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Equal-Time Commutator of Charge Densities in Quantum Electrodynamics

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We show that one-electron and one-photon expectation values of the equal-time commutator of charge densities in quantum electrodynamics, which are usually assumed to be zero, contain δ-function derivatives of order $n \geq 4$ when evaluated in fourth-order perturbation theory.

I. INTRODUCTION

A few years back Drell and Hearn[1] obtained a sum rule from dispersion relations and the low-energy theorem which relates the anomalous magnetic moment of the proton to an integral over photoabsorption cross sections. Many authors[2] have given an equal-time commutator method of derivation of this sum rule which is applicable to any spin-$\frac{1}{2}$ particle. In this derivation it is assumed that the electric charge densities commute at equal times. On the other hand, in the derivation given by Drell and Hearn, it is assumed that the spin-flip Compton amplitude satisfies an unsubtracted dispersion relation. As we shall show, this would be true if the electric current $\vec{j}(x)$ commutes with the potential $\vec{A}(y)$ at equal times. This assumption is equivalent to the vanishing of the commutator of the charge density with itself at equal times. The vacuum expectation value of this commutator can be shown to be zero. It is however, not clear whether its one-particle expectation value is also zero. A simple way to settle this question is to examine how far unsubtracted dispersion relations for Compton scattering as well as photon-photon scattering are satisfied when cross sections and amplitudes appearing in these relations are approximated by their perturbation-theory values. We find that these are not satisfied, showing thereby that $\vec{j}(x)$ does not commute with $\vec{A}(y)$ at equal times. Consequently $\rho(x)$ does not commute with $\rho(y)$ at equal times. We give explicit expressions for the one-electron and one-photon expectation values of

$$[\rho(x), \rho(y)]\delta(x_0 - y_0) .$$

II. ONE-ELECTRON EXPECTATION VALUE

To obtain the one-electron expectation value of the commutator under discussion, we start with the on-shell forward Compton scattering amplitude

$$f_{rs}(\omega) = t_{rs}(\omega) - i\int d^4x\, e^{ikx}\delta(x_0)\langle p|[j_s(0), \dot{A}_r(x)]|p\rangle + \omega\int d^4x\, e^{ikx}\delta(x_0)\langle p|[j_s(0), A_r(x)]|p\rangle , \tag{1}$$

where

$$t_{rs}(\omega) = i\int d^4x\, e^{ikx}\theta(x_0)\langle p|[j_s(0), j_r(x)]|p\rangle . \tag{2}$$

It follows from the structure of $t_{rs}(\omega)$ that the dispersive part $d_2(\omega^2)$, given by

$$f_{rs}(\omega)=\delta_{rs}d_1(\omega^2)+i\omega\epsilon_{rst}\sigma_t d_2(\omega^2),$$

and the cross section $[\sigma_P(\omega)-\sigma_A(\omega)]$ are related by

$$d_2(\omega^2)=\frac{1}{2\pi}\int_0^\infty \frac{d\omega'^2}{\omega'^2-\omega^2}[\sigma_P(\omega')-\sigma_A(\omega')]+\frac{1}{3i}\epsilon_{rst}\sigma_t\int d^4x\, e^{ikx}\delta(x_0)\langle p|[j_s(0),A_r(x)]|p\rangle . \tag{3}$$

Note that the commutator

$$\delta(x_0)\langle p|[j_s(0),\dot{A}_r(x)]|p\rangle ,$$

being proportional to δ_{rs}, contributes only to the spin-nonflip amplitude.[3] The last term on the right-hand side of Eq. (3) is usually taken to be zero. If this were so, one would have

$$d_2(0)=\frac{1}{2\pi}\int_0^\infty \frac{d\omega'^2}{\omega'^2-\omega^2}[\sigma_P(\omega')-\sigma_A(\omega')]\Big|_{\omega=0}, \tag{4}$$

$$\left[\frac{d^\eta}{d\omega^{2\eta}}d_2(\omega^2)\right]_{\omega=0}=\frac{1}{2\pi}\frac{d^\eta}{d\omega^{2\eta}}\left[\int_0^\infty \frac{d\omega'^2}{\omega'^2-\omega^2}[\sigma_P(\omega')-\sigma_A(\omega')]\right]\Bigg|_{\omega=0}. \tag{5}$$

In order to decide whether Eqs. (4) and (5) are valid or not, we calculate $d_2(\omega^2)$ in fourth-order perturbation theory. The fourth-order Compton amplitude can be written as[4]

$$T_{rs}^{(4)}(\omega)=\frac{ie^4}{(2\pi)^4}\sum_{l=1}^{4}\bar{u}(y_l^{rs}+y_l'^{rs})u, \tag{6}$$

where

$$y_1^{rs}=\gamma_s\frac{i(\hat{p}+\hat{k})-m}{(p+k)^2+m^2}\int\gamma_\mu\frac{i(\hat{p}+\hat{k}-\hat{q})-m}{(p+k-q)^2+m^2}\gamma_\mu\frac{d^4q}{q^2+\lambda^2}\frac{i(\hat{p}+\hat{k})-m}{(p+k)^2+m^2}\gamma_r, \tag{7}$$

$$y_2^{rs}=\int\frac{d^4q}{q^2+\lambda^2}\gamma_\mu\frac{i(\hat{p}-\hat{q})-m}{(p-q)^2+m^2}\gamma_s\frac{i(\hat{p}+\hat{k}-\hat{q})-m}{(p+k-q)^2+m^2}\gamma_\mu\frac{i(\hat{p}+\hat{k})-m}{(p+k)^2+m^2}\gamma_r, \tag{8}$$

$$y_3^{rs}=\gamma_s\frac{i(\hat{p}+\hat{k})-m}{(p+k)^2+m^2}\int\gamma_\mu\frac{i(\hat{p}+\hat{k}-\hat{q})-m}{(p+k-q)^2+m^2}\gamma_r\frac{i(\hat{p}-\hat{q})-m}{(p-q)^2+m^2}\gamma_\mu\frac{d^4q}{q^2+\lambda^2}, \tag{9}$$

$$y_4^{rs}=\int\gamma_\mu\frac{i(\hat{p}-\hat{q})-m}{(p-q)^2+m^2}\gamma_s\frac{i(\hat{p}+\hat{k}-\hat{q})-m}{(p+k-q)^2+m^2}\gamma_r\frac{i(\hat{p}-\hat{q})-m}{(p-q)^2+m^2}\gamma_\mu\frac{d^4q}{q^2+\lambda^2}. \tag{10}$$

Here p (k) are the electron (photon) momenta, and λ is the small mass given to the photon in order to eliminate the infrared divergence. The $y_l'^{rs}$ are obtained from y_l^{rs} by interchanging r and s, and by making the substitution $k\leftrightarrow -k$. The evaluation of these y's is fairly lengthy but straightforward; we give only the final result for the spin-flip Compton amplitude, which is[5]

$$\sum_{l=1}^{4}(y_l^{rs}+y_l'^{rs})=i\omega\epsilon_{rst}\sigma_l\sum_{s=1}^{\infty}\omega^{2s}m^{-2s}2^{2s+1}\left\{\left[\frac{s(4s^3+12s^2+15s+6)}{(s+1)^2(2s+1)^2}+\frac{2s^2(2s+3)}{(s+1)(2s+1)}\ln\frac{2\omega}{m}\right]\delta_{lt}-2\left(\delta_{lt}-\frac{k_lk_t}{\omega^2}\right)\left[\frac{s(3s+2)}{(s+1)^2(2s+1)^2}+\frac{2s^2}{(s+1)(2s+1)}\ln\frac{2\omega}{m}\right]\right\}. \tag{11}$$

Since $\epsilon_{rsk}\epsilon_{rst}\sigma_k\sigma_l(\delta_{lt}-k_lk_t/\omega^2)=0$, the second term does not contribute to $d_2(\omega^2)$. The spin-flip amplitude which follows from (11) is[6]

$$d_2(\omega^2)=r_0^{\,2}\sum_{s=1}^{\infty}\omega^{2s}m^{-2s}2^{2s+1}\left[\frac{s(4s^3+12s^2+15s+6)}{(s+1)^2(2s+1)^2}+\frac{2s^2(2s+3)}{(s+1)(2s+1)}\ln\frac{2\omega}{m}\right]. \tag{12}$$

We now use the spin-flip Compton scattering cross sections[7]

$$[\sigma_P(\omega)-\sigma_A(\omega)]=4\pi r_0^{\,2}\left[\frac{1+4\gamma+5\gamma^2}{\gamma(1+2\gamma)^2}-\frac{(1+\gamma)\ln(1+2\gamma)}{2\gamma^2}\right],\quad \gamma=\frac{\omega}{m}, \tag{13}$$

on the right-hand side and the expression (12) on the left-hand side of Eqs. (4) and (5) to see if these identities are satisfied. We find that, whereas identity (4) is satisfied (both sides being zero), Eq. (5) is not:

$$\left[\frac{d}{d\omega^2}\,d_2(\omega^2)\right]_{\omega=\lambda}=\frac{r_0^{\,2}}{m^2}\left(\frac{114}{9}+\frac{120}{9}\ln\frac{2\lambda}{m}\right), \tag{14}$$

while the right-hand side is

$$\frac{1}{2\pi}\frac{d}{d\omega^2}\left\{\int_0^\infty\frac{d\omega'^2}{\omega'^2-\omega^2}\left[\sigma_P(\omega')-\sigma_A(\omega')\right]\right\}\Bigg|_{\omega=\lambda}=\frac{r_0^{\,2}}{m^2}\left(\frac{194}{9}+\frac{120}{9}\ln\frac{2\lambda}{m}\right). \tag{15}$$

Clearly the right-hand side is different from the left-hand side, which shows that

$$\delta(x_0)\langle p|[j_s(0),A_r(x)]|p\rangle\neq 0\,.$$

Assuming

$$\delta(x_0)\langle p|[j_s(0),A_r(x)]|p\rangle=i\epsilon_{rst}\sigma_t\left[G_0^e+\sum_{\eta=1}^{\infty}G_\eta^e(-\vec{\nabla}^2)^\eta\right]\delta^4(x), \tag{16}$$

where G_0^e, G_1^e, G_2^e, ..., are constants, we find, upon using Eq. (16) in Eq. (3), the following formulas for the various coefficients G_0^e, G_1^e, ..., G_η^e:

$$G_0^e=d_2(0)-\frac{1}{\pi}\int_0^\infty\frac{d\omega'^2}{\omega'^2-\omega^2}\left[\sigma_P(\omega')-\sigma_A(\omega')\right]\Bigg|_{\omega=0}, \tag{17}$$

$$G_\eta^e=\left[\frac{d^\eta}{d\omega^{2\eta}}d_2(\omega^2)\right]_{\omega=\lambda}-\frac{d^\eta}{d\omega^{2\eta}}\left\{\frac{1}{2\pi}\int_0^\infty\frac{d\omega'^2}{\omega'^2-\omega^2}\left[\sigma_P(\omega')-\sigma_A(\omega')\right]\right\}\Bigg|_{\omega=\lambda}. \tag{18}$$

We see from Eq. (17) that the Drell-Hearn sum rule will be modified if G_0^e is different from zero. The coefficients G_η^e can be explicitly calculated in fourth-order perturbation theory by using Eqs. (12) and (13) in Eqs. (17) and (18). We find

$$G_0^e=0, \tag{19}$$

$$G_1^e=-\frac{80}{9}\frac{r_0^{\,2}}{m^2}, \tag{20}$$

$$G_2^e=-\frac{1984}{45}\frac{r_0^{\,2}}{m^4}. \tag{21}$$

Since the equal-time commutator $[j_s(0),A_r(x)]\delta(x_0)$ is related to $[E_s(0),E_r(x)]\delta(x_0)$ as well as $[\rho(0),\rho(x)]\times\delta(x_0)$ by virtue of Maxwell's equations, we have

$$\delta(x_0)\langle p|[E_s(0),E_r(x)]|p\rangle=i\epsilon_{rst}\sigma_t\left[G_0^e+\sum_{\eta=1}^{\infty}G_\eta^e(-\vec{\nabla}^2)^\eta\right]\delta^4(x), \tag{22}$$

$$\delta(x_0-y_0)\langle p|[\rho(y),\rho(x)]|p\rangle=i\epsilon_{rst}\sigma_t\partial_s^y\partial_r^x\left[G_0^e+\sum_{\eta=1}^{\infty}G_\eta^e(-\vec{\nabla}^2)^\eta\right]\delta^4(x-y), \tag{23}$$

as a consequence of which the equal-time commutation relation for dipole moment operator becomes

$$\langle p|[D_r(t),D_s(t)]|p\rangle=i\epsilon_{rst}\sigma_tG_0^e. \tag{24}$$

The modified Drell-Hearn sum rule given in Eq. (17) can be derived from the above commutation relation by employing techniques used in Ref. 2. As noted earlier, $G_0^e=0$ in fourth-order, which is in keeping with the fact that $d_2(0)$ as well as the integral appearing in Eq. (17) are also zero in this order. If G_0^e happens to be nonzero in sixth-or higher orders, the Drell-Hearn sum rule would no longer be valid; it would have to be replaced by our Eq. (17).

III. ONE-PHOTON EXPECTATION VALUE

We shall now show that in addition to the one-electron expectation value, the one-photon expectation value of the commutator of electric fields at equal times is also nonzero. To obtain this, we start with the one-shell forward photon-photon scattering amplitude represented by

$$M_{jris}(\omega)=t_{jrjs}(\omega)-i\omega\int d^4x\,e^{ikx}\delta(x_0)\langle k,e_j|[j_s(0),\dot{A}_r(x)]|k,e_j\rangle+\omega^2\int d^4x\,e^{ikx}\delta(x_0)\langle k,e_j|[j_s(0),A_r(x)]|k,e_j\rangle, \tag{25}$$

with

$$t_{jrjs}(\omega)=i\omega\int d^4x\, e^{ikx}\theta(x_0)\langle k, e_j|[j_s(0), j_r(x)]|k, e_j\rangle. \tag{26}$$

Here the polarization of one of the photons during collision changes from e_r to e_s, while that of the other remains unaltered. If we decompose the amplitudes in the manner

$$M_{jris}(\omega)=\delta_{jr}\delta_{is}F(\omega)^2+\delta_{js}\delta_{ir}F(-\omega^2)+\delta_{ij}\delta_{rs}H(\omega^2), \tag{27}$$

we find that Eq. (25) is equivalent to

$$\begin{aligned} D(\omega^2)&=\tfrac{1}{2}\mathrm{Re}[F(\omega^2)-F(-\omega^2)]\\ &=\frac{1}{2\pi}\int_0^\infty \frac{d\omega'^2}{\omega'^2-\omega^2}[\sigma_{++}(\omega')-\sigma_{+-}(\omega')]+\sum_{\eta=0}^{\infty}G_\eta^\gamma\omega^{2\eta}, \end{aligned} \tag{28}$$

where

$$\delta(x_0)\langle k, e_j|[j_s(0), A_r(x)]|k, e_j\rangle=i\epsilon_{rst}S_t\left[\sum_{\eta=0}^{\infty}G_\eta^\gamma(-\vec{\nabla}^2)^\eta\right]\delta^4(x), \tag{29}$$

$\vec{S}$ being photon spin and σ_{++} (σ_{+-}) being the total cross sections for collisions between two circularly polarized photons with parallel (antiparallel) polarizations. Note that the second term on the right-hand side of Eq. (25), being proportional to δ_{rs}, does not contribute to $D(\omega^2)$. It follows from Eq. (28) that

$$G_0^\gamma=D(0)-\frac{1}{\pi}\int_0^\infty\frac{d\omega}{\omega}[\sigma_{++}(\omega)-\sigma_{+-}(\omega)], \tag{30}$$

$$G_\eta^\gamma=\left[\frac{1}{\eta!}\frac{d^\eta}{d\omega^{2\eta}}D(\omega^2)\right]_{\omega=0}-\frac{1}{\pi}\int_0^\infty\frac{d\omega}{\omega^{2\eta+1}}[\sigma_{++}(\omega)-\sigma_{+-}(\omega)],\quad \eta=1,2,\ldots. \tag{31}$$

Let us now calculate these coefficients in fourth-order perturbation theory. The amplitude $D(\omega^2)$ to order e^4 given by Karplus and Neuman[8] is

$$D(\omega^2)=\frac{2r_0^2}{\gamma^2}\{3[B(\gamma^2)-B(-\gamma^2)]-[T(\gamma^2)-T(-\gamma^2)]\},\quad \gamma=\omega/m, \tag{32}$$

where the transcendental functions $B(u)$ and $T(u)$ are defined by

$$B(u)=\tfrac{1}{2}\int_0^1 dy\ln[1-i\epsilon-4uy(1-y)]\underset{|u|\ll 1}{\sim}-\tfrac{1}{3}u-\frac{2}{15}u^2-\frac{8}{105}u^3\cdots,$$

$$T(u)=\tfrac{1}{4}\int_0^1\frac{dy}{y(1-y)}\ln[1-i\epsilon-4uy(1-y)]\underset{|u|\ll 1}{\sim}-u-\tfrac{1}{3}u^2-\frac{8}{45}u^3\cdots.$$

The cross sections appearing on the right-hand side of Eqs. (30) and (31), to order e^4, are the pair production corss sections for collision of circularly polarized photons which are given by the Breit-Wheeler[9] formula

$$\sigma_{++}(\omega)-\sigma_{+-}(\omega)=\pi r_0^2\left[\frac{4}{\gamma^2}\cosh^{-1}\gamma-\frac{6}{\gamma^3}(\gamma^2-1)^{1/2}\right]. \tag{33}$$

Usings Eqs. (32) and (33) in Eqs. (30) and (31), we find $G_0^\gamma=0$, while all other G_η^γ are nonzero. The typical low-order coefficient G_1^γ is given by

$$G_1^\gamma=\frac{28r_0^2}{45m^2}. \tag{34}$$

We therefore conclude that the one-photon expectation value of the equal-time commutator of j_r with A_s, and hence that of E_r with E_s, or $\rho(y)$, is nonzero. It will be noticed that G_1^γ for the one-photon expectation value given above is different from the corresponding coefficient in the one-electron expectation value given in Eq. (20). We therefore conclude that the δ-function derivatives of arbitrary order $\eta\geq 4$ in the equal-time commutator of electric charge densities in quantum electrodynamics are q numbers.

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[5]For the calculation of the fourth-order Compton amplitude we need an explicit expression for $\Sigma_R^{(2)}(p)$ (regularized value of the electron self-energy). The expression for this given by Akhiezer and Berestetskii (Ref. 4) as well as by Jauch and Rohrlich, [*Theory of Photons and Electrons* (Addison-Wesley, Reading, Mass., 1955)] is incorrect. Professor Rohrlich is aware of this error. The correct expression for $\Sigma_R^{(2)}(p)$ is

$$\Sigma_R^{(2)}(p) = \frac{i\alpha}{2\pi}\frac{(\hat{P}-im)^2}{m} \times \left\{ \frac{1}{2(1-\rho)}\left(1-\frac{2-3\rho}{1-\rho}\ln\delta\right) + \frac{(i\hat{p}-m)}{m}\left[\frac{1}{2\rho(1-\rho)}\left(2-\rho+\frac{-4+4\rho+\rho^2}{1-\rho}\ln\rho\right) + \frac{1}{\rho}\left(1+\ln\frac{\lambda^2}{m^2}\right)\right]\right\} \qquad (\rho=-2\omega/m).$$

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Role of Positrons in Compton Scattering

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Projection operators are derived which split the electromagnetic interaction into a "pairlike" interaction and a "fieldlike" interaction. The separation is applied to lowest-order electromagnetic phenomena. All first-order electromagnetic scattering of charged particles is shown to arise from pairlike transitions. For Compton scattering, the pairlike interaction is shown to yield the Klein-Nishina formula. The primitive self-energy of an electron is shown to vanish for the pairlike interaction alone, if related boundary conditions are imposed on the photon propagator. The usual ultraviolet divergence can thus be attributed to the fieldlike interaction.

I. INTRODUCTION

The divergences of electrodynamics, in both classical and quantum theory, originate in the simplest electromagnetic interactions. In the classical theory, radiative reaction yields "runaway" or, alternatively, noncausal solutions to the equation of motion. In the quantum theory, the nonvanishing amplitude for forward Compton scattering may be identified as the (nonsummed) source of the infinite primitive self-energy of the electron. While the existence of the divergences suggests that the electromagnetic interaction is in some way "nonminimal" or redundant, the close relationship of the divergences to observed physical phenomena (including the Lamb shift in higher-order interactions) seems to imply that the divergences are essential elements of the theory, which cannot be avoided except by a procedure of renormalization. The overwhelming success of the renormalized theory, as evidenced by the precision of its agreement with numerous experiments, leads further to this conclusion that the postulated interactions are correct.

Nevertheless, we have sought a "more minimal" interaction to explain simple electromagnetic phenomena. We have been at least partially successful. We introduce projection operators to split the electromagnetic interaction into a "fieldlike" interaction and a "pairlike" interaction. The pairlike interaction is shown to account for lowest-order electromagnetic phenomena, and to yield a vanishing primitive self-energy of the electron.

The derivations are carried out in the formalism