrap model,[4]

$$\rho(E) \simeq \frac{A}{E^3} \exp\left(\frac{E}{m_\pi}\right), \tag{10}$$

so that one finds a cross section which falls very rapidly according to (9) after the initial rise from threshold.

Whether or not this behavior dominates then depends on the relative strength of $j = m$ and $j \neq m$ terms in Eq. (3′). As discussed in the Introduction, those reactions which are simply related to elastic scattering will have strong contributions from $j \neq m$ terms which will lead to Regge behaved amplitudes as in Eqs. (5′) and (6). The reason for this domination of $j \neq m$ terms is that in an energy interval from E to $E + dE$, the number of $j = m$ terms is proportional to $\rho(E)$ while the number of $j \neq m$ terms is proportional to $\rho^2(E)$. In cases where cancellations do not occur among the $j \neq m$ terms, the extra power of $\rho(E)$ leads to power behavior instead of a threshold enhancement. However, in other cases, e.g., $\bar{p}p \to \pi\pi$, there is no direct relationship with *forward* elastic scattering. Here, there is a strong threshold enhancement and the $j \neq m$ terms must therefore contribute relatively little. *This requires a randomness of sign between the couplings γ^{jl}_{ab} and γ^{jl}_{cd} as we go from resonance to resonance, or at least rapid oscillations in the relative sign.* Random sign in reduced widths is well known in nuclear reactions.[5,6]

III. APPLICATIONS TO PRODUCTION PROCESSES

According to the above, strong threshold enhancements may occur in reactions which can be represented by a sum of resonances, except elastic scattering and those reactions which are related to elastic scattering by symmetry considerations. First consider type-(A) reactions of the form $a + b \to c + M$, with $M \to d + e$. Reactions of this type are depicted graphically in Fig. 1(a). The cross section for these processes is[1]

$$\frac{d\sigma}{dM^2} \simeq \frac{1}{\lambda(s, m_a{}^2, m_b{}^2)} \int dt\, R(s,t,M) \sum_{l,l',j,m} [(2l+1)(2l'+1)]^{1/2} \int \frac{p}{M}\, d\Omega\, \frac{P_l(\cos\theta) P_{l'}(\cos\theta) \gamma^{ml}_{aR} \gamma^{jl'}_{de} \gamma^{jl'}_{aR} \gamma^{ml}_{de}}{(M - E^l_m + \frac{1}{2} i \Gamma^{lm})(M - E^{l'}_j - \frac{1}{2} i \Gamma^{jl'})} . \tag{11}$$

Here $\gamma^{ml}_{aR} R^{1/2}$ is the magnitude of the exchange amplitude and s is the square of the center-of-mass energy. t is the momentum transfer from a to M. p and Ω are the momentum and angular coordinates of particle d in the rest frame of M. γ^{ml}_{aR} is the coupling constant at the vertex 1 in Fig. 1(a) and $\gamma^{jl'}_{de}$ is the coupling constant at vertex 2. The coupling constants may be positive or negative. Let us assume that the terms with $j \neq m$ in Eq. (11) have random signs and there is therefore much cancellation. The sum will then be dominated by the terms with $j = m$. Doing the angular integration and dropping terms with $j \neq m$ gives

$$\frac{d\sigma}{dM^2} \simeq \frac{1}{\lambda(s, m_a{}^2, m_b{}^2)} \int dt\, R(s,t,M) \frac{p}{M} \sum_{j,l} (2l+1) \frac{\Gamma^{jl}_{aR} \Gamma^{jl}_{de}}{(M - E_{jl})^2 + \frac{1}{4}(\Gamma^{jl})^2} , \tag{12}$$

or, replacing the sum by an integral and performing the sum over l,

$$\frac{d\sigma}{dM^2} \simeq \frac{1}{\lambda(s, m_a{}^2, m_b{}^2)} \int dt\, R(s,t,M) \times \int dM' \frac{p}{M} \frac{\Gamma_{aR} \Gamma_{de} \rho(M')}{(M - M')^2 + \frac{1}{4}\Gamma^2} . \tag{13}$$

Approximating the integral over M' by its value at the peak of the Breit-Wigner form gives

$$\frac{d\sigma}{dM^2} \simeq \frac{\pi}{\lambda(s, m_a{}^2, m_b{}^2)} \times \int dt\, R(s,t,M) \frac{4p}{M} \frac{\Gamma_{aR}(M) \Gamma_{de}(M) \rho(M)}{\Gamma(M)} . \tag{14}$$

Γ_{de} is the partial width for the $d + e$ channel. In order to get an explicit expression for Γ_{de}, we note that the sum of partial widths is equal to the total width. We have then

$$\Gamma(M)=\sum_{k=2}^{\infty}\frac{1}{k!}\prod_{l=1}^{k}\int\frac{d^3p_l}{2E_l}dM_l\rho(M_l)\delta(M-\sum E_l)\times\delta^3(\sum\vec{p}_l)\Gamma(p_1,M_1,\ldots,p_k,M_k). \tag{15}$$

The sum is over 2-, 3-, 4-, ... body final states. $\rho(M_l)$ is the density of single-particle states at mass M_l and $\Gamma(p_1, M_1, \ldots, p_k, M_k)$ is the partial width into the k-body channel labeled by $p_1, M_1, \ldots, p_k, M_k$. The factorial is needed so that each final state is counted only once. The choice for $\Gamma(p_1, M_1, \ldots, p_k, M_k)$ is motivated by the statistical bootstrap[4] in which $\rho(M)$ satisfies the equation

$$\rho(M)=\sum_{k=2}^{\infty}\frac{V^{k-1}}{k!}\prod_{l=1}^{k}\int d^3p_l\,dM_l\rho(M_l)\delta(M-\sum E_l)\times\delta^3(\sum\vec{p}_l), \tag{16}$$

where $(2\pi)^3V$ is the hadronic volume. Let us now write

$$\Gamma(p_1,M_1,\ldots,p_k,M_k)=\frac{V^{k-1}\left[\prod_{l=1}^{k}(2E_l)_{\text{c.m.}}\right]\lambda(M)}{\rho(M)}. \tag{17}$$

The product is over the energies in the rest frame of M of the particles in the final state. Substituting (17) in (15) and using (16) gives $\Gamma(M)=\lambda(M)$. From (17) we have

$$\Gamma_{de}(M,m_d,m_e)=\frac{V\Gamma(M)}{\rho(M)}\,\frac{M^4-(m_d{}^2-m_e{}^2)^2}{M^2}. \tag{18}$$

We do not know the precise form of the coupling at vertex 1 involving the virtual exchange. It will be of the form

$$\Gamma_{aR}(M,m_a,t)=\frac{a(M,m_a,t)}{\rho(M)}, \tag{19}$$

where $a(M, M_a, t)$ is the continuation of $\rho(M)\Gamma_{aR}(M,m_a,m_b)$ from Eq. (18) to virtual masses for Regge exchange. For Pomeranchukon exchange, there is no straightforward method for obtaining $a(M, M_a, t)$.

In the case of diffractive dissociation,

$$P(M)\equiv\frac{1}{\lambda(s,m_a{}^2,m_b{}^2)}\int dt\,R(s,t,M)a(M,m_a,t)\xrightarrow[M^2\gg 1]{s\gg M^2}\frac{A}{M^2}. \tag{20}$$

Then from (14), (18), (19), and (20),

$$\frac{d\sigma}{dM^2}\simeq\frac{2\pi PV\lambda^{1/2}(M^2,m_d{}^2,m_e{}^2)}{\Gamma(M)M^4}\,\frac{[M^4-(m_d{}^2-m_e{}^2)^2]}{\rho(M)}. \tag{21}$$

This equation with P and Γ treated as constants was used in Ref. 1 to describe experimental distributions. Good agreement was obtained for numerous reactions.

We now turn our attention to processes of the form $a+b\to c+d$. As was shown in Sec. II, there is no sizeable threshold enhancement in elastic scattering because all coupling constants appear squared and therefore strong cancellations do not occur. The coherent resonance sum gives Regge behavior, as is well known. For amplitudes related to elastic scattering by symmetry considerations, one will also have Regge behavior. If no relation to elastic scattering exists, the diagonal terms can dominate. Then from Eqs. (8′) and (18),

$$\sigma(s)\approx\frac{2\pi^2}{k^2}\,\frac{[s^2-(m_a{}^2-m_b{}^2)^2][s^2-(m_c{}^2-m_d{}^2)^2]}{s^2}\times\frac{V^2\Gamma}{\rho(\sqrt{s})}. \tag{22}$$

This formula is compared with the experimental data for the reactions $\bar{p}+p\to\pi^++\pi^-$ (see Ref. 7) and $\bar{p}+p\to\rho^-+\pi^+$ (see Ref. 8) in Figs. 2 and 3. Good agreement with the data is obtained in each case. In Fig. 2 the data for $\bar{p}+p\to\bar{n}+n$ are also plotted. Since this reaction is related to $\bar{p}p$ and $\bar{n}n$ elastic scattering by isospin invariance, it is expected to have a high energy behavior given by Regge exchange. The curve through the data is $\sigma(s)\sim s^{-1}$, as expected for ρ exchange with $\alpha_\rho(0)=\frac{1}{2}$.

It is interesting to note the two data points at 5 and 8 GeV in the reaction $\bar{p}+p\to\rho^-+\pi^+$ of Fig. 3. These points are consistent with a high-energy tail following a power law in s consistent with baryon Regge exchange. This would come from the terms in (3) with $j\neq m$ if there were some nonrandom contribution, i.e., a certain amount of coherence. Since this reaction is related by line reversal to $\pi^++p\to\rho^++p$ at *backward* angles, one might get some idea of the size of the high-energy Regge

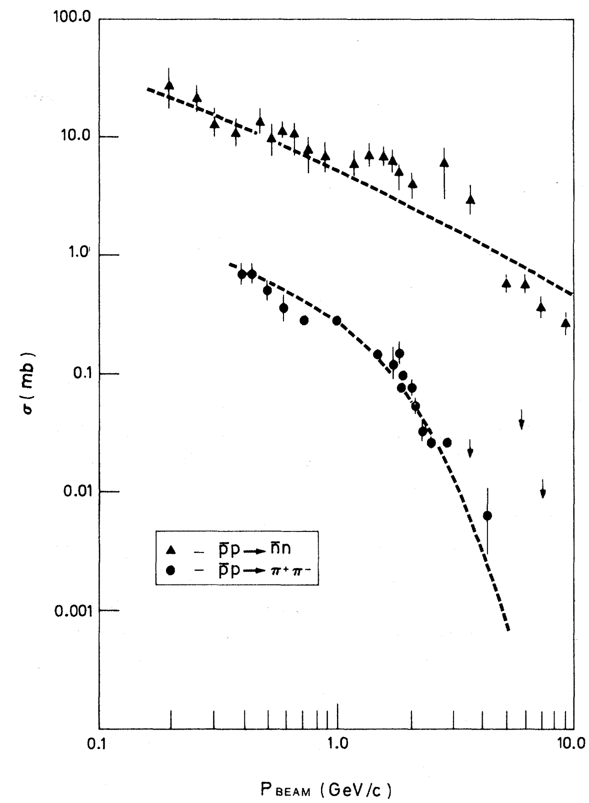

FIG. 2. Cross sections for the reaction $\bar{p}p\to\bar{n}n$ (▲) and $\bar{p}p\to\pi^+\pi^-$ (●) are shown as a function of energy, and their behaviors are very different. The curve through the data for $\bar{p}p\to\bar{n}n$ is given by $5P_{\text{beam}}^{-1}$, which is asymptotically suggestive of ρ exchange. Data for $\bar{p}p\to\pi^+\pi^-$ are compared with Eq. (22) in the text, which is inversely proportional to the density of states and characteristic of threshold enhancements. The theoretical curve is normalized to the data.

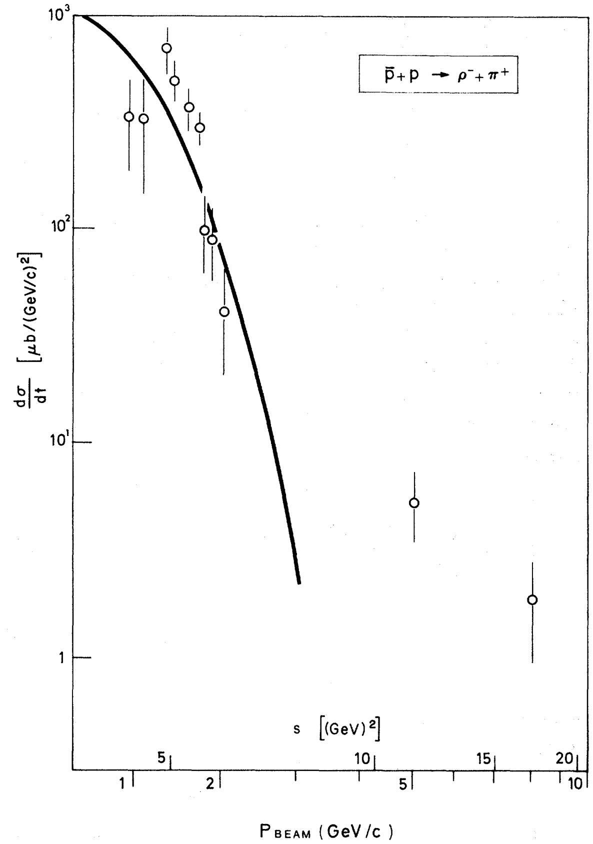

FIG. 3. Data for the reaction $\bar{p}p\to\rho^-\pi^+$ are compared with Eq. (22) in the text. The rapid falloff of the data from 1 to 2 GeV/c is explained by the threshold enhancement mechanism. The high-energy points at 5 and 8 GeV/c are suggestive of a small contribution from coherent interference of different resonances, as explained in the text. The data are essentially isotropic except for the two high-energy points. The theoretical curve is normalized to the data.

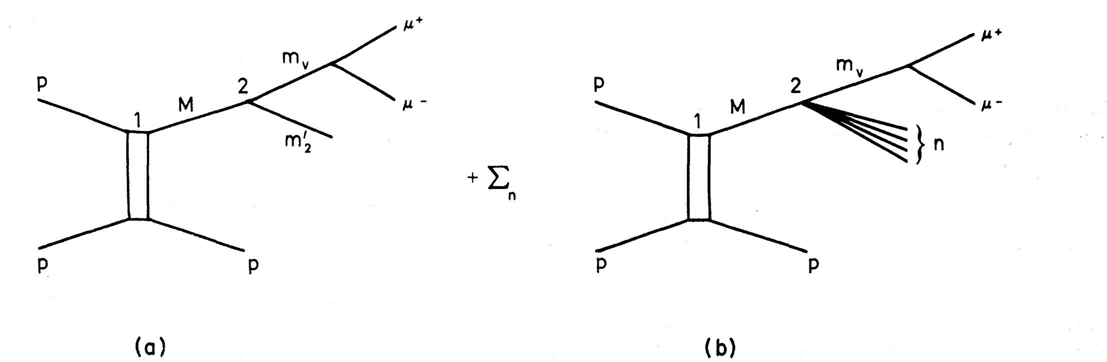

FIG. 4. Dynamical mechanism for the reaction $pp\to\mu^+\mu^-$ +anything is depicted graphically. A massive baryon is produced by Pomeranchukon exchange at vertex 1 and decays into a massive vector boson plus (a) a baryon or (b) a baryon plus other particles. The vector boson then decays by "generalized vector dominance" into a $\mu^+\mu^-$ pair.

tail assuming exchange degeneracy. Similarly, one can relate $\bar{p}+p \to \pi^- + \pi^+$ to backward $\pi^\pm p$ elastic scattering.[9]

Several reactions of class (A) were considered in Ref. 1. These include the threshold enhancements in the A_1, A_3, Q, and L regions. An example of an extension to the type of reactions in class (A), $a+b \to c+M$, $M \to d$ +anything, is given by $p+p \to \mu^+\mu^-$ +anything. The mechanism for producing $\mu^+\mu^-$ pairs is depicted graphically in Fig. 4. At vertex 1, a massive baryon is produced, which decays statistically into a number of constituents. It is assumed that the pairs arise from the decay of vector bosons of mass m_V and that this coupling is a slowly varying function of m_V. There will be a modification in the decay rate from that used in deriving Eq. (21) if the masses of the decay products are large enough (above ~1 GeV) and one considers all resonances produced at that mass. The decay rate $\omega(p_1, m_1, \ldots, p_l, m_l)$ will be the partial width given in Eq. (17) multiplied by the densities of constituents,

$$\omega(p_1, m_1, \ldots, p_k, m_k) = \rho(m_1)\cdots\rho(m_k) \times \Gamma(p_1, m_1, \ldots, p_k, m_k). \tag{23}$$

For the graph of Fig. 4(a) one calculates

$$\frac{d\sigma}{dm_V} \cong \text{constant} \times \int_{m_p+m_V}^{\sqrt{s}-m_p} \frac{d\sigma}{dM} F_V(M, m_V)\, dM\,. \tag{24}$$

$d\sigma/dM$ is the cross section for producing a fireball of mass M, and

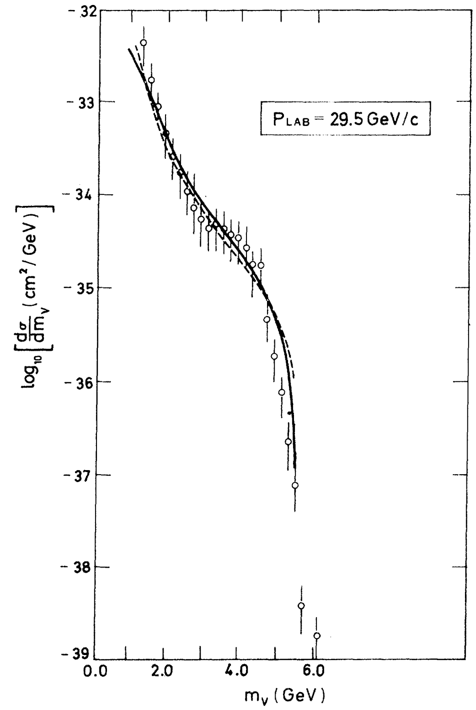

FIG. 5. Experimental data at 29.5 GeV/c for the reaction $pp \to \mu^+\mu^-$ +anything are plotted against the theoretical distribution given in Eq. (25). The solid curve is from an exact treatment of the graph in Fig. 1(a) and the dashed curve is from an approximate treatment of all terms. The shape is the same in both cases, though the scale is different. Effects of t_{min} which would give some energy dependence have not been included in the calculations. The theoretical curves are normalized to the data.

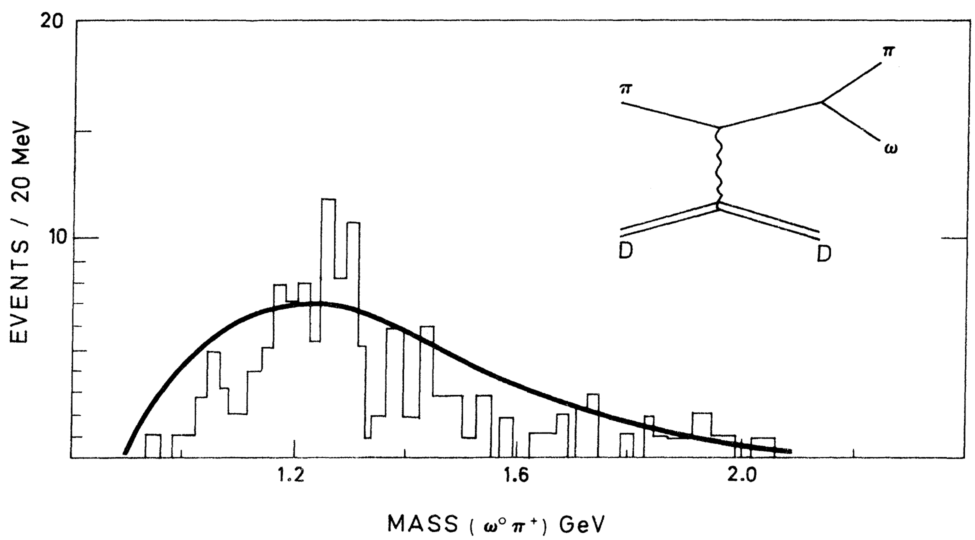

FIG. 6. The $\omega\pi$ mass distribution for the reaction $\pi d \to (\omega\pi)d$ is compared with Eq. (21) in the text. The theoretical curve is normalized to the data.

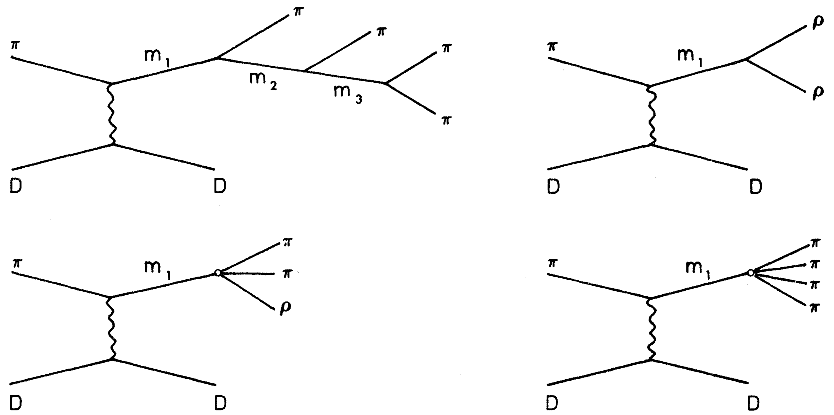

FIG. 7. Possible contributions to the reaction $\pi d \rightarrow (4\pi)d$ are depicted graphically.

$$F_V(M, m_V) = \tfrac{1}{2}\pi V \frac{\rho_V(m_V)}{\rho(M)} \int_{m_p}^{\sqrt{s}-m_p-m_V} dm_2\, \rho(m_2)\lambda^{1/2}(M^2, m_V^2, m_2^2)\left[\frac{M^4-(m_2^2-m_V^2)^2}{M^2}\right], \tag{25}$$

where

$$\lambda(x^2, y^2, z^2) = x^4 + y^4 + z^4 - 2x^2y^2 - 2x^2z^2 - 2y^2z^2.$$

Taking $\rho(M) = AM^{-3}\exp(M/m_\pi)$ (see Ref. 4) and the density of vector bosons $\rho_V(m_V) = cm_V^{-4.5}\exp(m_V/m_\pi)$ (see Ref. 10), the distribution in the μ-pair invariant mass m_V has been calculated. Comparison with experimental data[11] is given in Fig. 5. The higher-order terms of Fig. 4(b) have been estimated in several approximations and they do not alter the shape of the spectrum. For example, if one neglects momentum conservation in the statistical bootstrap, one readily shows

$$F_V(M, m_V) = \frac{\rho_V(m_V)\rho(M-m_V)}{\rho(M)}, \tag{26}$$

which is to be compared with Eq. (25). The form for $d\sigma/dm_V$ resulting from this approximation is also shown in Fig. 5 and agrees closely with that obtained using Eq. (6).

Finally we consider the class-(A) reaction $\pi + d \rightarrow 4\pi + d$. Almost all the reactions of type (A) considered in Ref. 1 involved Pomeranchukon exchange, which is forbidden in this case. Since this reaction is partially ω exchange, the $\pi + d \rightarrow \omega\pi + d$ portion of the data is of interest because it at least partly involves $\omega\pi$ elastic scattering

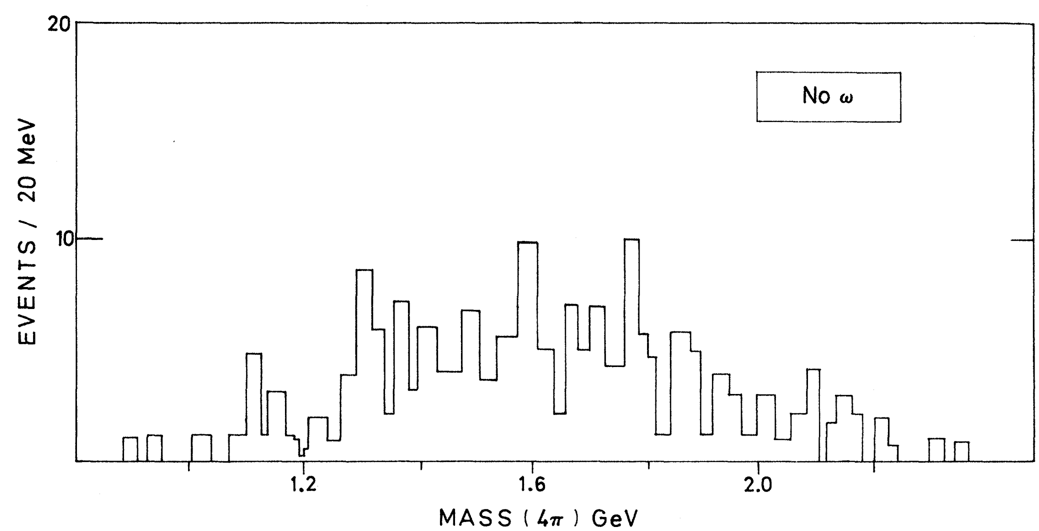

FIG. 8. Data for the reaction $\pi d \rightarrow (4\pi)d$ with no ω are given. Though no fit is given, the broad distribution seen may be explained by some combination of the graphs of Fig. 7.

with a virtual ω. If the sign correlation between the coupling constants at the $\omega\pi$ vertex and the virtual $\omega\pi$ vertex were the same as in elastic scattering, the distribution in the $\omega\pi$ invariant mass would show no strong threshold enhancement according to the results of Sec. II. A threshold enhancement would appear if this sign correlation were to be lost as the ω mass moved off shell. There is clear evidence for a threshold enhancement, the "B enhancement," as seen in Fig. 6. The theoretical curve is calculated using Eq. (21) and compared with the experimental data.[12]

The four-pion distribution with no ω may also be calculated using the decay matrix in Eq. (23). The several branchings which must be considered are shown in Fig. 7. All of these contributions have been calculated,[13] and the distributions in M_1 from the graphs in Fig. 7 are broad and relatively flat. The experimental data shown in Fig. 8 (see Ref. 12) have this same broad, flat behavior, in contrast with the $\omega\pi$ mode.

IV. CONCLUSIONS

We have presented a picture in which resonance dominance produces threshold enhancements in the cases where coupling constants have random signs or at least rapid sign oscillation. To the extent that there is some coherence in the summed Breit-Wigner amplitude, one can expect a high-energy tail with Regge behavior. A Regge tail of some magnitude may be expected for all reactions which permit t-channel Regge-pole exchanges. Certainly for those amplitudes related to forward elastic scattering by symmetries this Regge tail is the dominant behavior. Reactions like $\bar{p}p \rightarrow \bar{\Sigma}^+\Sigma^-$ and backward K^-p scattering, which do not allow non-exotic single exchanges, may have no high-energy inverse-power-law tail, or perhaps a high-power-of-s tail characteristic of two or more Regge exchanges[14] (Regge cuts) or of exotic exchange—if exotic resonances exist.

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