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APPENDIX

Using the DGS representation, it is easy to show[2,7] that

$$\nu W_2 \equiv \overline{W}(\kappa, \nu)$$
$$= \kappa\nu \int_{-1}^{1} db \int_0^\infty da\, \sigma(a, b)\delta(\kappa + 2b\nu - a)\epsilon(\nu + b)\,.$$

Scaling then requires that

$$\int_0^\infty da\, \sigma(a, b) = 0\,,$$

$$F_2(\omega) = -\tfrac{1}{2}\omega \int_0^\infty da\, a \frac{\partial}{\partial \omega}\sigma(a, \omega)\,.$$

From this it follows that

$$\int_0^1 d\omega\, F_2(\omega)/\omega = -\tfrac{1}{2}\int_0^\infty da\, a[\sigma(a, 1) - \sigma(a, 0)]\,.$$

Mass spectrum condition and smoothness require $\sigma(a, 1) = 0$. In the non-Pomeron case (or for the non-Pomeron part), Brandt and Ng incorrectly assume that $F(\omega) \to 0$ as $\omega \to 0$ requires $\overline{\sigma}(a, 0) = 0$ and they get the sum rule

$$\int_0^1 d\omega\, F_2(\omega)/\omega = 0\,,$$

where they argue that $\overline{F}_2(\omega) = F_2(\omega) - F_2(0)$.

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$J = 0$ Fixed Pole in the Virtual Compton Amplitude

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We show that the residue of the $J = 0$ fixed pole in the virtual Compton amplitude has itself a pole with the positive residue at the value of t where $\alpha(t) = 0$; here $\alpha(t)$ is a Regge trajectory appearing in the t channel and this point is the so-called sense-nonsense point. Hence the residue of the $J = 0$ fixed pole has a positive sign at $t = 0$ [where $\alpha(t) > 0$], which is the sign opposite to the Born term. The experimental relevance of our result to the electroproduction off nucleons is discussed.

I. INTRODUCTION

It is well known that nonstrong amplitudes can develop J-plane fixed poles at nonsense points due to the linear unitarity relation in t channel.[1] These fixed poles are characteristic of current amplitudes, and are distinguished from wrong-signature fixed poles due to third-double-spectral-function effects. In the virtual Compton amplitude the fixed poles may appear at $J = 1, 0, -1, -2, \ldots$ in a t-channel helicity double-flip amplitude. The fixed pole at the wrong-signature nonsense point $J = 1$ has been extensively studied by many authors in connection with the coupling of the Pomeron to photons. In fact, it has been argued that the $J=1$ fixed pole (possibly including the fixed pole due to the effect of the third double spectral function) serves to eliminate the nonsense factor, $\alpha(t)-1$ at $t=0$, from the Pomeron residue in the Compton amplitude, thus allowing a constant high-energy photoabsorption cross section.[2]

The $J=0$ fixed pole has the right signature and hence is directly measurable in the inelastic scattering of electrons off hadrons through dispersion relations or, equivalently, finite-energy sum rules (FESR). There has been much controversy about its existence and about the peculiar behavior of its residue as a function of $-q^2$, the mass squared of the virtual photon.[3-5]

Let us denote by $\epsilon^{\mu *}T_{\mu\nu}(q,p)\epsilon^{\nu}$ the forward virtual Compton amplitude, where q and p the four-momenta of the photon and nucleon, respectively, and ϵ^{ν} is the polarization vector of the photon. Spin averaging, with respect to the nucleon spin, should always be understood. The amplitude $T_{\mu\nu}$ is decomposed in the usual manner,

$$T_{\mu\nu}=T_1(\nu,q^2)\,(g_{\mu\nu}-q_\mu q_\nu/q^2)+T_2(\nu,q^2)\times(p_\mu-q_\mu p\cdot q/q^2)(p_\nu-q_\nu p\cdot q/q^2)/M^2\,, \tag{1.1}$$

where $\nu=-p\cdot q/M$, with M the nucleon mass. Using the s-t crossing relation for the helicity amplitude,[6] one can express invariant amplitudes T_1 and T_2 in terms of t-channel helicity amplitudes. At $t=0$ we have

$$\begin{aligned}T_1&=T^t_{++}+T^t_{+-}\,,\\ T_2&=2T^t_{+-}/(1+\nu^2/q^2)\,,\end{aligned} \tag{1.2}$$

where $T^t_{\lambda\mu}$ is the t-channel helicity amplitude, with λ and μ the helicities of the virtual photons. The helicity double-flip amplitude T^t_{+-} may have fixed poles at $J=1$ or 0, while the helicity-nonflip amplitudes T^t_{++} and T^t_{00} may not. It should, however, be noted that, due to the constraint at pseudothreshold $t=0$,

$$T^t_{++}+T^t_{+-}=T^t_{00}\,,$$

these helicity-nonflip amplitudes must have Kronecker δ's at $J=1$ or 0.[7] According to the Regge-pole analysis, one obtains the following asymptotic form as $\nu\to\infty$ with q^2 fixed:

$$\nu T_2\sim\sum_j\frac{-1-e^{-i\pi\alpha_j(0)}}{\sin\pi\alpha_j(0)}\,\beta_j(q^2)\nu^{\alpha_j(0)-1}+\beta_F(q^2)/\nu+O(\nu^{-3/2})\,, \tag{1.3}$$

where the index j is either P or $P'(A_2)$ and where β_j is the residue of the trajectory j and β_F is the residue of the $J=0$ fixed pole. Because of exchange degeneracy the P' and A_2 trajectories are the same. We have not considered the $J=1$ fixed pole since it has the wrong signature.[8] Because νT_2 is the s-u crossing-odd amplitude, we can write down the following superconvergence relation:

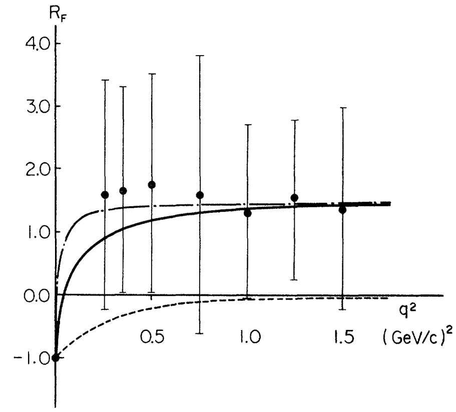

FIG. 1. The q^2 dependence of R_F from Ref. 4. The solid line corresponds to our phenomenological fit with $C=1.5$ and $a^2=0.03$. See Eq. (4.1). The dashed and dot-dashed curves correspond to the Born and parton terms, respectively.

$$\frac{G_E{}^2+(q^2/4M^2)G_M{}^2}{1+q^2/4M^2}+\frac{2M}{q^2}\int_{\nu_0}^{\infty}d\nu(\nu W_2-R)=-R_F\,, \tag{1.4}$$

where $G_E(q^2)$ and $G_M(q^2)$ are the charge and magnetic form factors, respectively, $W_2=(1/\pi)\,\mathrm{Im}T_2$, and

$$\begin{aligned}R&=(1/\pi)\sum_j\beta_j(q^2)\nu^{\alpha_j(0)-1}\,,\\ R_F&=M\beta_F(q^2)/q^2\,.\end{aligned} \tag{1.5}$$

At $q^2=0$ Eq. (1.4) reduces to the FESR for the real photoabsorption cross section. Some analyses of experimental data suggest that $R_F<0$ and $R_F\sim-1$ at $q^2=0$.[9] For large q^2, i.e., $q^2\gtrsim 2$ (GeV/c)^2, Eq. (1.4) becomes

$$\int_1^{\infty}d\omega[F_2(\omega)-R(\omega)]=-R_F\,, \tag{1.6}$$

where $\omega=2M\nu/q^2$ and $F_2(\omega)$ and $R(\omega)$ are the Bjorken scaling limits of νW_2 and R, respectively. We have assumed the existence of the limit of R_F as $q^2\to\infty$. From the experimental data on the structure function given by deep-inelastic scattering, some authors have concluded that $R_F>0$ and $R_F\sim1.5$.[3] For the nonscaling region, i.e., for $q^2\lesssim1.5$ (GeV/c)^2, the same analysis has been done on Eq. (1.4), and it has been concluded that $R_F>0$ and possibly $R_F\sim$ const (~1.5) (see Fig. 1).[4] This result seems to support the so-called polynomial residue[10-12] ($\beta_F\propto q^2$) for not-too-small q^2, but the sign of R_F there is opposite to that of the Thomson-limit value at $q^2=0$.

If the zero intercepts of the Pomeron trajectory $\alpha_P(t)$ and of the $P'(A_2)$ trajectory $\alpha_{P'}(t)$ were negative, the amplitude $\nu T_2 - \beta_F/\nu$ would be superconvergent, and R_F in Eq. (1.4) would be negative-definite. As $\alpha_P(0)$ and $\alpha_{P'}(0)$ come up to the physical values 1 and $\frac{1}{2}$, respectively, we have to subtract Regge terms from $\nu T_2 - \beta_F/\nu$ in order to get the superconvergence relation. The above experimental situation suggests that for not-too-small q^2 the Regge terms overcompensate the other terms in the left-hand side of Eq. (1.4) to result in the positive R_F. In the present paper we shall give a theoretical justification[13] of the positive constant R_F for relatively large q^2. The problem of how this positive constant R_F squares with the negative value of R_F at $q^2=0$ which is consistent with the Born term will not be resolved, but some speculations on the problem will be given.

In Sec. II we examine the sign of R_F in the dual model for the virtual Compton amplitude proposed by Landshoff and Polkinghorne.[14] We find that $R_F>0$ if $\alpha(0)>0$, where $\alpha(t)$ is a Regge trajectory appearing in the t channel. The mechanism producing the positive R_F is clarified: The $J=0$ fixed-pole residue R_F for arbitrary t has a pole at the nonsense point, $1/\alpha(t)$, with positive residue. The model also suggests that the fixed poles at $j=1,0,-1,-2,\ldots$ in T_2 are dual to the meson poles connected with the virtual photons.[15]

In Sec. III the results obtained in the model in Sec. II are generalized to a certain extent by applying the parton picture as discussed by Landshoff, Polkinghorne, and Short.[16] In fact, we show that the mechanism producing the positive R_F as shown in the model is quite general.

In Sec. IV concluding remarks are given and, in particular, speculation about the sign change of R_F as a function of q^2 is presented.

II. MODEL CALCULATION

In this section we investigate the behavior, in particular the sign, of the $J=0$ fixed-pole residue R_F in the region $q^2 \gtrsim 1$ (GeV$/c)^2$ in the Veneziano-like parametrized model of Landshoff.[14] The arguments in this section and in Sec. III are restricted to the region of q^2 quoted above, because the model in this section and the parton picture in Sec. III are designed to reproduce properties characteristic in the deep-inelastic region. As a result, we shall obtain a positive sign for R_F, $R_F>0$. We also investigate what mechanism gives the positive sign to R_F in Landshoff's model.[14]

In the present model $T_2(s,t,q^2)$ is given by[14]

$$T_2(s,t,q^2)=2M[A(s,t,q^2)+A(u,t,q^2)], \tag{2.1}$$

with

$$A(s,t,q^2)=\alpha' N^2 \int_0^\infty \frac{dv\,dw\,dz}{vwz}\left(1+\frac{z(v+w+vw)}{1+z+vw}\right)^{-m}\left(1+\frac{1}{z}\right)^{\alpha(t)-2}\left(1+\frac{z}{1+vw/(1+z+v+w)}\right)^{\beta(s)-1/2}$$
$$\times\left[\left(1+\frac{1}{v}\right)\left(1+\frac{1}{w}\right)\right]^{\alpha(-q^2)-1}\left(1+\frac{N(v+w)}{1+z}\right)^{-1}, \tag{2.2}$$

where $\alpha(t)=\alpha' t+\alpha(0)$ and $\beta(s)=\alpha' s+\beta(0)$ are the Regge trajectories exchanged in the t and s channels, respectively, and N is the normalization constant fixed by the Fubini–Dashen–Gell-Mann sum rule[17] so that $N=1/B(m,1-\alpha(0))$, where B is the beta function. In Eq. (2.2), m is a positive parameter which is related to the asymptotic behavior of the elastic form factor $F(-q^2)$, given in this model by

$$F(-q^2)=NB(m,1-\alpha(-q^2)) \underset{q^2\to\infty}{\sim} N(\alpha' q^2)^{-m}. \tag{2.3}$$

In order to extract the fixed-pole terms of $A(s,t,q^2)$ with $J=1,0,-1,\ldots$ from Eq. (2.2), we look for the fixed-power terms which behave like $s^{-1}, s^{-2}, s^{-3},\ldots$ as $s\to-\infty$. We easily see that such terms arise from the region $v,w\sim\infty$ in Eq. (2.2). Hence we make a change of variables from v and w to v' and w' such that $v=-\beta(s)/v'$ and $w=-\beta(s)/w'$. We perform v' and w' integrations to get

$$A(s,t,q^2)\Big|_{\text{fixed pole}} \sim -\frac{F(t)}{s}-\frac{Nq^2}{s^2}\int_0^\infty \frac{dz}{z^3}\left(1+\frac{1}{z}\right)^{\alpha(t)-2}(1+z)^{-m+1}+O(s^{-3}) \tag{2.4a}$$

$$=-\frac{F(t)}{s}+\frac{mF(t)}{\alpha(t)}\frac{q^2}{s^2}+O(s^{-3}), \tag{2.4b}$$

where we have neglected a term independent of q^2 in the coefficient of $1/s^2$, since such a term should be canceled out when gauge invariance is correctly taken into account.[18] It is apparent in Eqs. (2.4b) and (2.3) that the residues of the fixed poles of $A(s,t,q^2)$ at $J=1,0,\ldots$ have nonsense poles at $\alpha(t)=1,0,\ldots$, respectively. These poles are not present in $A(s,t,q^2)$, and should be canceled by the nonsense poles appearing in the ordinary Regge term, $\beta(t,q^2)s^{\alpha(t)-2}$. It is easy to see that the cancellation takes place in this model. The cancellation mechanism works generally if the fixed poles only occur multiplying the Regge poles. For the case of $J=1$, this mechanism has been extensively investigated.[19] Noting that $\nu=(s-u)/4M$ we get

$$R_F(q^2)=\frac{m}{\alpha(0)}-1. \tag{2.5}$$

The second term of the right-hand side of Eq. (2.5) comes from the term $-F(t)/s$ of Eq. (2.4b) and the corresponding term $-F(t)/u$ of $A(u,t,q^2)$. Obviously $R_F>0$ if $0<\alpha(0)<m$ and $R_F<0$ if $\alpha(0)<0$. It should here be remembered that the integral representation of Eq. (2.4a) converges only when $\alpha(t)<0$, i.e., $t<t_0$ where $\alpha(t_0)=0$. The case where $\alpha(t)>0$ with $\alpha(0)>0$ should be reached by an analytic continuation of this term in t from $t<t_0$ to $t=0$ (see Fig. 2). Equation (2.4b), when $t>t_0$, represents the analytic function obtained by this continuation. The sign of the second term of Eq. (2.4b) is the same as that of $\alpha(t)$ in the region $t<t_1$, where $\alpha(t_1)=1$. Thus this term changes sign as $\alpha(t)$ changes its sign in the continuation process. The sign of the second term of Eq. (2.4b) at $t=0$, in which we are interested, is *positive* [because $\alpha(0)>0$]. *Thus the nonsense pole factor* $1/\alpha(t)$ *in the* $J=0$ *fixed-pole residue of* $A(s,t,q^2)$ *is essential for the positivity of* R_F.

Another important point to be noted is that the fixed-pole terms of $A(s,t,q^2)$, Eq. (2.4), are obtained by estimating the large v and w contribution in Eq. (2.2). The region $v,w\sim\infty$ is, in duality language, dual to the region $v,w\sim 0$ which contributes to poles in q^2 at $\alpha(-q^2)=1,2,3,\ldots$. Hence we notice that the fixed poles at $J=1,0,-1,\ldots$ are dual to the meson poles at $\alpha(-q^2)=1,2,3,\ldots$.[20] Since we are picking up only the ground state at $v=w=\infty$, we recognize that Eq. (2.4) may be expressed diagrammatically by Fig. 3 with the ground-state particles coupled to the photons.

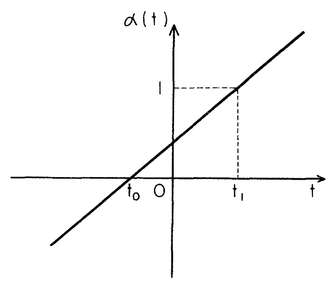

FIG. 2. The Regge trajectory $\alpha(t)$, where $\alpha(t_0)=0$ and $\alpha(t_1)=1$.

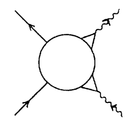

FIG. 3. Feynman diagram relevant to the fixed pole.

In Sec. III we study whether the mechanism described here works generally; we discuss whether the nonsense pole factor in the $J=0$ fixed-pole residue of $A(s,t,q^2)$ exists in the general framework.

III. GENERAL CONSIDERATION: NONPERTURBATIVE PARTON PICTURE

We now try to make more general the above model-dependent argument. As suggested by the preceding discussion, we assume that the amplitude corresponding to the diagram in Fig. 3 is the dominant contribution to the fixed-pole term. It is the same diagram as the one discussed by Landshoff, Polkinghorne, and Short.[16] We adopt their nonperturbative parton model as a more general framework in the sense that the underlying parton-proton amplitude is not restricted to a specific form.

The Feynman diagram of Fig. 3, with a six-point parton-proton amplitude, contains the diagrams with four-point amplitudes in Fig. 4 which are dominant over others when q^2 is large enough, say $q^2 \gtrsim 1$ $(\mathrm{GeV}/c)^2$ where scaling is realized. In fact, the contributions to the fixed-pole term of the gluon-exchange diagrams (shown in Fig. 5) contained in the connected six-point amplitude are independent of q^2 and are of order $1/\nu^{n+1}$ as $\nu\to\infty$ with q^2 fixed, where n is the number of exchanged gluons.

Here only spinless partons are dealt with for simplicity. The generalization to the spin-$\frac{1}{2}$ case is straightforward, and we get the same result for the fixed pole as in the spinless case, except for a slight modification of the invariant parton-proton amplitude.

The diagrams which contribute to the amplitude T_2 are Figs. 4(a) and 4(b). The seagull diagram in Fig. 4(c) contributes only to T_1. We evaluate the contribution of Figs. 4(a) and 4(b) to the $J=0$

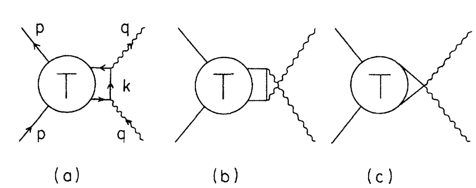

FIG. 4. Dominant diagrams for large q^2 contained in Fig. 3.

fixed-pole terms in T_2 by using the Sudakov parametrization for k in Fig. 4(a): $k=xp+yp+\kappa$, where κ is a vector orthogonal to p and q, and is spacelike. In terms of these variables T_2 can be expressed as the coefficient of $p_\mu p_\nu/M^2$ in the expansion of $T_{\mu\nu}$, and the contribution of Fig. 4(a) to T_2 is

$$T_2^{(4a)} \sim \frac{-i}{(2\pi)^4} M^2 \int J dx dy d^2\kappa\, d\sigma \frac{\rho(\sigma)}{\sigma+k^2} \times \left(4x^2 + \frac{2\kappa^2}{M^2(1+\nu^2/q^2)}\right) \times T(s', \mu^2), \quad (3.1)$$

where J is the Jacobian and behaves as $M\nu$ when ν tends to infinity, $\rho(\sigma)$ is the Lehmann spectral function, and $T(s', \mu^2)$ is the forward off-shell parton-proton amplitude (nonamputated, i.e., including parton propagators) with $s'=-(p+q-k)^2$ and $\mu^2=-(q-k)^2$ the parton mass squared. The variables s' and μ^2 may be expressed by x, y, and κ as follows:

$$s' = (x-1)^2 M^2 - (y-1)^2 q^2 + 2(x-1)(y-1) M\nu - \kappa^2, \quad (3.2)$$

$$\mu^2 = x^2 M^2 - (y-1)^2 q^2 + 2x(y-1) M\nu - \kappa^2. \quad (3.3)$$

In the following we extract the dominant part of Eq. (3.1) in the limit $\nu\to\infty$ with q^2 fixed. Since it is assumed in this model that $T(s', \mu^2)$ decreases sufficiently rapidly as $\mu^2 \to -\infty$, the dominant contribution arises from the part of the integration where μ^2 is finite. As $\nu\to\infty$ with q^2 fixed this region is concentrated around the point $y=1$. Hence we introduce the new variable y' with

$$y = 1 + y'/2M\nu \quad (3.4)$$

and change the y integral into a y' integral. By studying the cut structure of $T(s', \mu^2)$ in the y' plane[16] we easily derive an expression for the dominant part of Eq. (3.1). The contribution of Fig. 4(b) can be estimated in the same way except that q is replaced by $-q$, and the dominant contribution in the integral comes from the region $y\approx -1$. Combining the contributions of both diagrams we get

$$T_2\big|_{\text{fixed pole}} \sim -\frac{2q^2}{\nu^2} \int d^2\kappa \left(\int_0^1 \frac{dx}{1-x} \int_0^\infty ds' \, \mathrm{Im} T(s', \mu^2) + \int_{-1}^0 \frac{dx}{1+x} \int_0^\infty du' \, \mathrm{Im} T(u', \mu^2) \right). \quad (3.5)$$

Note that terms independent of q^2 are neglected in Eq. (3.5) because of the gauge-invariance requirement.[21]

Since ImT is proportional to the parton-proton cross section with a positive proportionality constant, it is apparent that the right-hand side of Eq. (3.5) is negative-definite for spacelike q^2. We must, however, beware of its singular structure. If we require that ImT be a smooth function of μ^2 with s' fixed, the integral (3.5) converges at the ends of the ranges of the variables x and κ. In fact ImT depends on these variables only through μ^2 and

$$\mu^2 = -(xs' + \kappa^2)/(1-x) + xM^2, \quad (3.6)$$

where we used Eqs. (3.2), (3.3), and (3.4) and neglected the terms of order $1/\nu^2$. It is also reasonable to assume that the s' integral does not diverge at the threshold of the parton-proton scattering. Then only the end point $s'=\infty$ need be handled carefully. Let us now concentrate on the contribution to Eq. (3.5) coming from the region near this end point. Since if μ^2 is bounded then xs' is also bounded for large s' as seen in Eq. (3.6), the region of small x is important in the x integral. Hence we make a change of variable, $\bar{x}=s'x$. If $T(s', \mu^2)$ is essentially the same as the hadronic amplitude in its structure, we may parametrize it in Regge form for large s':

$$\mathrm{Im} T(s', \mu^2) \sim \beta(\mu^2)(s')^{\alpha(0)},$$

with $\alpha(0)$ the zero-intercept of the Regge trajectory exchanged in the t channel of the parton-proton scattering. We finally obtain from Eq. (3.5) for the contribution to R_F coming from the region $s'\sim 0$ and $x\sim 0$

$$R_{FL} = -2M \int d^2\kappa \int_0^\infty d\bar{x}\, \beta(-\bar{x}+\kappa^2) \int_L^\infty ds' (s')^{\alpha(0)-1}, \quad (3.7)$$

where L is some large number. We see that Eq. (3.7) is convergent only when $\alpha(0)<0$ and then $R_{FL}<0$. Since $\alpha(t)$ is expected to be an ordinary Regge trajectory, the positive intercept is reason-

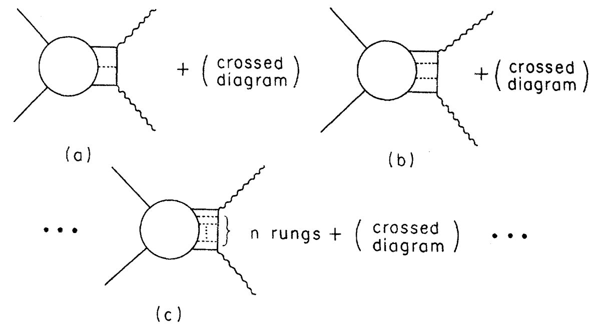

FIG. 5. Gluon exchange diagrams.

able. For such a case we must first perform the integral in Eq. (3.7) with $\alpha(0)<0$ and continue the result analytically to the region $\alpha(0)>0$. By this procedure we get $R_{FL}=B/\alpha(0)$, where B is a positive constant. Hence for $\alpha(0)>0$ we obtain $R_{FL}>0$. If the contribution to R_F coming from the other region in Eq. (3.7) is unimportant, the mechanism changing the sign of R_F is just the same as the one discussed in the Landshoff model.

We have shown that the contribution of the parton diagram (Fig. 4) to the $J=0$ fixed-pole term is of the form

$$q^2R_F(q^2)=Cq^2+C_1, \quad C>0$$

where the constant C_1 was neglected in Eq. (3.5). As we have remarked earlier in this section, the diagrams with gluon lines as shown in Fig. 5 contribute to the $J=0$ fixed-pole term as constant terms: $q^2R_F(q^2)=C_2$. Since our framework here is gauge-invariant, the constants C_1 and C_2 cancel when we sum up all the contributions of the diagrams including those in Figs. 4 and 5 to the $J=0$ fixed-pole terms. Hence we finally have $R_F(q^2)=\text{const}>0$. One may notice that, as we have considered all possible Feynman daigrams in deriving the above result, it should hold for any q^2 and hence predicts a positive value at $q^2=0$ which contradicts the experimental result. The following comment, however, is in order: A hadronic Born term with form factor, if it persists at high energy, contributes to the $J=0$ fixed-pole terms; its residue is, in fact, the first term in Eq. (1.4). This form of the q^2 dependence of R_F cannot be accommodated by our result in this section. The example suggests that our way of calculation in the parton picture is incomplete in the sense that it cannot generate the hadronic one-particle state as a bound state of many partons (or our framework itself might not be suitable for describing the hadron as a bound state of many partons). Accordingly we note that our result on R_F in this section should not be taken seriously in the small-q^2 region where many diagrams other than those in Fig. 4 contribute. In Sec. IV we speculate on the behavior of R_F for smaller q^2.

We have investigated the above-mentioned mechanism in other available models and obtained the following results: First, in the model of Ademollo and Giudice,[22] the relevant amplitude involves a term which shows our sign-change mechanism. Second, the situation is the same for the model of Kikkawa and Sato[23] in the planar and one-loop approximations, and the diagram which contributes to the fixed-pole term in this model is Fig. 6 alone. Third, in the dynamical resonance model of Manassah and Matsuda,[24] the same mechanism works, provided L, the dimension of

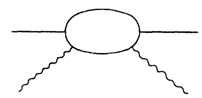

FIG. 6. The planar one-loop diagram in the model of Kikkawa and Sato which is responsible for the fixed-pole term.

the new harmonic oscillator introduced by them, is equal to unity. This can be easily understood by noting that the zero intercept of the Regge trajectory exchanged in the t channel is $1-\frac{1}{2}L$ and it is positive only when $L=1$.

IV. CONCLUDING REMARKS

We have given a theoretical basis for the fact that the residue of the $J=0$ fixed pole for relatively large q^2 is positive and constant. On the other hand, experimental analyses of FESR (1.4) at $q^2=0$ seem to favor the Thomson-limit value for R_F, i.e., $R_F\sim -1.0$. This result may be a formidable challenge to our conclusion. There are several ways of trying to resolve the discrepancy between the negative R_F at $q^2=0$ and the positive constant R_F for relatively large q^2. One of the possible solutions is that $R_F(q^2)$ is discontinuous at $q^2=0$. High-energy e-p scattering experiments at very small q^2 are essential to test this possibility. Another possibility is that, for small q^2, R_F is not constant, i.e., the fixed-pole residue is not polynomial, and R_F for $q^2\neq 0$ is smoothly continued to its value at $q^2=0$. This somewhat peculiar behavior of R_F may be possible if diagrams other than the ones in Fig. 4 contribute for smaller q^2 to modify the constant R_F at larger q^2. In fact the nucleon pole term in the virtual Compton amplitude cannot be generated only by such parton diagrams as those in Fig. 4, while, as is well known, the nucleon pole term, if it persists at high energy, contributes to $R_F(q^2)$. Thus one may push forward the following speculation: The fixed pole has two parts, one coming from the nucleon pole term which is dominant at smaller q^2 and one coming from the parton diagrams as in Fig. 4. These two components get together to show the behavior of $R_F(q^2)$ for all q^2:

$$R_F(q^2)=-\frac{G_E{}^2+(q^2/4M^2)G_M{}^2}{1+q^2/4M^2}+\frac{Cq^2}{q^2+a^2}, \qquad (4.1)$$

with $C>0$ and a^2 the constant to be determined. Here we have modified the constant C coming from the parton diagrams by the factor $q^2/(q^2+a^2)$ purely phenomenologically in order to suppress the parton contribution at $q^2=0$. The experimental data in Fig. 1 can be reproduced reasonably by Eq.

(4.1) if $C=1.5$ and $a^2=0.03$ $(\mathrm{GeV}/c)^2$, where we have used experimental data for G_E and G_M. The functional form (4.1) may also be applied to the case of the neutron target. The neutron pole term in Eq. (4.1) vanishes at $q^2=0$, and so $R_F(q^2)$ should be zero at $q^2=0$.[25] The constant C may be different in magnitude from that of the proton target.

Finally we emphasize that in order to go further we definitely need more experimental data on electroproduction off the proton and deuteron at very small q^2 and high energies.

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[18]In this model, gauge invariance is not taken into account as in all other Veneziano-like parametrized models.

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[20]We are informed that this viewpoint has also been pointed out by Miransky, Shelest, and Zinovjev. See Ref. 15.

[21]Gauge invariance is indeed satisfied when we sum up all the contributions of the Feynman diagrams included in Fig. 3 (Ref. 12). Further we know that it must be satisfied in each order of the strong interactions (the parton-parton interactions). We may consider the case where the sum of the contributions of Figs. 4(a)–4(c) alone satisfies gauge invariance. In this case the parton-proton amplitude is restricted by the gauge condition and it can be shown easily that the $J=0$ fixed pole decouples from the longitudinal photon.

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