PHYSICAL REVIEW D VOLUME 8, NUMBER 6 15 SEPTEMBER 1973

Goldstone Bosons in Chiral SU(4) × SU(4)

P. Dittner and S. Eliezer
Department of Physics, Imperial College, Prince Consort Road, London SW7, England
(Received 22 January 1973)

We suggest an approximate chiral SU(4)×SU(4) symmetry for the strong Hamiltonian with the vacuum only SU(3)-invariant in the symmetry limit. Two exact sum rules are obtained. Within this scheme both chiral SU(2)×SU(2) and SU(3) are good symmetries.

In recent years the group SU(4)×SU(4) has assumed importance in several different theories of elementary particles.[1–3] Among the theoretical motivations for considering this group, it is worth mentioning the following: First, there exists the possibility of incorporating a quark-lepton symmetry by regarding the four basic leptons as a fundamental representation of SU(4). Then, in recent attempts to include hadrons in unified theories of weak and electromagnetic interactions, SU(4) has been introduced in order to overcome problems of renormalizability and to avoid sizable strangeness-changing ($\Delta S = 1$) neutral currents.[2,3] In addition we can try to use this larger group to shed light on existing ideas on SU(3) and SU(2)×SU(2) symmetries of strong interactions. In this way we may hope to open up the possibility of an integration of symmetries of strong interactions with renormalized theories of electromagnetic and weak interactions.

However, experimentally, an SU(4) theory receives little confirmation, mainly because the existing particles fit into multiplets of SU(3) rather than SU(4). Indeed, there has been only one known event to suggest the existence of a new particle with a new quantum number, namely the observation by Niu *et al.*[4] in cosmic-ray showers; they suggest that this particle (assuming it to be a meson) decayed into $\pi^0\pi^+$ and had a mass of 1.78 GeV and a relatively long lifetime of 2.2×10^{-14} sec.

Previous attempts have been made to regard SU(4) as a symmetry group of strong interactions.[1] But in a straightforward generalization of SU(3) other difficulties arise, for instance with the vector-meson mass formula, and these appear over and above the multiplet structure. The former problems can be eliminated[2] but the main problem of the SU(4) multiplets is a serious drawback unless many new particles are to be discovered.

We suggest that these difficulties can be overcome by still taking SU(4)×SU(4) as an approximate symmetry of the strong Hamiltonian density, $\mathcal{H}$, but allowing the vacuum to be symmetric only under SU(3) in the symmetry limit. Thus our fundamental assumptions are as follows:

(i) $\mathcal{H} = \mathcal{H}_0 + \epsilon\mathcal{H}_{SB}$, (1)

where $\mathcal{H}_0$ is the SU(4)×SU(4)-symmetric part of the Hamiltonian density, and $\mathcal{H}_{SB}$ the symmetry-breaking part. (ii) In Eq. (1), ϵ is a small formal parameter and we assume that it is sufficient to work to first order in ϵ for the purposes of checking the theory. (iii) In the limit of chiral SU(4)×SU(4) symmetry ($\epsilon \to 0$) the vacuum is taken to be *only an SU(3) scalar*. Thus the full chiral group is realized with SU(3) multiplets of particles and a pseudoscalar octet, triplet, and singlet (ϕ_i; $i = 1, \ldots, 15$), and a scalar triplet and singlet (χ_i; $i = 9, \ldots, 15$) of Goldstone bosons.[5] (iv) For $\mathcal{H}_{SB}$ we take the simplest generalization of the model of Gell-Mann, Oakes, and Renner[6] (GMOR), namely,

$$\mathcal{H}_{SB} = u_0 + cu_8 + du_{15}, \tag{2}$$

where the u_j together with v_j ($j = 0, 1, \ldots, 15$) are the scalar and pseudoscalar densities transforming as the $(4, \bar{4}) + (\bar{4}, 4)$ representation of SU(4)×SU(4), i.e.,

$$\begin{aligned} &[Q_i^V, u_j] = i f_{ijk} u_k, \quad [Q_i^V, v_j] = i f_{ijk} v_k, \\ &[Q_i^A, u_j] = -i d_{ijk} v_k, \quad [Q_i^A, v_j] = i d_{ijk} u_k, \end{aligned} \tag{3}$$

where $i = 1, \ldots, 15$ and $j, k = 0, 1, \ldots, 15$. The coefficients f_{ijk} and d_{ijk} are listed in Table I. (v) We can use soft-meson theorems, so that for our Goldstone bosons in the theory the mass-squared matrices are given by[7]

$$f_i f_j m^2{}_{ij} = -\langle 0 | [Q_i^A, [Q_j^A, \epsilon\mathcal{H}_{SB}]] | 0\rangle \quad \text{(no sum on } i \text{ or } j\text{)}, \tag{4}$$

$$F_i F_j M^2{}_{ij} = -\langle 0 | [Q_i^V, [Q_j^V, \epsilon\mathcal{H}_{SB}]] | 0\rangle \quad \text{(no sum on } i \text{ or } j\text{)}, \tag{5}$$

where in Eq. (4) $i, j = 1, \ldots, 15$, and in Eq. (5) $i, j = 9, \ldots, 15$, and f_i and F_i are the appropriate meson decay constants. (vi) We adopt the following particle assignment: $\phi_1, \ldots, \phi_7$ are π and K; ϕ_8 and ϕ_{15} mix to give η and $X(958)$; $\phi_9, \ldots, \phi_{12}$ are ξ and $\bar{\xi}$, the isospin doublets from the new triplet and antitriplet; ϕ_{13} and ϕ_{14} give λ and $\bar{\lambda}$, the isospin-singlet parts of these triplets. For the scalar mesons we assume that χ_{15}, the SU(3)-singlet scalar Goldstone boson, mixes with the eighth component of an octet of scalar bosons ($\chi_1, \ldots, \chi_8$, which are not Goldstone bosons) to give the physical $\eta_N(S^*)$ and ϵ particles. The remaining seven particles in this octet are π_N and κ.

By virtue of assumption (iii) above, u_{15}, as well as u_0, has a nonvanishing vacuum expectation value to zeroth order in ϵ. This is the crucial point which distinguishes our model from previous SU(4) × SU(4) models. Relations between masses are obtained by writing Eq. (4) explicitly for the 15 pseudoscalar mesons. As a first approximation we assume that all the f_i are equal here.[8] The η-X mixing is achieved by diagonalizing the mass matrix (the only nondiagonal term to appear is $m^2_{8,15}$). Solving these equations we obtain the sum rule

$$(m_\eta^2 - m_\pi^2)(m_X^2 - m_\pi^2) = \tfrac{2}{3}(m_K^2 - m_\pi^2)(2m_\eta^2 + 2m_X^2 - m_\pi^2 - 3m_K^2)\,, \quad (6)$$

which is satisfied exactly (with $m_X = 958$ MeV) to within the electromagnetic mass differences. We should like to mention that this sum rule has been obtained previously[2] in an SU(4) model; it is interesting to see that the same sum rule still holds here despite the fact that we have one more unknown, namely $\langle u_{15}\rangle_0$. A further point of interest is that this sum rule holds equally well for linear masses.

Taking the new particle observed by Niu *et al.*[4] to be ξ, the isospin-$\frac{1}{2}$ particle from the SU(3) triplet of pseudoscalar mesons,[9] and inserting its mass into the formulas for the mass matrix [Eqs. (4)], together with the masses of the π, K, η, and X, two sets of solutions are obtained for c, d, and $\langle u_{15}\rangle_0/\langle u_0\rangle_0$:

Solution I:

$$c = -0.065, \quad d = -1.63, \quad \frac{\langle u_{15}\rangle_0}{\langle u_0\rangle_0} = 0.46;$$

Solution II:

$$c = -1.44, \quad d = +0.56, \quad \frac{\langle u_{15}\rangle_0}{\langle u_0\rangle_0} = -1.72\,.$$

Using these results we arrive at two possible values for the mass of the λ, namely, 1.8 GeV and 8.3 GeV. Bearing in mind that these particles are supposed to be Goldstone bosons, the large value of m_λ in the second case leads us to discard the second set of solutions in favor of the first.

It is remarkable to consider our preferred solution from the point of view of the subgroups SU(3) and chiral SU(2)×SU(2) ($\partial_\mu A_i^\mu = 0$ for $i = 1, 2, 3$). Experimentally, besides being a good chiral SU(2) ×SU(2) symmetry (typified by smallness of pion mass, accuracy of Goldberger-Treiman relation, etc.), SU(3) seems to be a good symmetry of nature (e.g., baryon masses, vector masses, etc.) except for the mass splitting of the pseudoscalar octet. In our model we have in general (even for the pseudoscalar mass) a good SU(3) symmetry ($c \ll 1$) as well as SU(2)×SU(2); the difficulty with SU(3) for the pseudoscalars is overcome by having two SU(3) singlets in the Hamiltonian whose contributions partly cancel in the masses, so that the octet and the over-all singlet contribution are of

TABLE I. Nonvanishing values of f_{ijk} and d_{ijk}.

i	j	k	f_{ijk}
1	2	3	1
1	4	7	$\frac{1}{2}$
1	5	6	$-\frac{1}{2}$
1	9	12	$\frac{1}{2}$
1	10	11	$-\frac{1}{2}$
2	4	6	$\frac{1}{2}$
2	5	7	$\frac{1}{2}$
2	9	11	$\frac{1}{2}$
2	10	12	$\frac{1}{2}$
3	4	5	$\frac{1}{2}$
3	6	7	$-\frac{1}{2}$
3	9	10	$\frac{1}{2}$
3	11	12	$-\frac{1}{2}$
4	5	8	$\frac{1}{2}\sqrt{3}$
4	9	14	$\frac{1}{2}$
4	10	13	$-\frac{1}{2}$
5	9	13	$\frac{1}{2}$
5	10	14	$\frac{1}{2}$
6	7	8	$\frac{1}{2}\sqrt{3}$
6	11	14	$\frac{1}{2}$
6	12	13	$-\frac{1}{2}$
7	11	13	$\frac{1}{2}$
7	12	14	$\frac{1}{2}$
8	9	10	$1/2\sqrt{3}$
8	11	12	$1/2\sqrt{3}$
8	13	14	$-(\frac{1}{3})^{1/2}$
9	10	15	$(\frac{2}{3})^{1/2}$
11	12	15	$(\frac{2}{3})^{1/2}$
13	14	15	$(\frac{2}{3})^{1/2}$
i	j	0	0

TABLE I *(Continued)*

i	j	k	d_{ijk}	i	j	k	d_{ijk}
1	1	8	$(\frac{1}{3})^{1/2}$	5	5	8	$-1/2\sqrt{3}$
1	1	15	$(\frac{1}{6})^{1/2}$	5	5	15	$(\frac{1}{6})^{1/2}$
1	4	6	$\frac{1}{2}$	5	9	14	$-\frac{1}{2}$
1	5	7	$\frac{1}{2}$	5	10	13	$\frac{1}{2}$
1	9	11	$\frac{1}{2}$	6	6	8	$-1/2\sqrt{3}$
1	10	12	$\frac{1}{2}$	6	6	15	$(\frac{1}{6})^{1/2}$
2	2	8	$(\frac{1}{3})^{1/2}$	6	11	13	$\frac{1}{2}$
2	2	15	$(\frac{1}{6})^{1/2}$	6	12	14	$\frac{1}{2}$
2	4	7	$-\frac{1}{2}$	7	7	8	$-1/2\sqrt{3}$
2	5	6	$\frac{1}{2}$	7	7	15	$(\frac{1}{6})^{1/2}$
2	9	12	$-\frac{1}{2}$	7	11	14	$-\frac{1}{2}$
2	10	11	$\frac{1}{2}$	7	12	13	$\frac{1}{2}$
3	3	8	$(\frac{1}{3})^{1/2}$	8	8	8	$-(\frac{1}{3})^{1/2}$
3	3	15	$(\frac{1}{6})^{1/2}$	8	8	15	$(\frac{1}{6})^{1/2}$
3	4	4	$\frac{1}{2}$	8	9	9	$1/2\sqrt{3}$
3	5	5	$\frac{1}{2}$	8	10	10	$1/2\sqrt{3}$
3	6	6	$-\frac{1}{2}$	8	11	11	$1/2\sqrt{3}$
3	7	7	$-\frac{1}{2}$	8	12	12	$1/2\sqrt{3}$
3	9	9	$\frac{1}{2}$	8	13	13	$-(\frac{1}{3})^{1/2}$
3	10	10	$\frac{1}{2}$	8	14	14	$-(\frac{1}{3})^{1/2}$
3	11	11	$-\frac{1}{2}$	9	9	15	$-(\frac{1}{6})^{1/2}$
3	12	12	$-\frac{1}{2}$	10	10	15	$-(\frac{1}{6})^{1/2}$
4	4	8	$-1/2\sqrt{3}$	11	11	15	$-(\frac{1}{6})^{1/2}$
4	4	15	$(\frac{1}{6})^{1/2}$	12	12	15	$-(\frac{1}{6})^{1/2}$
4	9	13	$\frac{1}{2}$	13	13	15	$-(\frac{1}{6})^{1/2}$
4	10	14	$\frac{1}{2}$	14	14	15	$-(\frac{1}{6})^{1/2}$
				15	15	15	$-(\frac{2}{3})^{1/2}$
				i	j	0	$(1/\sqrt{2})\delta_{ij}$

the same size.

We conclude with a sum rule for the scalar mesons. Equation (5) gives $M^2{}_{15,15}=0$, and diagonalization of the mass matrix therefore yields $M^2{}_{8,8}=M_{\eta_N}{}^2+M_\epsilon{}^2$. Combining this with the Gell-Mann–Okubo mass formula for the octet, we obtain

$$3(M_\epsilon{}^2+M_{\eta_N}{}^2)=4M_\kappa{}^2-M_{\pi_N}{}^2. \tag{7}$$

By taking[10] $M_{\pi_N}=1$ GeV, $M_{\eta_N}=1$ GeV, and $M_\kappa=1.2$ GeV, M_ϵ is given by Eq. (7) as 0.77 GeV. Thus this sum rule is also in excellent agreement with the commonly accepted values of the scalar masses.

We are grateful to Dr. R. Delbourgo, Professor T. W. B. Kibble, and Professor A. Salam for helpful discussions. One of us (P. D.) would like to acknowledge financial support from the Science Research Council.

[1]D. Amati, H. Bacry, J. Nuyts, and J. Prentki, Nuovo Cimento 34, 1732 (1964); Z. Maki, T. Maskawa, and I. Umemura, Prog. Theor. Phys. 47, 1682 (1971) and references therein.

[2]J. D. Bjorken and S. L. Glashow, Phys. Lett. 11, 255 (1964).

[3]S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970); J. D. Bjorken and C. H. Llewellyn Smith, *ibid.* 7, 887 (1973).

[4]K. Niu, E. Mikumo, and Y. Maeda, Prog. Theor. Phys. 46, 1644 (1971).

[5]This means that one can use the Wigner-Eckart Theorem only for SU(3) and not for SU(4). Thus within this scheme SU(4) provides no new information except for the pseudoscalar and scalar bosons. Thus for example we introduce no difficulties with the vector-meson mass formula.

[6]M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).

[7]R. Dashen, Phys. Rev. 183, 1245 (1969).

[8]To first order in ϵ, the f_i in our scheme can be proved equal within the SU(3) multiplets. Taking all the f_i equal, however, is indeed a further assumption which seems to be justified by our exact sum rule [see Eq. (6)].

[9]We choose the doublet member ξ^+ of the SU(3) triplet rather than the singlet λ for the following reason: The decay products were $\pi^+\pi^0$, which from Bose statistics and angular momentum considerations should be in a state $|I, I_3\rangle = |2, 1\rangle$. Therefore, depending on whether the new particle is the ξ or the λ we have a $\Delta I = \frac{3}{2}$ or a $\Delta I = 2$ transition, respectively. Assuming a generalized current-current weak Hamiltonian,

$$\mathcal{H}_W = \sum_{i,j=1}^{15} G_{ij} J_\mu^i J_\mu^j ,$$

to be responsible for this decay [the new currents with $i = 9, \ldots, 15$ will not affect the results of the known semileptonic processes because of their SU(3) transformation properties], the $\Delta I = 2$ transition is not allowed, since one of the currents in $\mathcal{H}_W$ must be from the triplet.

[10]Particle Data Group, Phys. Lett. 39B, 5 (1972).

PHYSICAL REVIEW D VOLUME 8, NUMBER 6 15 SEPTEMBER 1973

Compton Scattering by Polarized Electrons as a Test of Quantum Electrodynamics

R. C. Stabler
R. D. 2, Princeton, New Jersey 08540
(Received 12 January 1972)

The pairlike cross section for Compton scattering from polarized electrons is derived, and shown to differ slightly from the cross section obtained from the conventional matrix elements. The differences may enable an experimental test of the existence of the fieldlike interaction.

If the electromagnetic interaction is viewed from the rest frame of the interacting photon, it divides naturally into a pairlike transition—a pair-creation or a pair-annihilation event, and a fieldlike transition—an event in which the photon is absorbed or emitted by an electron (or positron). In the pairlike transition an electron-positron pair with equal and opposite momenta is created or annihilated. In the fieldlike transition, the electron absorbs or emits the photon with no change in its charge or momentum.

It is remarkable that most familiar electromagnetic phenomena result from the pairlike interaction, but that the nonobservable self-force on a free electron results from the fieldlike interaction.[1] These results are obtained by the introduction of the projection operator

$$P_p(q;n) = \frac{-n\cdot q + n\circ q + (\not{q} - m)\not{n}}{2n\circ q} \tag{1}$$

and its transpose P_p^t at each vertex where a photon of momentum k interacts. In Eq. (1), $n \equiv (p\cdot k)k$, $q \equiv p + k$, and the quantity $n\circ q$ is proportional to the electron energy in the photon's rest frame:

$$n\circ q \equiv [(n\cdot q)^2 - n^2(q^2 - m^2)]^{1/2} . \tag{2}$$

The application of these projection operators identifies ordinary electromagnetic scattering as a pairlike transition and yields the Klein-Nishina formula for Compton scattering from unpolarized electrons.

For the case of polarized electrons, however, the Compton cross section differs slightly. We evaluate the pairlike cross section using the doubly projected propagators:

$$K_p(q;n_1,n_2) = P^t(q;n_2)\frac{\not{q} + m}{q^2 - m^2 + i\delta} P(q;n_1) . \tag{3}$$

Using this propagator for both the "direct" and "exchange" diagrams, the matrix element for pairlike interactions only reduces to