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PHYSICAL REVIEW D VOLUME 8, NUMBER 1 1 JULY 1973

Quark-Fragmentation Models for Very-High-Energy Production Processes

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(Received 2 October 1972)

The qualitative features of models for high-energy production processes based on the fragmentation of the dressed quarks assumed to make up hadrons are discussed. It appears that, if quarks are very massive (i.e., quark mass ≈ 10 GeV/c^2), such models could explain in a simple way the experimental elusiveness of quarks, the limitations on transverse momenta observed in production reactions, the average inelasticity observed in cosmic-ray proton interactions, the abundance of pions produced in high-energy collisions, and the large transverse momenta observed in air showers for primary particle energies above 10^6 GeV.

INTRODUCTION

There are indications that known hadrons are composites of more fundamental particles. If a theory of strong interactions is to be developed in terms of these particles, called quarks for convenience, all strong-interaction processes must be described in terms of quarks, and ordinary hadrons should appear only as quark bound states.

We present a qualitative discussion of a class of models for very-high-energy production reactions in which baryons are composed of three quarks, mesons are composed of a quark (Q) and an antiquark ($\overline{Q}$), and the only processes which can take place are the production and absorption of $Q\overline{Q}$ pairs. The interactions between quarks are due to a direct self-interaction and $Q\overline{Q}$ pair (meson) exchange. We assume that quarks are very massive and we arbitrarily set the quark mass M at 10 GeV/c^2 in this paper. It does not matter if there are three or nine quarks in these models, as long as the baryons are tightly bound states of three massive particles and mesons are tightly bound states of a massive particle and antiparticle.

QUARK-FRAGMENTATION MODELS

The class of production models we consider, which we call quark-fragmentation models, also make the following set of assumptions:

(1) Hadrons are composed of "dressed" quarks, so each quark is surrounded by a virtual $Q\overline{Q}$ cloud. It is assumed that virtual $Q\overline{Q}$ pairs are produced isotropically with an average momentum of about 300 MeV/c in the rest frame of the quark. By the uncertainty principle this leads to a characteristic radius of the virtual $Q\overline{Q}$ cloud of a dressed quark of around $\frac{2}{3}$ F.

(2) At very high energies, hadronic collisions usually consist of a single scattering of a quark in one hadron and a quark in the other.[1,2] Thus, the hadronic scattering amplitude is the sum of contributions from all diagrams in which a (dressed) quark in one colliding hadron scatters off a dressed quark in the other colliding hadron, while the other quarks making up the hadrons continue undisturbed as spectators.[3]

(3) The only production mechanism is the isotropic production of virtual $Q\overline{Q}$ pairs with low momentum (≈0.3 GeV/c) in the quark rest frame.

(4) Quark-quark scattering is predominantly forward ($p_T \approx 0$) and results in the fragmentation of the colliding dressed quarks. In this fragmentation, the dressed quarks share their momentum with the constituents of their virtual $Q\overline{Q}$ clouds. Thus, the mechanism by which a hadron loses momentum in a high-energy collision is through the sharing of the momentum of one dressed quark in

the hadron with the fragments of that quark's virtual $Q\bar{Q}$ cloud. After the collision, the fragments (quarks and antiquarks) of the virtual cloud move in a cluster with some fraction of the momentum of the incident hadron. The momentum of the virtual quarks and antiquarks relative to the cluster c.m. will be about 300 MeV/c. Notice that the more virtual $Q\bar{Q}$ pairs a cluster contains, the more massive it will be and the slower it will move. Because the quark and antiquark momenta are isotropically distributed in the cluster c.m. frame, the mesons formed from the cluster will be "radiated" isotropically in the cluster c.m. frame. Mesons will be formed because the characteristic lifetime for the decay $Q+\bar{Q}\to$ meson should be of the order of $\tau \approx \hbar/2M$, and the distance which the quarks in the cluster can travel in this time is approximately

$$\frac{\bar{p}}{M}\,\frac{\hbar}{2M} \approx 3\times 10^{-4}\ \mathrm{F}\,.$$

Thus, something on the order of 10^3 characteristic decay times must pass before the virtual quarks and antiquarks produced by the fragmentation process get out of range of the $Q\bar{Q}$ (meson) exchange force which can bind them into mesons. So, the quarks and antiquarks in a cluster are effectively confined and their decay into $Q\bar{Q}$ bound states (mesons) should be strongly favored even in those cases where there is enough energy in the cluster c.m. system to allow the existence of the quarks and antiquarks as real physical particles.

(5) Subsequent to the collision, the quarks which have collided are picked up by the spectator quarks to form the leading particles.[3] The momentum of the quark which is picked up will in general be low, since it has shared its momentum during the fragmentation process with the constituents of its virtual $Q\bar{Q}$ cloud. This deceleration of a quark in the leading particle may excite the leading particle into a resonant state (isobar), and if so, the leading isobar will decay (following the subsequent formation of virtual $Q\bar{Q}$ pairs) into two or more hadrons. Clearly the leading particle will always be a meson or meson isobar if the incident particle is a meson, and a hadron or hadron isobar if the incident particle is a hadron.

(6) Specific models for the production of virtual $Q\bar{Q}$ pairs and the fragmentation of dressed quarks in QQ scattering must be employed to obtain the average multiplicity $\langle n\rangle$ as a function of energy; the dependence of the n-prong production cross section σ_n on n and the energy; unitarity; the precise form of limiting fragmentation or scaling expected; and details of the longitudinal momentum distribution.

Notice that, because of the additive nature of this class of models, most of the "theoretical" questions involving unitarity, scaling, etc. in hadronic collisions are transferred to the underlying quark interactions. Thus, a unitary model of $Q\bar{Q}$ pair production in QQ collisions should lead to a unitary model of meson production in hadronic collisions.

PROPERTIES OF PRODUCTION PROCESSES AT VERY HIGH ENERGIES

At very high energies, quark-fragmentation models should lead to a picture of hadronic production reactions with the following qualitative properties:

(1) Free quarks will be very difficult to observe.

Production of real $Q\bar{Q}$ pairs by the usual types of production mechanisms is *very* strongly suppressed.[4] Production of *real* $Q\bar{Q}$ pairs by diffraction dissociation[5] (as opposed to the virtual $Q\bar{Q}$ pairs forming a meson) should also be strongly suppressed. This can be seen as follows. Consider a proton and the diffractively dissociated state of a proton plus a real $Q\bar{Q}$ pair with mass $M^* = 2M + m$, where m is the proton mass. If the laboratory momentum P of these two states is such that $P \gg M^*$, the difference in energy of the two states is

$$\frac{1}{2P}(M^{*2} - m^2)\,.$$

Then, when $P \gg \frac{1}{2}(M^{*2} - m^2)$, the proton and the diffractively dissociated state (proton plus $Q\bar{Q}$ pair) are essentially degenerate in energy, and diffraction-dissociation-type processes are possible.[6] If the quark mass is 10 GeV, diffractive dissociation into a proton plus a $Q\bar{Q}$ pair becomes possible when $P \gg 220$ GeV/c (say, around $P \approx 2000$ GeV/c). Such an equivalent lab energy is out of reach of the CERN Intersecting Storage Rings, and the cosmic-ray flux of protons with energies above 2000 GeV/c is only about $10^6\ \mathrm{m}^{-2}\mathrm{sr}^{-1}\mathrm{yr}^{-1}$.[7] Even at energies above the threshold for diffraction dissociation, $Q\bar{Q}$ pair production should be low, because the production amplitude is expected to be proportional to the probability of finding a proton in the state (proton plus $Q\bar{Q}$ pair).[8] But this probability should be low because of the large number of competing proton-plus-n-meson (bound $Q\bar{Q}$ pair) states of lower energy. Finally, if a $Q\bar{Q}$ pair is to be formed by diffraction dissociation, recombination into a meson is strongly favored energetically.

(2) The additivity relations for high-energy hadronic processes have been built into the model.

We assume that hadronic transition amplitudes are the sum of the transition amplitudes for the various possible interactions of their constituent quarks. Then, by the optical theorem, the hadronic total cross sections are the sum of the total cross sections of the various quark-quark interactions.

(3) Generally, transverse momenta in production processes will be of the order of 300 MeV/c.

When a dressed quark shatters in a hadronic collision, it leaves a cluster of virtual quarks and antiquarks (the fragments of its virtual $Q\overline{Q}$ cloud). The entire cluster moves with some fraction of the incident hadron's momentum, but in the cluster rest frame the fragments comprise a "gas" of virtual quarks and antiquarks moving with an average momentum of around 300 MeV/c. Because the characteristic decay time of a $Q\overline{Q}$ pair to a meson should be of the order of $\hbar(2M)^{-1}$ and some 10^3 characteristic times must pass before a quark and antiquark in the cluster can get out of range of the $Q\overline{Q}$ (meson) exchange force which acts between them, the formation of a meson cloud is strongly favored at all energies. In fact, at lower energies there will be insufficient energy to allow quarks to exist as real particles, and mesonic final states are an energetic necessity. A quark-model meson wave function given in an earlier paper,[9] which is in fact independent of the details of the model used in that paper, suggests that the average relative $Q\overline{Q}$ momentum in a meson is around $23M$ MeV/c, where M is the quark mass in GeV/c^2. For a quark mass of 10 GeV/c^2, the average relative $Q\overline{Q}$ momentum in a meson is about 230 MeV/c, and vector addition in an "average" case shows that the characteristic momentum (in the cluster rest frame) of the mesons formed by recombination of the virtual $Q\overline{Q}$ cloud should be around 550 MeV/c. Since the quarks and antiquarks in the virtual $Q\overline{Q}$ cloud were produced isotropically, the mesons formed by recombination should be produced isotropically with an average momentum $\overline{p} \approx 550$ MeV/c. Averaging over all directions yields an average transverse momentum $\langle p_T \rangle = \overline{p}/\sqrt{3} = 320$ MeV/c in the cluster rest frame. Since the transverse momentum is a Lorentz invariant, this implies an average transverse momentum of 320 MeV/c in high-energy production processes.

(4) Above a laboratory energy of about 10^6 GeV, events with large transverse momentum should be observed.

At an energy where real (as opposed to virtual) free quarks can be produced by the fragmentation process, some of these free quarks can decay into hadrons ($Q\overline{Q}$ or QQQ bound states) and a lighter quark. This decay would lead to larger transverse momenta in some cosmic-ray events. Furthermore, annihilation of a real quark and antiquark could lead to particles with transverse momenta of the order of 10 GeV/c. In a quark-fragmentation model of pp scattering this will first occur when there is enough energy in the pp c.m. frame to allow the existence of a free physical quark and antiquark and the two colliding protons, all with the requisite momenta, after the fragmentation process has taken place. The initial c.m. energy of the colliding protons is $2(p^2+m^2)^{1/2}$ and the requisite final energy is

$$2[(\tfrac{1}{9}p)^2+M^2]^{1/2}+[(\tfrac{7}{9}p)^2+m^2]^{1/2}+(p^2+m^2)^{1/2}\,.$$

When $p \gg 9M$, both the initial and final energy are $\approx 2p$ and the fragmentation (as opposed to diffraction dissociation) into a free quark and antiquark is energetically possible. The condition $p \gg 9M$ implies $p \gg 90$ GeV/c and

$$P_{\text{lab}} \gg 2(9M)^2 m^{-1}$$

or

$$P_{\text{lab}} \gg 1.6\times10^4 \text{ GeV}/c\,.$$

Thus, we would not expect large transverse momenta to appear in air showers much below a primary energy of about 2×10^5 GeV if the quark mass is around 10 GeV/c^2. The occurrence of large transverse momenta might be looked on as evidence for the existence of quarks, and arguments similar to the above might be used to determine the quark mass required in a fragmentation-type model from the onset of the occurrence of large transverse momenta in air showers.

Since real $Q\overline{Q}$ pairs should not be produced in significant numbers at cosmic-ray energies much below 10^6 GeV in a quark-fragmentation model with a quark mass around 10 GeV/c^2, we can set an upper limit on the flux of quarks produced by cosmic rays in the atmosphere. The flux of cosmic-ray protons with energies above 10^6 GeV (10^{15} eV) is[7] about 10^{-9} cm^{-2}sec^{-1}sr^{-1}. Only a small fraction of these protons would be expected to produce real $Q\overline{Q}$ pairs, and this gives an upper limit on the flux of fractionally charged particles in cosmic rays near sea level of 10^{-10} cm^{-2}sec^{-1}sr^{-1}.[10]

(5) The inelasticity in extremely high-energy nucleon-nucleon collisions with a leading nucleon must lie between 0.4 and 0.56.

The coefficient of inelasticity K is the fraction of the primary particle energy transferred to secondary particles. K is defined by $K=\sum_i E_i E_0^{-1}$, where E_0 is the initial energy of the primary and $\sum_i E_i$ is the sum of the energies transferred to the secondaries. $\sum_i E_i = E_0 - E_a$, where E_a is the final energy of the primary, so

$$K = 1 - E_a E_0^{-1}\,.$$

Experimentally, K seems to be around 0.4 to 0.5

for NN collisions up to energies around 3×10^4 GeV.[11]

The momenta $\vec{p}_1$ and $\vec{p}_2$ of colliding protons in the c.m. frame is related to the laboratory momentum $\vec{P}$ of a proton incident upon another proton at rest by

$$|\vec{p}_1| = |\vec{p}_2| = |\vec{P}|\, m(s)^{-1/2},$$

where s is the square of the c.m. energy. At very high energy and momentum $s \rightarrow 2mE_{\text{lab}}$ and the laboratory energy $E_{\text{lab}} \simeq P$, so

$$P = 2p^2/m\,.$$

Consider a quark-fragmentation model where the c.m. momenta of the scattering protons are sufficiently high that the lifetime of the virtual $Q\overline{Q}$ pairs in the colliding protons is much longer than the collision time.[12] Each dressed quark in the protons will have a c.m. momentum $\frac{1}{3}p$. Now, the only way a proton can lose momentum is when one of its dressed-quark constituents collides with a dressed quark in the other proton and shatters. The quark which shatters shares its momentum with the fragments of its virtual $Q\overline{Q}$ cloud, and is picked up again to form a proton with reduced momentum. The greatest momentum loss and thus the maximum possible inelasticity permitted in a quark-fragmentation model for an event with a leading nucleon occurs when the multiplicity (that is, the number of virtual $Q\overline{Q}$ pairs in the cloud clothing the quark) is so large that essentially all of the momentum of the colliding quark is lost from the proton, resulting in a final c.m. proton momentum of $\frac{2}{3}p$. We then have, at very high energies,

$$E_0 \approx P \approx \frac{2p^2}{m}$$

and

$$E_a \approx \frac{2}{m}\left(\frac{2p}{3}\right)^2,$$

so

$$K = 1 - E_a E_0^{-1} = 0.56$$

is the *upper limit* on the inelasticity allowed in a quark-fragmentation model. The lower limit on the inelasticity occurs when there is only one $Q\overline{Q}$ pair to share the momentum of the colliding quark in the incoming proton. In that case, the final c.m. momentum of the three quarks comprising the incident proton is $\frac{7}{9}p$ and the inelasticity is

$$K = 1 - (\tfrac{7}{9})^2 = 0.4\,.$$

So, the inelasticity at high energies in a quark-fragmentation model *must* lie between 0.4 and 0.56.

Suppose we assume for the moment that the total average multiplicity of charged plus neutral produced mesons $\langle n(E)\rangle$ is known as a function of energy (we neglect baryon production). Then we should have on the average $\frac{1}{2}\langle n(E)\rangle$ mesons or $\langle n(E)\rangle$ quarks and antiquarks per cluster at a given energy. After one of its constituent quarks loses momentum by shattering and sharing its momentum with the fragments of the quark's virtual $Q\overline{Q}$ cloud, the final proton momentum after a scattering event should be, on the average,

$$\tfrac{2}{3}p + \tfrac{1}{3}[\langle n(E)\rangle + 1]^{-1}p = [2\langle n(E)\rangle + 3] \times [3\langle n(E)\rangle + 3]^{-1}p,$$

whence the average inelasticity as a function of energy is

$$K = 1 - \left(\frac{2\langle n(E)\rangle + 3}{3\langle n(E)\rangle + 3}\right)^2.$$

If, for example, $\langle n\rangle \sim \ln s$, K would be a very slowly varying function of s which approached 0.56 as $s \rightarrow \infty$.

(6) Pions should comprise approximately 80% of the particles produced in very-high-energy reactions.

Soon after a dressed quark shatters in a collision, the virtual quarks and antiquarks which are the fragments of its $Q\overline{Q}$ cloud will form $Q\overline{Q}$ bound states (mesons). In an SU_3-degenerate approximation, the bound $Q\overline{Q}$ system has energy levels m_π, m_K, etc., corresponding to the various meson masses. To get a rough idea of the mass distribution of produced mesons, assume that each bound $Q\overline{Q}$ system is in thermodynamic equilibrium with a hot gas of pions in the cluster rest frame and that the only allowed $Q\overline{Q}$ bound states are the pion (ground state) and the kaon (excited state). In the cluster rest frame, the average pion energy $\langle E\rangle = (\overline{p}^2 + m^2)^{-1/2}$, with $\overline{p} = 550$ MeV/c. This average energy corresponds to a temperature kT of

$$kT = \tfrac{2}{3}[(\overline{p}^2 + m_\pi^2)^{-1/2} - m_\pi] = 290\ \text{MeV},$$

or $T = 3.4\times10^{12}$ °K. At the temperature kT, the occupation probability of the energy levels of the bound $Q\overline{Q}$ system is

$$P(E) = \exp(-E/kT)\,.$$

Therefore, the relative probability of the $Q\overline{Q}$ bound state being a pion or a kaon is

$$P_\pi P_K^{-1} = \exp[(m_\pi - m_K)(kT)^{-1}],$$

where m_K is the kaon mass. Since we have assumed that there are only two energy levels, P_K

$+P_\pi = 1.0$. This gives a 22% probability of producing kaons, an estimate which is, however, somewhat sensitive to the value of the average momentum assumed. Nevertheless, since kaons seem to be less than 20% of the pionic component of cosmic rays, it appears that quark-fragmentation models might provide a simple qualitative picture of the mechanisms behind the observed secondary particle abundances *without* the need for postulating a "hadronic temperature."

(7) If production processes are symmetric in the QQ c.m. frame, there will be a basic asymmetry in high-energy production reactions observed in meson-baryon scattering which can be eliminated by boosting to a Lorentz frame where $p_{\text{baryon}}/p_{\text{meson}} = \frac{3}{2}$.[3]

This asymmetry will, however, be modified by any asymmetry in the production process in the c.m. frame of the underlying QQ reactions. Because of phase-space limitations, this effect may not be visible in low-energy or high-multiplicity events. At very high multiplicities, the fragments of the shattered virtual $Q\overline{Q}$ cloud will have very low momenta in the c.m. frame of the colliding hadrons and we should see a pionizationlike event symmetric in the hadron c.m. system.

(8) Individual cosmic-ray events can be described by an "isobars plus clusters" picture.

Individual cosmic-ray events will show evidence for fireball-like clusters, the total cluster c.m. momentum being some fraction of the initial hadron c.m. momentum. However, the mass of the clusters rises as the multiplicity rises. So, at very high multiplicites, the cluster and the quarks comprising them move very slowly in the c.m. frame of the colliding hadrons, giving something which looks like pionization. Thus, the dip in the longitudinal momentum spectrum of inclusive reactions at $p_L = 0$ which is expected in a simple two-fireball model should *not* appear since the region around $p_L = 0$ will be filled up with high-multiplicity events.[13]

(9) We expect a constant ratio of charged to neutral prongs in production events, so that a large charged multiplicity implies a large neutral multiplicity.

This can easily be seen if we assume that only pions are produced, for then the probability of any given $Q\overline{Q}$ pair forming a π^0, π^-, or π^+ should be roughly $\frac{1}{3}$, and the ratio of charged to neutral particles should be constant at around 2:1 in all events at all energies.

(10) Associated production of strange particles is built into the model.

In the usual quark model,[14] the λ quark is the only one with nonzero strangeness. Strange-meson production then involves $\lambda\overline{\lambda}$ pair production in the virtual $Q\overline{Q}$ cloud of a colliding quark. When the virtual cloud shatters during a collision, the λ and $\overline{\lambda}$ must be bound into different mesons to secure nonzero strangeness in the produced mesons. Thus, strange mesons can only be produced in associated production reactions.

(11) Because of the basic "parton" nature of the quark-fragmentation models, they should display limiting fragmentation as defined by Benecke, Chou, Yang, and Yen.[15]

(12) Processes which correspond to photon dissociation are allowable.[16]

Photon–vector-meson equivalence shows that photons can have virtual states involving $Q\overline{Q}$ pairs. These virtual states can also shatter and share momentum when colliding with hadrons.

Note Added in Proof

(13) A break in the primary cosmic-ray energy spectrum can be expected at high energies.

It has been shown[7,17] that an increase in the nucleon-nucleon interaction cross section or an increase in the inelasticity K (fraction of the primary particle energy transferred to secondaries) at high energy can cause a break in the primary cosmic-ray energy spectrum. For a quark mass of 10 GeV/c^2, diffraction dissociation of a proton into its three constituent quarks becomes energetically possible when the lab momentum P of the proton satisfies $P \gg 450$ GeV/c. However, the diffraction-dissociation amplitude should be proportional to the probability of finding the proton in the high-mass three-quark virtual state. Because of the large number of competing lower-mass virtual states, the chances of finding the three-quark virtual state occupied should be low until the fractional change in c.m. energy required to reach the three-quark virtual state from the proton ground state is small. This requires

$$\{3[(p/3)^2 + M^2]^{1/2} - (p^2 + m^2)^{1/2}\}/(p^2 + m^2)^{1/2} \approx 0$$

or $p \gg 3M$, where p is the c.m. proton momentum. Thus if the quark mass is 10 GeV/c^2, significant dissociation of protons into their constituent quarks requires a laboratory momentum P which satisfies $P \gg 18M^2m^{-1}$ or $P \gg 1.8 \times 10^3$ GeV/c. This suggests that events in which the proton is totally dissociated into its constituent quarks and no leading nucleon is found will only occur above a primary proton energy of around 2×10^4 GeV.

The asymptotic cross section for diffraction dissociation of protons into quarks can be roughly estimated at 5 mb from a simple nonrelativistic model of the proton wave function.[9] This would increase the nucleon-nucleon total cross section from its constant value of approximately 35 mb at lower energies to approximately 40 mb, an increase of about 15%. When this asymptotic cross section is reached, about $\frac{5}{40}$ or $\frac{1}{8}$ of all cosmic-ray protons will dissociate completely into quarks, giving up all their energy to the quark "secondaries." The remaining $\frac{35}{40}$ or $\frac{7}{8}$ of the events will be the usual "leading-nucleon"-type event. If we assume that the inelasticity K equals $\frac{1}{2}$ for the leading-nucleon processes, we will find that the average energy transferred to secondaries for all processes after the diffraction breakup cross section reaches its asymptotic limit is

$$KE = \tfrac{1}{8}E + \tfrac{7}{8}(\tfrac{1}{2}E) \text{ or } KE = \tfrac{9}{16}E\,,$$

which yields an asymptotic average inelasticity of 0.56, an increase of 11%.

An increase in the interaction cross section of 15% and an increase in the average inelasticity of 11% in nucleon-nucleon scattering would be sufficient, according to Naranan,[17] to cause the experimentally observed break in the primary cosmic-ray energy spectrum at about 10^{15} eV.

(14) The detailed picture of high-energy production reactions will be affected by the time sequence of the fragmentation process.

If the cascade multiplication effect in high-energy nucleon-nucleus scattering is low[18] (indicating the absence of pionization-type processes), this would suggest that most of the dressed quarks involved in high-energy collisions break up after a time delay, which allows them to escape from the nucleus before fragmenting.

If, on the other hand, pionization processes are found to be a frequent feature of high-energy production reactions and this can be reconciled with the absence of cascade multiplication in nucleon-nucleus scattering,[18] a different picture would emerge. Pionization processes could develop in a quark fragmentation model if a significant fraction of the dressed quarks involved in high-energy production reactions shattered as soon as the two colliding quarks came within the range of the strong-interaction force which acts between them (i.e., at a separation of about 1 F). Then pionization effects might be produced by the interaction of the hadronic clouds produced from the fragments of the dressed quarks.

RESTRICTIONS ON QUARK-FRAGMENTATION MODELS

Acceptable quark-fragmentation models must exhibit unitarity, produce virtual $Q\overline{Q}$ pairs with low momentum in the quark rest frame, and produce reasonable predictions for the average multiplicity and the variation of the topologic cross section σ_n with energy and the number of prongs n.

There may be several approaches which could lead to useful quark-fragmentation models. Models similar to the diffraction-excitation model championed by Hwa[19] would envision diffractive excitation of the colliding quarks into massive states called fireballs, with the production cross section for fireballs of mass M_f varying as $d\sigma/dM_f \propto M_f^{-2}$. This fireball or excited state consisting of a quark plus virtual $Q\overline{Q}$ pairs would then break up into a cluster of quarks and antiquarks. These quarks and antiquarks would share the longitudinal momentum of the colliding quark and retain their low (~300-MeV/c) momentum with respect to the cluster rest frame leading to a limitation on transverse momenta in the secondary particles. The number of virtual $Q\overline{Q}$ pairs in a cluster of mass M_f (and thus the number of mesons produced from that cluster) will be proportional to M_f, leading to $d\sigma/dn \sim n^{-2}$ and an average multiplicity $\langle n\rangle \sim \ln s$. Single or double diffractive excitation is possible, and individual events need not be symmetric in the QQ c.m. frame.

If, on the other hand, we consider models in which

(1) virtual $Q\overline{Q}$ pairs are produced with low momenta in the rest frame of a quark,

(2) the topologic cross sections σ_n vary as the nth power of the coupling strength, and

(3) the total cross section in QQ scattering displays a leading power behavior,

$$\sigma_{\text{tot}}(\lambda) \sim \beta(\lambda) s^{\alpha(\lambda) - 1}\,,$$

well-known arguments lead to a logarithmic increase in average multiplicity with energy and a Poisson distribution for the topologic cross sections.[20]

A four-fermion self-interaction model considered[21] in the course of investigations of Heisenberg's unified theory of elementary particles reveals some interesting characteristics when applied to quarks. In this theory, with the basic interaction $V(\bar{\psi}\psi)(\bar{\psi}\psi)$, it is found that enforcing the absence of light-cone singularities leads to a strongly attractive short-range force in $Q\overline{Q}$ scattering which can produce tightly bound states and a strongly repulsive core in QQ scattering which may be responsible for saturation of the quark and the nuclear forces. In this model, no tightly bound QQ state exists. It remains to be seen whether three-quark bound states can exist in this model and what the momentum distribution of the virtual $Q\overline{Q}$ pairs clothing a quark will be. Furthermore, it will be necessary to predict σ_n vs n and the average multiplicity with this model before it can be used in the framework of quark-fragmentation models.

Finally, a simplistic (but difficult to extend) physical picture might assume that the probability of finding a quark in a virtual state with n $Q\overline{Q}$ pairs is proportional to the lifetime of that state, whence

$$P_n \sim T_n \sim \frac{\hbar}{n2M}$$

or

$$P_n \sim \frac{1}{n}\,.$$

If it is assumed further that at high energies only forward scattering is important and that both quarks are decelerated by the same amount in a collision, the only way momentum can be lost by the quark is by sharing it with the components of its virtual $Q\overline{Q}$ cloud. Since both quarks lose the same momentum, the same number of $Q\overline{Q}$ pairs must be in each colliding cloud and the joint probability of finding both in an n $Q\overline{Q}$ pair state is n^{-2}, implying $\sigma_n \sim n^{-2}$ and an average multiplicity which grows as $\ln s$. Such a picture, of course would lead to a strict asymmetry in high-energy meson-baryon scattering which could only be eliminated by boosting to a Lorentz frame where $p_{\text{baryon}}/p_{\text{meson}} = \frac{3}{2}$.

CONCLUSION

It appears that quark-fragmentation models of high-energy hadronic production processes may provide a natural explanation of the inelasticity, the limitations on transverse momenta, and the abundance of pions observed in production reactions. If, in addition, the quarks are assumed to be very massive ($M \simeq 10$ GeV/c^2), the experimental elusiveness of quarks and the existence of cosmic-ray events with large transverse momenta at energies above 10^6 GeV can be explained.

ACKNOWLEDGMENT

We would like to thank Professor Leon Kaufman, who suggested this line of inquiry to one of us (T.R.M.). His encouragement and criticism provided the impetus for much of this work.

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[12] This will only occur at very high energies and momenta, so our model is, strictly speaking, only good for very-high-energy cosmic-ray collisions. The momentum required to justify our argument can be roughly estimated as follows. Suppose the collision time is the time it takes in the QQ c.m. frame for the Lorentz-contracted dressed quarks to pass through each other. This time is $t = 2l_0/v\gamma$, where l_0 is ~1 F. Since $p = Mv\gamma$ we have $t = 2Ml_0 p^{-1}$. The lifetime of an n-virtual-$Q\overline{Q}$ state in the c.m. frame is, from the uncertainty relation and the relativistic time dilation, approximately $\tau = \gamma\hbar(2nM)^{-1}$ or $\tau = (p^2+M^2)^{1/2}\hbar(2nM^2)^{-1}$. We now require $t \ll \tau$, which implies $4nM^3 l_0 \ll \hbar p(p^2+M^2)^{1/2}$. For large p, which we shall certainly require, the condition on the quark c.m. momentum becomes $4nM^3 l_0 \ll \hbar p^2$. Since the proton momentum in the QQ frame (which is also the pp c.m. frame) is $3p$, this requires a laboratory proton momentum of $P \gg 72nM^3 l_0 (m\hbar)^{-1}$ or $P \gg n(3.7\times 10^5)$ GeV/c. *Note added in Proof.* The model can be justified at lower momenta if the time required for a dressed quark to break up into fragments is very short. Then, the lifetime of virtual $Q\overline{Q}$ pairs in the dressed quark can be longer than the fragmentation time at moderate momenta and the model should be valid at momenta below 10^6 GeV/c.

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