Remark on Baryon Conservation*

A. Pais
Rockefeller University, New York, New York 10021
(Received 27 March 1973)

The Higgs mechanism can serve to implement baryon conservation via an extension of the local weak-electromagnetic gauge group by a local factor U(1) without conflict with the Eötvös experiments.

(1) Charge conservation and baryon conservation are believed to be equally absolute.[1] The former law emerges in the dynamical context of a strict local gauge invariance. On the other hand, no convincing dynamical framework has been found so far for baryon conservation. To be sure, one may postulate a local gauge invariance for this purpose, in straight analogy to the electromagnetic case. This implies the existence of a neutral massless vector field and of a long-range repulsive force between baryons, proportional to Γ^2, where Γ (the analog of e) is the baryonic charge. However, the important observation was made long ago[2] that Γ is severely bounded by the experimental limits on the variance from substance to substance of the gravitational to inertial mass ratio as observed in the Eötvös experiments.[3] From the recent improved measurements by Roll, Krotkov, and Dicke[4] one deduces

$$\frac{\Gamma^2}{4\pi\hbar c} \lesssim 10^{-43}\,\alpha, \tag{1}$$

where $\alpha = \frac{1}{137}$. Equation (1) (which will be referred to as the Eötvös constraint) follows from the fact that a Coulombic force arises between massive bodies which of course carry macroscopic baryonic charges ΓA (A = the mass number). One can admit an infinitesimal Γ which satisfies Eq. (1), but this does not seem attractive. Or one can dispense altogether with the idea of a long-range field and put in baryon conservation by hand as a gauge invariance of the first kind.[5] One may speculate whether a dynamical clue is lost in doing so. This raises the following question: Can one give a dynamical context to baryon conservation such that no infinitesimal couplings are introduced and yet such that the Eötvös constraint is circumvented?

It is the purpose of this note to point out that baryon conservation can be associated with a *local* U(1) gauge group, provided one makes use of the Higgs mechanism.[6] This can be done in such a way that this conservation remains absolute while yet it is associated with a vector field of short range. Before giving the simple details, it is useful to comment first on another facet of the problem, namely the quantization of charge.

(2) As is well known, the Abelian nature of the electromagnetic gauge group precludes any insight into the problem of why charge is quantized.[7] In simplest terms, if we treat electromagnetism as a separate phenomenon, then the equality of the positron and the proton (bare) charge is to be put in by hand (after which the ratio is stable under renormalization). In the recent attempts to formulate a renormalizable unified field theory of weak and electromagnetic phenomena,[8] where one assumes the existence of a local "weak-electromagnetic" gauge group $\mathcal{G}$, the possibility arises for an understanding of charge quantization. However, if $\mathcal{G}$ contains an Abelian factor U(1) and if the charge operator contains the U(1) generator (as happens in numerous models) then charge quantization continues to be an ingredient extraneous to the group structure.

We are faced with the same quantization problem if we attempt to associate baryon conservation with a U(1) group. Clearly one will want to introduce a number of fundamental fermion fields with a *common* baryonic charge. It is a limitation on what follows that the U(1) mechanism described below provides as little reason for the commonness of baryonic charge as does either the "classical" U(1) description of electromagnetism or a number of variants of the unified $\mathcal{G}$ description for electric charge. In any event, we shall confine the discussion in this note to the following limited objectives.

(i) To extend $\mathcal{G}$ to $\mathcal{G} \times$U(1), where U(1) is to be the local gauge responsible for baryon conservation.

(ii) To assume the existence of a number of fundamental fermion fields Q_n (commonly thought of as quark fields) which enter in the representations deployed within $\mathcal{G}$ and which carry a common baryonic charge Γ.

(3) As a result of the extension of $\mathcal{G}$ an extra gauge field U'_μ appears, scalar with respect to $\mathcal{G}$. The only fields coupling to U' are taken to be the Q_n mentioned above[9] and one electrically neutral complex scalar field ϕ coupled with strength Γ_ϕ. Let $\mathcal{L}(\mathcal{G})$ be the strictly renormalizable Lagrangian for $\mathcal{G}$.[10] Then the full Lagrangian $\mathcal{L}$ equals $\mathcal{L}(U'_\mu, \phi)$

$+\mathcal{L}(\mathcal{G})$, where

$$\mathcal{L}(U'_\mu, \phi) = -\tfrac{1}{4}G_{\mu\nu}{}^2 - |D_\mu\phi|^2 - \mu^2|\phi|^2 - f|\phi|^4 - J_\mu U'_\mu ,$$
$$D_\mu = \partial_\mu - i\Gamma_\phi U'_\mu ,\qquad (2)$$
$$J_\mu = i\Gamma \sum_n \overline{Q}_n \gamma_\mu Q_n ,$$

with $G_{\mu\nu} = \partial_\mu U'_\nu - \partial_\nu U'_\mu$, $\mu^2 < 0$, $f > 0$. $\mathcal{L}$ is strictly renormalizable since ϕ *cannot couple invariantly and renormalizably* to any fermions.[11] This absence of ϕ-fermion couplings is crucial to the argument. It implies that the ϕ current $J_\mu^{(\phi)} = i\Gamma_\phi[\phi^\dagger D_\mu\phi - (D_\mu\phi)^\dagger\phi]$ is separately conserved. Thus there exist two nonvanishing charges, the fermion charge $\int J_0 d^3x$ and the ϕ charge $\int J_0^{(\phi)} d^3x$. In going to the asymmetric ϕ vacuum we shall lose the ϕ charge but retain the fermion charge, which now, however, will be associated with a vector field the quanta of which are massive.

We proceed by familiar steps to the unitarity gauge to see the particle content: Set $\phi = 2^{-1/2} \times(\lambda + S)\exp(i\,\theta/\lambda)$, with $\lambda = (-\mu^2 f^{-1})^{1/2}$; let $U'_\mu = U_\mu + (\Gamma_\phi\lambda)^{-1}\partial_\mu\theta$; regauge the Q_n appropriately; drop a constant and find that $\mathcal{L} = \mathcal{L}(\mathcal{G}) + \mathcal{L}(U_\mu, S)$, where

$$\mathcal{L}(U_\mu, S) = -\tfrac{1}{4}G_{\mu\nu}{}^2 - \tfrac{1}{2}M^2U_\mu{}^2 - \tfrac{1}{2}(\partial_\mu S)^2 - \tfrac{1}{2}m^2S^2 - \tfrac{1}{2}\Gamma_\phi^2 U_\mu{}^2(2\lambda S + S^2) - f(\lambda S^3 + \tfrac{1}{4}S^4) - J_\mu U_\mu , \qquad (3)$$

with $M^2 = \Gamma_\phi^2\lambda^2$, $m^2 = \mu^2 + 3f\lambda^2$. $S(x)$ is a real "baryonically neutral" scalar field. Equation (3) no longer contains any separate scalar-particle conservation. The only remaining baryonic fields are the Q_n, and $\mathcal{L}$ manifestly conserves baryon number. [The factor U(1) forbids from the outset the introduction of $\mathcal{G}$ representations with mixed baryon-lepton content.]

(4) A few additional comments: (a) The argument is independent of the magnitude of Γ. Thus the interaction $J_\mu U_\mu$ may be identified with a strong vector-gluon coupling. (b) If this is done, the strong-interaction part of $\mathcal{L}$ [that is, $\mathcal{L}(U, S)$ augmented with the kinetic-energy terms of the Q fields, borrowed from $\mathcal{L}(\mathcal{G})$] is invariant under a global group $U(N)\times U(N)$, where N is the number of Q fields. In other words, the assumption (ii) which is accidental [that is, not dictated by the $\mathcal{G}\times U(1)$ group structure], when combined with the existence of $\mathcal{G}$, leads to an "induced" global chiral hadronic symmetry, spontaneously broken via the $\overline{Q}Q$ Higgs couplings contained in $\mathcal{L}(\mathcal{G})$. It is an interesting question whether some such inductions of hadron symmetries are possible in less accidental ways. (c) Obviously, the present argument does not exclude the possibility that hadron symmetries find their origins, in whole or in part, in further local gauges.[12] (d) Important constraints may arise for any vector-gluon picture as a result of the anomaly problem, as emphasized by Georgi and Glashow.[13] (e) The S particles are coupled to U and thus to the Q fields. I have so far not seen any physical consequences of these S couplings which may be considered as a distinct signature of the model. Under these circumstances, the present note should therefore be considered only as methodological in character. (f) One may contemplate a similar Abelian device for lepton conservation.

Added notes: (1) An alternative dynamical approach to the baryon gauge problem was suggested some time ago by Schwinger.[14] (2) I thank J. Logan for drawing my attention to the recent work of Braginskii and Panov[15] which strengthens the bound in Eq. (1) to $3\times 10^{-45}\,\alpha$. (3) Bounds for couplings of long-range fields to lepton number and to muon number have been given by Okun'.[16]

Upon the completion of this work I learned that the idea that the vector gluon can acquire mass through spontaneous breakdown of baryon gauge invariance without loss of the usual global baryon conservation was also known to S. Weinberg. I thank Professor Weinberg for a discussion of his ideas and Professors B. W. Lee and I. Bars for valuable comments and criticism.

---

*Work supported in part by the U. S. Atomic Energy Commission under Contract No. AT(11-1)2232.

[1]Cf. G. Feinberg and M. Goldhaber, Proc. Natl. Acad. Sci. U. S. A. 45, 1301 (1959).

[2]T. D. Lee and C. N. Yang, Phys. Rev. 98, 1501 (1955).

[3]R. Eötvös, E. Fekete, and D. Pekar, Ann. Physik 68, 11 (1922).

[4]P. G. Roll, R. Krotkov, and R. H. Dicke, Ann. Phys. (N.Y.) 26, 442 (1964).

[5]W. Pauli, Rev. Mod. Phys. 13, 203 (1941).

[6]P. W. Higgs, Phys. Lett. 12, 132 (1964); Phys. Rev. Lett. 13, 508 (1964); Phys. Rev. 145, 1156 (1966). Cf. also B. Zumino, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 2, p. 285.

[7]See, e.g., C. N. Yang, Phys. Rev. D 1, 2360 (1970). I thank B. W. Lee for a discussion of this point.

[8]See B. W. Lee, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, 1972* (Ref. 6), Vol. 4, p. 249.

[9]The relative scale of the baryonic charge of the Q_n and

of ϕ can always be set so as to forbid trilinear Q-lepton-ϕ couplings. This question arises if and only if $\mathcal{G}$ contains Q and lepton representations which are abstractly identical. (Among the numerous other ways to forbid such couplings one may note the possible existence of lepton-number gauge fields.) I want to thank H. Pagels for a discussion on this point.

[10]For the technical meaning of this term and for earlier references see, e.g., T. Hagiwara and B. W. Lee, Phys. Rev. D 7, 459 (1973). The symbol $\mathcal{L}(\mathcal{G})$ is meant to imply that specific choices for lepton, quark, and scalar field representations have been made.

[11]To be precise, one more term should be added to $\mathcal{L}$, namely, $|\phi|^2$ times a linear superposition of all $H^{(i)\dagger}H^{(i)}$, where the $H^{(i)}$ are the Higgs fields of representation type (i), as they appear in $\mathcal{L}(\mathcal{G})$. These terms do not affect the argument.

[12]Cf. I. Bars, M. B. Halpern, and M. Yoshimura, Phys. Rev. D 7, 1233 (1973); Y. Achiman, Nucl. Phys. B (to be published); J. C. Pati and A. Salam, Phys. Rev. D 8, 1240 (1973).

[13]H. Georgi and S. Glashow, Phys. Rev. D 6, 429 (1972).

[14]J. Schwinger, Phys. Rev. 125, 397 (1962); see also B. Zumino, Phys. Lett. 10, 224 (1964).

[15]V. B. Braginskii and V. I. Panov, Zh. Eksp. Teor. Fiz. 61, 873 (1971) [Sov. Phys.–JETP 34, 463 (1972)].

[16]L. B. Okun', Yad. Fiz. 10, 358 (1969) [Sov. J. Nucl. Phys. 10, 206 (1969)].

PHYSICAL REVIEW D VOLUME 8, NUMBER 6 15 SEPTEMBER 1973

High-Energy Expansions of Scattering Amplitudes*

Stephen J. Wallace

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

(Received 12 April 1973)

The partial-wave series is converted without approximation to a Fourier-Bessel expansion based on a new (infinite series) expansion for the Legendre function. The direct connection between the Fourier-Bessel phase shift and the partial-wave interpolating phase shift is established as an infinite series in powers of K^{-2} (K = wave number). The series contains the Glauber eikonal approximation as a leading term and reproduces the results of an eikonal expansion about the Glauber propagator. Corrections to the eikonal approximation are developed and rules are given for an unambiguous interpretation of the eikonal expansion. The relativistic eikonal expansion is discussed for forward and backward scattering without small-angle approximations.

I. INTRODUCTION

The problem of obtaining high-energy limits of scattering amplitudes is one of general interest in physics. In this paper, attention is focused on the high-energy expansion of the Fourier-Bessel representation of scattering amplitudes, as it has been obvious for a long time that high-energy scattering through small angles is very conveniently treated by means of eikonal (or straight-line path) approximations. One of the simplest and most successful theories of this type was developed by Glauber,[1] who noted the advantages of a straight-line path parallel to the average momentum. By introducing such a path, Glauber obtained a Fourier-Bessel representation of the scattering amplitude which embodies approximate unitarity. The Fourier-Bessel representation is advantageous because its existence can be justified for all angles on general grounds of analyticity in the momentum transfer as shown by Blankenbecler and Goldberger.[2]

However, all derivations of eikonal approximations require a small scattering angle in some sense. As a result many variants of the approximation[3] have arisen in attempts to extend the angular range of validity. In principle, the number of possible variants of the eikonal approximation is unlimited. This is because, for nonforward scattering, the set of rays which represents the eikonal approximation to the scattering wave function can be imagined to propagate through the interaction in innumerable ways, each of which generates a new variant of the approximation. Since the question of angular range of validity for any particular variant of the approximation has remained open, there has been no compelling reason to believe any of them was good for large-angle scattering.[4]

Several studies of the high-energy limit of scattering have been made by means of converting the partial-wave sum to an integral over impact parameters. For example, Glauber showed that his average-momentum-direction eikonal approximation could be obtained in such a manner if the Legendre polynomials were replaced by a Bessel function $J_0(qb)$, where $q = 2K\sin\frac{1}{2}\theta$ and $b = (l+\frac{1}{2})/K$. Similar methods have been used to examine for-