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# Power Series, Analyticity, and Current-Hadron Scattering Amplitudes

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We propose to expand the current-hadron scattering amplitude as a double power series in the mass of the current and its laboratory energy. The series is summed by a double Sommerfeld-Watson transformation. Scaling is seen to provide a powerful constraint on the location of singularities in a complex parameter plane associated with the mass variable.

## I. INTRODUCTION

Recently there has been considerable interest in the properties of current-particle scattering amplitudes in connection with light-cone or null-plane commutators[1] and Bjorken scaling.[2] The analytic properties of the virtual current-hadron scattering amplitude as a function of the current mass and its laboratory energy figure significantly in such considerations. suri[3] made a systematic investigation of the analytic properties of the forward virtual Compton scattering amplitude in perturbation theory. Abarbanel *et al.*[4] pointed out that if scaling is to be consistent with Regge behavior, then the Regge residue function must behave in a specific manner as a function of the current mass when the latter grows very large.

In this paper we propose to study the current-hadron scattering amplitude by an approach advanced by Khuri[5] some time ago in connection with strong-interaction amplitudes. Our analysis involves a double-power-series representation of the current-particle scattering amplitude in the laboratory energy variable $\nu$ and the mass $\sigma$ of the current. We consider nonforward scattering, but restrict ourselves to a pairwise equal-mass configuration. The representation is extended outside the domain of convergence of the double power series by summing the series using a double Sommerfeld-Watson transform. As we remarked before, Khuri has applied this procedure earlier to strong-interaction amplitudes treated as functions of the Mandelstam variables. Fubini[6] applied the same technique, but starting from a single-power-series expansion in the energy variable to current amplitudes. By analogy with the Khuri plane we are led to introduce a complex parameter plane associated with the mass variable. We find that the property of scaling puts a stringent condition on the location of singularities in the latter plane.

Section II is devoted to the construction of the representation for the scattering amplitude, while in Sec. III we give some discussions and conclusions.

## II. REPRESENTATION FOR THE SCATTERING AMPLITUDE

We consider the process

$$J_1(q_1)+p_1\to J_2(q_2)+p_2\,, \tag{2.1}$$

where $J_{1,2}$ is a scalar current operator and $p_1, p_2$ designate hadronic states of the indicated momenta. We shall assume for the current-hadron scattering amplitude describing the process (2.1) the analytic structure embodied in the Deser-Gilbert-Sudarshan (DGS) representation.[7] As is well known this representation satisfies the usual requirements associated with the general postulates of quantum field theory: local commutativity, Lorentz invariance, and the existence of a complete set of physical states with positive energy. The representation satisfies all these requirements but is not dictated by them. We take the scattering amplitude to be given by the Fourier transform of the $T$ product of the local operators $J_i(x)$ ($i=1,2$):

$$T=i\int d^4x\, e^{iQ\cdot x}\langle p_2\,|\,T(J_1(\tfrac{1}{2}x)J_2(-\tfrac{1}{2}x))\,|\,p_1\rangle\,, \tag{2.2}$$

where we define

$$Q=\tfrac{1}{2}(q_1+q_2)\,,$$
$$P=\tfrac{1}{2}(p_1+p_2)\,,$$
$$\Delta=p_2-p_1\,.$$

The DGS representation for $T$ also involves the four-vector $K$, where

$$\overline{K}=\Delta-\frac{\Delta\cdot P}{P^2}\,P\,,$$
$$K=\frac{\overline{K}}{(\overline{K}^2)^{1/2}}\,,$$

$K$ being orthogonal to $P$. We now write the DGS representation for the nonforward time-ordered Green's function as[7]

$$T(P,Q,K) = \int_{\mu_0}^{\infty} d\mu \int d\beta d\gamma \frac{H(\mu,\beta,\gamma,)}{Q^2+2P\cdot Q\beta+2Q\cdot K\gamma-\mu+i\epsilon}. \tag{2.3}$$

Here we shall confine ourselves to equal-mass hadronic states, $p_1^2=p_2^2=m^2$, and we also take the current "masses" to be equal: $q_1^2=q_2^2=q^2=\sigma$. Defining $\nu=2P\cdot Q$, $t=\Delta^2$, we see that in this case $T$ can be written as

$$T(P,Q,\Delta) = \int_{\mu_0}^{\infty} d\mu \int d\beta d\gamma \frac{H(\mu,\beta,\gamma)}{\sigma-\frac{1}{4}t+\nu\beta-\mu+i\epsilon}. \tag{2.4}$$

We consider nonforward scattering in general and moreover restrict ourselves to the "normal" case discussed by DGS[7] which corresponds to $\mu_0>0$. The integration variable $\beta$ varies over the compact interval $[-1,1]$. This enables us to restrict ourselves to values of $\sigma$ and $\nu$ such that

$$|\sigma+\nu\beta| < \mu_0+\tfrac{1}{4}t$$

or

$$|\sigma\pm\nu| < \mu_0+\tfrac{1}{4}t \tag{2.5}$$

for some fixed small $t$ such that

$$|\tfrac{1}{4}t| < \mu_0. \tag{2.6}$$

Because of (2.5) and (2.6) we can drop the $i\epsilon$ prescription in (2.4) and expand the denominator as

$$\frac{1}{\mu+\frac{1}{4}t-(\sigma+\nu\beta)} = \sum_{\lambda=0}^{\infty} \frac{(\sigma+\nu\beta)^{\lambda}}{(\mu+\frac{1}{4}t)^{\lambda+1}}. \tag{2.7}$$

Next we expand

$$(\sigma+\nu\beta)^{\lambda} = \sum_{z=0}^{\lambda} (\nu\beta)^z (\sigma)^{\lambda-z} \binom{\lambda}{z}, \tag{2.8}$$

where

$$\binom{\lambda}{z} = \frac{\lambda!}{z!(\lambda-z)!}.$$

Rearranging sums we can now write

$$\frac{1}{\mu+\frac{1}{4}t-(\sigma+\nu\beta)} = \sum_{z=0}^{\infty}\sum_{\lambda=z}^{\infty} (\nu\beta)^z \sigma^{\lambda-z} (\mu+\tfrac{1}{4}t)^{-\lambda-1}\binom{\lambda}{z}. \tag{2.9}$$

The amplitude $T(P,Q,\Delta)$ is a function of the scalar variables $\nu$, $\sigma$, and $t$. Substituting the expansion (2.9) into (2.4), we express $T(\nu,\sigma,t)$ as a double power series:

$$T(\nu,\sigma,t) = \sum_{z=0}^{\infty}\sum_{\lambda=z}^{\infty} \nu^z\sigma^{\lambda-z} \frac{\Gamma(\lambda+1)}{\Gamma(z+1)\Gamma(\lambda-z+1)} \times C(\lambda,z;t), \tag{2.10}$$

where

$$C(\lambda,z;t) = -\int_{\mu_0}^{\infty} d\mu \int d\beta d\gamma (\mu+\tfrac{1}{4}t)^{-\lambda-1} \times \beta^z H(\mu,\beta,\gamma). \tag{2.11}$$

The series (2.10) converges when the inequalities (2.5) and (2.6) hold and provided $C(\lambda,z;t)$ exists for large $z$ and $\lambda$. No problems arise from the behavior at large $\lambda$. The large-$z$ behavior is connected to the support properties of the spectral function $H(\mu,\beta,\gamma)$. From the analysis of DGS[7] we know that the support of $H(\mu,\beta,\gamma)$ in $\beta$ is given by $|\beta|<1$ for $\gamma\neq 0$ and is bounded at its widest parts, which occur for $\gamma=0$, by the lines $\beta=\pm 1$. Thus if we require $H(\mu,\beta,0)=0$ for $\beta=\pm 1$, we are assured of the existence of the integral representation for $C(\lambda,z;t)$ for large $z$. We can now proceed to sum up the double power series (2.10) using a double Sommerfeld-Watson transform. Double and multiple Sommerfeld-Watson transforms have been considered before in the case of the two-particle and multiparticle amplitudes by Khuri,[5] White, Goddard and White, Weis, and Abarbanel and Schwimmer.[8]

We take the right-hand side of (2.11) to provide the analytic interpolation for $C(\lambda,z;t)$ in the complex $\lambda$ and $z$ planes. It is clear from the above discussion that there are no difficulties associated with the limit $\mathrm{Re}\, z\to\infty$ at fixed $\lambda$. It is also evident from (2.11) that $C(\lambda,z;t)\sim(\mu_0)^{-\mathrm{Re}\lambda}$ as $\mathrm{Re}\,\lambda\to\infty$ at fixed $z$. We also require suitable behavior as $\mathrm{Im}\,\lambda\to\infty$ and $\mathrm{Im}\, z\to\infty$ so that we can neglect all the contours at infinity. We can now write (2.10) as

$$T(\nu,\sigma,t) = (\tfrac{1}{2}i)^2 \int_{C_z} \frac{dz}{\sin\pi z} \int_{C_\lambda} \frac{d\lambda}{\sin\pi(\lambda-z)} (-\nu)^z(-\sigma)^{\lambda-z} \frac{\Gamma(\lambda+1)}{\Gamma(z+1)\Gamma(\lambda-z+1)} C(\lambda,z;t), \tag{2.12}$$

where $C_z$ and $C_\lambda$ are the contours in the complex $z$ and $\lambda$ planes, respectively, the nature of which will be discussed presently. Now in the case of double-power-series expansions in the Mandelstam variables $s$, $t$, and $u$, Khuri[5] has shown that the properties associated with the Froissart-Gribov projection ensured that the required analyticity conditions hold for the expansion coefficients in the complex index planes. Moreover Khuri's results would lead us to expect that the singularities in the complex $z$ plane are analogous to those believed to reside in the complex angular momentum or $J$ plane, i.e., poles and cuts. The correspondence between poles in the $z$ plane (Khuri poles) and those in the $J$ plane (Regge poles) has been established in Ref. 5. The question now is what kind of singularities reside in the complex $\lambda$ plane. We shall assume that the only singularities in the $\lambda$ plane are poles and cuts. We can think of the $\lambda$ plane as being associated with the mass variable $\sigma$ and our assumption can be regarded as some sort of a maximal analyticity postulate regarding the singularity structure in the complex plane associated with this variable. We thus require the two index planes to possess a not fundamentally different structure. In the ensuing analysis we shall neglect the presence of cuts for ease of discussion.

Going back now to (2.12) and using the relation

$$-\frac{\pi}{\sin\pi z}=\Gamma(-z)\,\Gamma(1+z)\,,$$

we can write (2.12) as

$$T(\nu,\sigma,t)=\left(\frac{i}{2\pi}\right)^2\int_{C_z}dz\int_{C_\lambda}d\lambda\,\Gamma(-z)\Gamma(-\lambda+z)\Gamma(\lambda+1)\times(-\nu)^z(-\sigma)^{\lambda-z}C(\lambda,z;t)\,. \tag{2.13}$$

We now discuss the location of contours in the $\lambda$ and $z$ planes. Roughly speaking, the contour of integration is such that the singularities needed to reproduce the expansion (2.10) lie to the right [e.g., the singularities of $\Gamma(-z)$ and $\Gamma(-\lambda+z)$], and the "dynamical" singularities in $C(\lambda,z;t)$ to the left. Figures 1(a) and 1(b) display the contours in the $\lambda$ and $z$ planes. If we carry out the $\lambda$ integration in (2.13) first, then clearly $C(\lambda,z;t)$ cannot have any singularities in $\lambda$ to the right of the contour because closing the contour to the right must produce the series expansion in $\lambda$ embodied in (2.10). Next we consider the integration of the resultant function over $z$. Again there can be no singularities to the right of the contour because closing the contour to the right must reproduce the expansion contained in (2.10). Such singularities will evidently be absent if $C(\lambda,z;t)$ does not possess singularities the location of which depends on $z$. However, this is not a necessary condition. In principle one can have singularities the position of which depends on $\lambda$ since they will be washed up in the $\lambda$ integration if they cannot pinch against

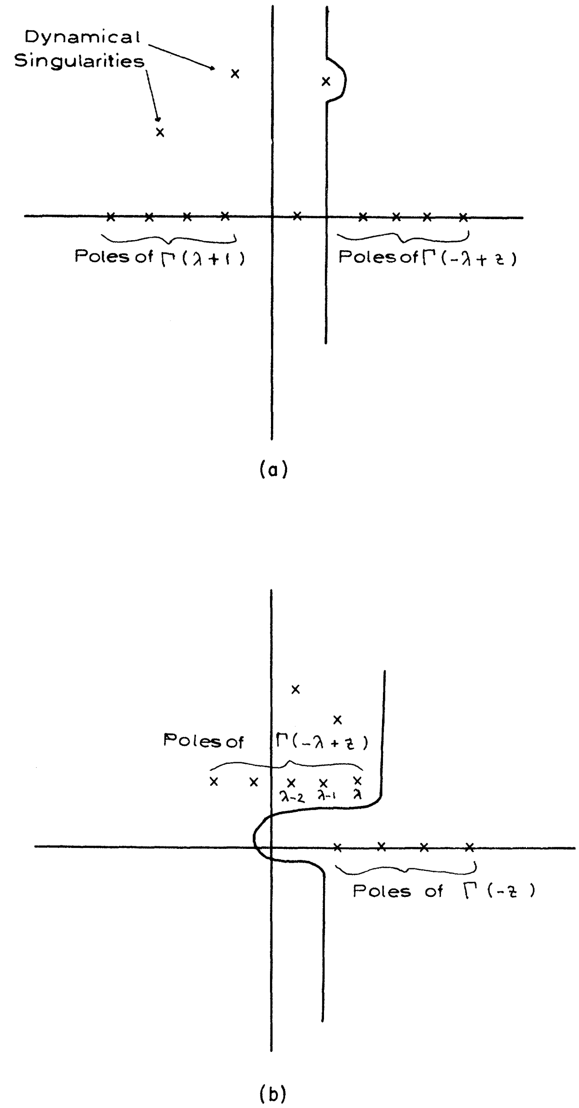


FIG. 1. (a) The contour $C_\lambda$; (b) the contour $C_z$.

$\Gamma(-\lambda+z)$.[9] However, our experience with the Khuri or $z$ plane (or the Regge plane) does not lead us to expect the existence of these kinds of singularities. Clearly singularities to the left of the contour exist and will contribute to the asymptotic behavior. So far we have been keeping $\nu$ and $\sigma$ small so that the inequalities (2.5) and (2.6) hold. Using (2.13) we can now analytically continue outside this range of values.

Our central result is then Eq. (2.13) expressing the current-hadron scattering amplitude as a double Sommerfeld-Watson transform. Let us now push the contour $C_\lambda$ to the left, picking up contributions from the poles of $\Gamma(\lambda+1)$ plus those from possible dynamical poles. We have

$$\int_{C_\lambda} d\lambda\ \Gamma(-z)\Gamma(-\lambda+z)\Gamma(\lambda+1)(-\nu)^z(-\sigma)^{\lambda-z}C(\lambda,z;t)$$
$$=\int_{X_\lambda} d\lambda\ \Gamma(-z)\Gamma(-\lambda+z)\Gamma(\lambda+1)(-\nu)^z(-\sigma)^{\lambda-z}C(\lambda,z;t)$$
$$-2\pi i\Big[\sum_{k=1}^{[X_\lambda]}\frac{(-)^{k-1}}{(k-1)!}\Gamma(-z)\Gamma(k+z)(-\nu)^z(-\sigma)^{-k-z}C(-k,z;t)$$
$$+\sum_i\Gamma(-z)\Gamma(-\zeta_i+z)\Gamma(\zeta_i+1)(-\nu)^z(-\sigma)^{\zeta_i-z}R_i(z;t)\Big],\quad (2.14)$$

where $X_\lambda$ is a vertical line parallel to the imaginary axis, $\mathrm{Re}\,\lambda=0$, and lying in the left half-plane. $[X_\lambda]$ is an integer such that $-[X_\lambda]$ is the nearest integer lying to the right of the vertical line $X_\lambda$. Dynamical poles in the $\lambda$ plane are taken to occur at $\lambda=\zeta_i$ and we define

$$R_i(z;t)=\operatorname*{res}_{\lambda=\zeta_i}C(\lambda,z;t).\quad (2.15)$$

Substituting (2.14) into (2.13) we obtain

$$T(\nu,\sigma,t)=\left(\frac{i}{2\pi}\right)^2\int_{C_z}dz\int_{X_\lambda}d\lambda\ \Gamma(-z)\Gamma(-\lambda+z)\Gamma(\lambda+1)(-\nu)^z(-\sigma)^{\lambda-z}C(\lambda,z;t)$$
$$-\frac{i}{2\pi}\int_{C_z}dz\Big[\sum_{k=1}^{[X_\lambda]}\frac{(-)^{k-1}}{(k-1)!}\Gamma(-z)\Gamma(k+z)(-\nu)^z(-\sigma)^{-k-z}C(-k,z;t)$$
$$+\sum_i\Gamma(-z)\Gamma(-\zeta_i+z)\Gamma(\zeta_i+1)(-\nu)^z(-\sigma)^{\zeta_i-z}R_i(z;t)\Big].\quad (2.16)$$

In the second term in (2.16) we now push the contour $C_z$ to the left and write

$$T(\nu,\sigma,t)=\left(\frac{i}{2\pi}\right)^2\int_{C_z}dz\int_{X_\lambda}d\lambda\ \Gamma(-z)\Gamma(-\lambda+z)\Gamma(\lambda+1)(-\nu)^z(-\sigma)^{\lambda-z}C(\lambda,z;t)$$
$$-\frac{i}{2\pi}\int_{X_z}dz\Big[\sum_{k=1}^{[X_\lambda]}\frac{(-)^{k-1}}{(k-1)!}\Gamma(-z)\Gamma(k+z)(-\nu)^z(-\sigma)^{-k-z}C(-k,z;t)$$
$$+\sum_i\Gamma(-z)\Gamma(-\zeta_i+z)\Gamma(\zeta_i+1)(-\nu)^z(-\sigma)^{\zeta_i-z}R_i(z;t)\Big]$$
$$+\sum_{k=1}^{[X_\lambda]}\sum_j\frac{(-)^{k-1}}{(k-1)!}\Gamma(-\alpha_j)\Gamma(k+\alpha_j)(-\nu)^{\alpha_j}(-\sigma)^{-k-\alpha_j}\beta_j(-k;t)$$
$$+\sum_i\sum_j\Gamma(-\alpha_j)\Gamma(-\zeta_i+\alpha_j)\Gamma(\zeta_i+1)(-\nu)^{\alpha_j}(-\sigma)^{\zeta_i-\alpha_j}\gamma_{ij}(t),\quad (2.17)$$

where we have taken the poles in the $z$ plane to occur at $z=\alpha_j$ and defined

$$\beta_j(-k;t)=\operatorname*{res}_{z=\alpha_j}C(-k,z;t),\quad (2.18)$$

$$\gamma_{ij}(t)=\operatorname*{res}_{z=\alpha_j}R_i(z;t).\quad (2.19)$$

In all of the above equations the pole trajectory functions $\alpha_j$ and $\zeta_i$ are of course functions of $t$. In (2.17), $X_z$ is a vertical line parallel to the imaginary axis lying between $-1$ and 0. We have not included in (2.17) contributions from the poles in $\Gamma(-\zeta_i+z)$, which occur at $z=\zeta_i-m$, $m=0,1,2,\ldots$. This is because, as we shall argue in the next section, $\mathrm{Re}\,\zeta_i\leq-1$ if $\nu\,\mathrm{Im}T$ scales in the manner of Bjorken.[2] By moving $X_z$ to the left we pick up contributions from these poles as well as contributions from the poles in $\Gamma(k+z)$. If, on the other hand, it is $\mathrm{Im}T$ that scales, then $\mathrm{Re}\,\zeta_i\leq0$ and some of the poles in $\Gamma(-\zeta_i+z)$ move into the interval $-1<\mathrm{Re}\,z<0$ and their contribution must be added to the right-hand side of (2.17). Finally, fixed poles may also be present and their contribution can be easily included.

## III. DISCUSSION AND CONCLUSIONS

We have obtained a representation for the current-hadron scattering amplitude in which the dependence on the energy $\nu$ and the mass $\sigma$ is made explicit. For the new index plane associated with the mass variable, we made the natural assumption that the only singularities that reside in it are poles and cuts. So far the location of singularities in the $\lambda$ plane is quite arbitrary. We now invoke the property of Bjorken scaling.[2] Suppose then that $\nu \operatorname{Im} T \to F(\omega)$ in the Bjorken limit defined by $\nu \to \infty$, $|\sigma| \to \infty$, with $-\sigma/\nu = \omega$ held fixed. From our representation (2.17) we see that this is possible if and only if

$$\operatorname{Re}\zeta_i \leq -1 \text{ for all } i\ . \tag{3.1}$$

Thus the property of scaling provides a powerful constraint on the location of singularities in the $\lambda$ plane. The relation (3.1) should be contrasted with the constraint $\alpha_j \leq 1$ which unitarity places upon the singularities in the angular momentum plane. Relation (3.1) tells us that if dynamical singularities exist in the $\lambda$ plane then they cannot appear to the right of $\operatorname{Re}\lambda = -1$. However, there is no compelling reason to have singularities at all in the $\lambda$ plane. The scaling law $\nu \operatorname{Im} T \to F(\omega)$ does not require them, and we see from (2.17) that we can have the scaling law in this form in their absence. This situation is particularly appealing, and the absence of singularities here is analogous to the situation with multiparticle amplitudes[10] where singularities in helicity do not occur independently of the angular momenta but are determined by the singularities in the angular momentum planes. In this circumstance we can say that scaling has a kinematical rather than a dynamical origin.

If, however, the scaling law has the form $\operatorname{Im} T \to F(\omega)$ in the Bjorken limit, then dynamical singularities in the $\lambda$ plane are essential and instead of (3.1) we will have the constraint

$$\operatorname{Re}\zeta_i \leq 0\ ,$$

with the position of the leading pole being $\zeta_l = 0$ for nontrivial scaling to occur. Scaling is then necessarily of a dynamical origin.

Recently the question of light-cone structure and asymptotic momentum-space properties of two- and three-point functions has been investigated by Andersson[11] using Mellin-transform techniques. The same techniques, in the context of the Jost-Lehmann-Dyson (JLD) representation,[12] have been employed by Bhaumik *et al.*[13] to analyze virtual Compton scattering. We would like to comment a little on the relationship of our work to theirs. These authors treated only the absorptive part of the virtual Compton amplitude and confined their analysis to the forward direction, which resulted in some simplification. On the other hand, in this paper we consider the full scattering amplitude and treat the general nonforward case. Our final results can be specialized to the absorptive part by taking the discontinuity. We used the DGS representation, rather than the JLD representation, to provide an expression for the coefficients of the double power series in the domain (2.5) and (2.6). Our approach is closer in spirit to the way in which the Regge plane was introduced in particle physics following the expansion of the scattering amplitude into a single series of partial waves. In fact, as we mentioned earlier, our analysis parallels that of Khuri,[5] who generalized the Regge analysis to the case of double-power-series expansion. However, in place of double dispersion relations given by the Mandelstam representation for strong-interaction amplitudes, which provided Khuri with an analytic expression for the expansion coefficients, here we had to use the $(\nu, \sigma)$ connection provided by the DGS representation. This is because a double dispersion representation in $\nu$ and $\sigma$ cannot be justified in general for the current-hadron scattering amplitude. Our basic result is the representation (2.13) in which the dependence of $T(\nu, \sigma, t)$ on $\nu$ and $\sigma$ is disentangled in a multiplicative manner.[14] Moreover the Regge behavior of the amplitude is clearly exhibited. The analysis of Ref. 13 is concerned with scaling properties and not with Regge properties.[15] Thus our representation is seen to make direct contact with the work of Abarbanel *et al.*[4] In fact the conjecture made by these authors concerning the scaling properties of Regge pole terms is seen to arise naturally in our representation, and the extra terms in Eq. (2.17) are seen to provide "corrections" to the form written in Ref. 4. The observation we made about the scaling behavior being kinematical in origin is also apparent in the results of Ref. 13.

Our representation may be of value in the analysis of current-algebra sum rules.[16,17] Its utility there, however, requires the assumption of the exchange of the integral entering the sum rules with those over $\lambda$ and $z$ occurring here. It would also be of interest to examine the situation with vector or axial-vector currents and spin states. The invariant amplitudes in such a situation are obtained by derivatives with respect to $\nu$ and the external current masses[6] from the amplitude describing a scalar situation like the one we considered here.

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# Regge-Slope Expansion in the Dual Resonance Model

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The expansion, in the Regge-slope parameter $\alpha'$, of simplified meson Born amplitudes comprising the Euler $B$ function and its multiparticle generalization is considered in order to proceed towards a Lagrangian density $\mathcal{L}$ which goes beyond the zero-slope limit of Scherk. A discussion of the Laurent-Taylor expansion of the four-meson amplitude leads, at least formally, to a Lagrangian density in terms of a scalar field $\phi$; this density is, however, nonlocal since it contains an infinite number of derivatives. Certain approximation schemes, in terms of higher-mass poles of the full amplitude, are introduced. Dimensionality arguments applied also to the multiparticle extension lead naturally to the expansion of $\mathcal{L}$ in the Regge-slope parameter $\alpha'$. The full Lagrangian density is given, up to order $\alpha'^2$ relative to the zero-slope limit. At this order it is necessary to consider only the four- and five-point functions.

## I. INTRODUCTION

The zero-slope limit of the scalar-boson Born amplitudes in the dual resonance model was first investigated by Scherk.[1] He has shown that the four-point amplitude represented by an Euler beta function and the $N$-point amplitude represented by the generalization of the beta function reduce, in the zero-slope limit, to the conventional $\phi^3$ Lagrangian theory. This demonstration and further work[2] along these lines are of considerable interest because they provide a linkage between the