

## Real Parts of Amplitudes and Polarization of $\pi N$ Charge-Exchange Scattering

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Using fixed- $t$  dispersion relations, low-energy phase shifts (or  $s$ -channel resonances), and the imaginary parts of the  $s$ -channel helicity amplitudes of analyses at 6 GeV, we determine the real parts, the differential cross sections, and the polarizations for  $\pi^- p \rightarrow \pi^0 n$  at several energies. The approach is parameter-free and almost model-independent, and leads to results in agreement with experiment and with amplitude analyses.

### I. INTRODUCTION

An important recent development in hadron physics has been the amplitude analysis of two-body reactions.<sup>1,2</sup> Its main conclusions for  $\pi^- p \rightarrow \pi^0 n$  are the following:

(1) The imaginary parts of both the  $s$ -channel helicity-nonflip ( $F_{++}$ ) and helicity-flip ( $F_{+-}$ ) amplitudes are peripheral, i.e., considered as functions of the momentum transfer  $\sqrt{-t}$  they have the structure of the Bessel functions  $J_0(R\sqrt{-t})$  and  $J_1(R\sqrt{-t})$ ,  $R \approx 1$  fermi, in accord with the demands of the dual absorption model (DAM).<sup>3</sup>

(2) The real part of  $F_{+-}$  is rather nonperipheral, but is related to the corresponding imaginary part by

$$\operatorname{Re} F_{+-}(s, t) \approx \operatorname{Im} F_{+-}(s, t) \tan \frac{1}{2} \pi \alpha(t)$$

where  $\alpha(t)$  is the  $\rho$  Regge trajectory. Thus, in a first approximation,  $F_{+-}(s, t)$  can be described by simple exchange of the  $\rho$  Regge pole [with a non-sense wrong-signature zero (NWSZ) in its residue].

(3) The real part of  $F_{++}$  is strongly nonperipheral, and is not simply related to  $\operatorname{Im} F_{++}$ .

The purpose of the present work is the following: Using as input the imaginary parts of the amplitude analyses of  $\pi^- p \rightarrow \pi^0 n$  at 6 GeV, it tries to deduce the essential features (in magnitude and structure) of the real parts  $\operatorname{Re} F_{+\pm}$ ; and, moreover, to calculate the differential cross sections and polarizations at 6 GeV and at higher energies. The approach is based on the following tools:

(a) fixed- $t$  dispersion relations;

(b) at low energy, information on the imaginary parts of the amplitudes in terms of either phase-shift analysis or pion-nucleon resonances;

(c) at high energy, a simple parametrization of  $\operatorname{Im} F_{+\pm}(s, t)$ , consistent with the spirit of DAM.

A calculation along these lines [at the specific values  $t = t_+ \approx -0.175$  GeV<sup>2</sup> and  $t = t_- \approx -0.5$  GeV<sup>2</sup>, where  $\operatorname{Im} F_{+\pm}(s, t_{\pm}) = 0$ , respectively, and using

only  $\pi N$  resonances for (b)] with satisfactory results has already been reported.<sup>4</sup> Here the approach is extended to all values of  $t$  ( $\geq -0.625$  GeV<sup>2</sup>) and is carried in terms of both phase shifts and resonances.

Section II contains the formulation of the basic relations and Sec. III presents our results, discusses their main aspects, and compares them with independent calculations carried along similar lines.<sup>5,6</sup>

### II. BASIC RELATIONS

The  $s$ -channel helicity amplitudes  $F_{++}$  and  $F_{+-}$  for the reaction  $\pi^- p \rightarrow \pi^0 n$  are given in terms of the usual invariant amplitudes  $A$  and  $B$  by<sup>7</sup>

$$\frac{4\pi\sqrt{s}}{M} \left( \frac{1+z_s}{2} \right)^{-1/2} F_{++}(\nu, t) = A(\nu, t) + \left( \nu - \frac{t}{4M} \right) B(\nu, t) \quad (2.1)$$

and

$$\begin{aligned} \frac{4\pi\sqrt{s}}{M} \left( \frac{1-z_s}{2} \right)^{-1/2} F_{+-}(\nu, t) \\ = \left( \frac{M}{4s} \right)^{1/2} \left( \frac{4M\nu - t + 4M^2}{2M^2} A + \frac{4M\nu - t + 4\mu^2}{2M} B \right), \end{aligned} \quad (2.2)$$

where  $M(\mu)$  is the nucleon (pion) mass,  $z_s$  the cosine of the  $s$ -channel c.m. scattering angle, and

$$\nu = (s - u)/4M. \quad (2.3)$$

$A$  and  $B$  satisfy the (unsubtracted) fixed- $t$  dispersion relations<sup>8</sup>

$$\begin{aligned} \operatorname{Re} A(\nu, t) = \frac{1}{\pi} P \int_{\nu_0}^{\infty} d\nu' \operatorname{Im} A(\nu', t) \\ \times \left( \frac{1}{\nu' - \nu} - \frac{1}{\nu' + \nu} \right) \end{aligned} \quad (2.4)$$

and

$$\begin{aligned} \operatorname{Re} B(\nu, t) = & B_p(\nu, t) + \frac{1}{\pi} P \int_{\nu_0}^{\infty} d\nu' \operatorname{Im} B(\nu', t) \\ & \times \left( \frac{1}{\nu' - \nu} + \frac{1}{\nu' + \nu} \right), \end{aligned} \quad (2.5)$$

with  $B_p$  the nucleon pole term:

$$B_p(\nu, t) = -\frac{\sqrt{2} g^2}{2M} \left( \frac{1}{\nu_B - \nu} + \frac{1}{\nu_B + \nu} \right); \quad (2.6)$$

in these expressions

$$\nu_0 = \mu + \frac{t}{4M}, \quad \nu_B = -\frac{\mu^2}{2M} + \frac{t}{4M}, \quad (2.7)$$

and<sup>9</sup>

$$g^2/4\pi = 14.6.$$

The helicity amplitudes are so normalized that the differential cross section  $d\sigma/dt$  and the polarization  $P$  for  $\pi^- p \rightarrow \pi^0 n$  at high energy are given by

$$\begin{aligned} \frac{d\sigma}{dt} = & \frac{4\pi}{s} (|F_{++}|^2 + |F_{+-}|^2), \\ P \frac{d\sigma}{dt} = & \frac{8\pi}{s} \operatorname{Im}(F_{++} F_{+-}^*). \end{aligned} \quad (2.8)$$

Finally, since we are interested in energies  $\geq 6$  GeV and  $t$  fixed in  $0 \leq -t \leq 0.7$  GeV<sup>2</sup> it will be sufficient to use in (2.8) the approximate expressions

$$\frac{4\pi\sqrt{s}}{M} F_{++}(\nu, t) = A(\nu, t) + \nu B(\nu, t) \quad (2.9)$$

and

$$\frac{4\pi\sqrt{s}}{M} F_{+-}(\nu, t) = \frac{(-t)^{1/2}}{s} [(\nu + M)A(\nu, t) + \nu B(\nu, t)]. \quad (2.10)$$

Our approach consists in assuming the following forms for the imaginary parts of the helicity amplitudes:

$$\begin{aligned} \operatorname{Im} \left[ \left( \frac{1+z_s}{2} \right)^{-1/2} F_{++}(\nu, t) \right] \\ = -\frac{\sqrt{2}M}{4\pi\sqrt{s}} \beta_+(t) \left( \nu - \frac{t}{4M} \right)^{\alpha_+(t)} \end{aligned} \quad (2.11)$$

to be used for all<sup>10</sup>  $\nu \geq \nu_0$ , and

$$\operatorname{Im} F_{+-}(\nu, t) = -\frac{\sqrt{2}M\sqrt{-t}}{4\pi\sqrt{s}} \beta_-(t) \nu^{\alpha_-(t)} \quad (2.12)$$

to be used for  $\nu \geq \nu_M$ , where  $\nu_M$  will be specified below. In these expressions  $\alpha_+(t)$  and  $\alpha_-(t)$  represent effective Regge trajectories; in general there is no need to have  $\alpha_+(t) = \alpha_-(t)$ . Nevertheless, for simplicity, we shall take them equal, and, in fact, identical to the  $\rho$  trajectory<sup>11</sup>:

$$\alpha_+(t) = \alpha_-(t) = 0.47 + 0.9t. \quad (2.13)$$

Next we notice that the quantity  $A(\nu, t) + \nu B(\nu, t)$  has definite  $s \leftrightarrow u$  crossing symmetry and satisfies a fixed- $t$  dispersion relation. Equations (2.4) and (2.5) lead to

$$\begin{aligned} \operatorname{Re} [A(\nu, t) + \nu B(\nu, t)] = & \nu B_p(\nu, t) + \frac{1}{\pi} P \int_{\nu_0}^{\infty} d\nu' \operatorname{Im} \left[ A(\nu', t) + \left( \nu' - \frac{t}{4M} \right) B(\nu', t) \right] \left( \frac{1}{\nu' - \nu} - \frac{1}{\nu' + \nu} \right) \\ & + \frac{t}{4M} \frac{1}{\pi} P \int_{\nu_0}^{\infty} d\nu' \operatorname{Im} B(\nu', t) \left( \frac{1}{\nu' - \nu} - \frac{1}{\nu' + \nu} \right). \end{aligned} \quad (2.14)$$

In view of Eqs. (2.1) and (2.11):

$$\operatorname{Im} \left[ A(\nu, t) + \left( \nu - \frac{t}{4M} \right) B(\nu, t) \right] = -\sqrt{2} \beta_+(t) \left( \nu - \frac{t}{4M} \right)^{\alpha_+(t)} \quad (2.15)$$

so that the first integral of (2.14) can be readily calculated. The second integral we break into two pieces:  $\nu_0 \leq \nu \leq \nu_M$  and  $\nu_M \leq \nu < \infty$ . The low-energy part is calculated through known phase shifts or by saturating  $\operatorname{Im} B$  with known resonances; thus  $\nu_M$  is specified by the limiting energies of the present phase-shift analyses ( $\sqrt{s_M} \approx 2$  GeV). To calculate the high-energy part ( $\nu_M \leq \nu < \infty$ ) we use (2.9) and (2.10) to express  $\operatorname{Im} B(\nu, t)$  in terms of the functions  $\beta_{\pm}(t)$  of (2.11) and (2.12); the result is:

$$\operatorname{Im} B(\nu, t) = -\sqrt{2} (\nu + M) \beta_+(t) \nu^{\alpha_+(t)-2} + 2\sqrt{2} M \beta_-(t) \nu^{\alpha_-(t)-2} \quad (2.16)$$

(here  $\nu \gg \nu_0$ ). With the definitions

$$B_{\text{low}}(\nu, t) = B_p(\nu, t) + \frac{t}{4M\nu\pi} P \int_{\nu_0}^{\nu_M} d\nu' \operatorname{Im} B(\nu', t) \left( \frac{1}{\nu' - \nu} - \frac{1}{\nu' + \nu} \right), \quad (2.17)$$

and

$$S_M(\nu, \alpha) \equiv \frac{1}{\pi} P \int_{\nu_M}^{\infty} d\nu' \nu'^{\alpha} \left( \frac{1}{\nu' - \nu} - \frac{1}{\nu' + \nu} \right), \quad (2.18a)$$

$$S_0(\nu, \alpha) \equiv \frac{1}{\pi} P \int_{\nu_0}^{\infty} d\nu' \left( \nu' - \frac{t}{4M} \right)^{\alpha} \left( \frac{1}{\nu' - \nu} - \frac{1}{\nu' + \nu} \right), \quad (2.18b)$$

we finally have

$$\begin{aligned} \operatorname{Re} F_{++}(\nu, t) = & \frac{M}{4\pi\sqrt{s}} \left\{ \nu B_{\text{low}}(\nu, t) - \sqrt{2} \beta_+(t) S_0(\nu, \alpha_+) \right. \\ & \left. - \frac{\sqrt{2} t}{4M} [\beta_+(t) S_M(\nu, \alpha_+ - 1) + M \beta_+(t) S_M(\nu, \alpha_+ - 2) - 2M \beta_-(t) S_M(\nu, \alpha_- - 1)] \right\}. \end{aligned} \quad (2.19)$$

For the helicity-flip amplitude Eq. (2.10) gives

$$\operatorname{Re} F_{+-}(\nu, t) = \frac{M\sqrt{-t}}{4\pi s^{3/2}} [M \operatorname{Re}(A + \nu B) + \nu \operatorname{Re} A].$$

Through (2.9)  $\operatorname{Re}(A + \nu B)$  is related directly to  $\operatorname{Re} F_{++}$ . For  $\operatorname{Re} A$  we again break the integral of (2.4) into a low-energy and a high-energy part; in the latter ( $\nu_M \leq \nu < \infty$ ) we use

$$\operatorname{Im} A(\nu, t) = -2\sqrt{2} M \beta_-(t) \nu^{\alpha_-(t)} + \sqrt{2} M \beta_+(t) \nu^{\alpha_+(t)-1}. \quad (2.20)$$

With the definition

$$A_{\text{low}}(\nu, t) = \frac{1}{\pi} \int_{\nu_0}^{\nu_M} d\nu' \operatorname{Im} A(\nu', t) \left( \frac{1}{\nu' - \nu} - \frac{1}{\nu' + \nu} \right), \quad (2.21)$$

we obtain

$$\operatorname{Re} F_{+-}(\nu, t) = \frac{M\sqrt{-t}}{4\pi s^{3/2}} [4\pi\sqrt{s} \operatorname{Re} F_{++} + \nu A_{\text{low}} - 2\sqrt{2} M \beta_-(t) \nu S_M(\nu, \alpha_-) + \sqrt{2} M \beta_+(t) \nu S_M(\nu, \alpha_+ - 1)]. \quad (2.22)$$

Our calculation of  $\operatorname{Re} F_{\pm\pm}$  proceeds with (2.19) and (2.22).

### III. RESULTS AND DISCUSSION

We calculate the imaginary parts of the integrals of  $B_{\text{low}}$  and  $A_{\text{low}}$ , Eqs. (2.17) and (2.21), in two different ways: (I) in terms of phase-shift analysis<sup>12</sup> and (II) by saturating through  $\pi N$  resonances. In the latter case we work in the narrow-resonance approximation, where the contribution of a resonance of spin  $J$ , mass  $M_J$ , width  $\Gamma_J$ , and elasticity  $x_J$  to the imaginary part of the partial wave  $f_{(J \pm 1/2)^\mp}$  is given by

$$\operatorname{Im} f_{(J \pm 1/2)^\mp} = \frac{\pi}{q} M_J \Gamma_J x_J \delta(s - M_J^2)$$

( $q = \text{c.m. momentum}$ ; notation of Ref. 7). Our resonance parameters are the same as in Ref. 4 and will not be repeated here.

The imaginary parts of  $F_{\pm\pm}$  used as inputs in our calculations are shown in Fig. 1. The calculated real parts at 6 GeV are given in Fig. 2; the resulting differential cross sections at 6, 9.8, and 13.3 GeV in Fig. 3 and the polarizations at 6, 8, and 11.2 GeV in Fig. 4.

The first observation is that the two ways of calculating  $B_{\text{low}}$  and  $A_{\text{low}}$ , through phase shifts and resonances, lead to very similar results.<sup>13</sup> This supports the usual assumption that the imaginary parts of the low-energy amplitudes of nondiffractive reactions are dominated by direct-channel resonances.

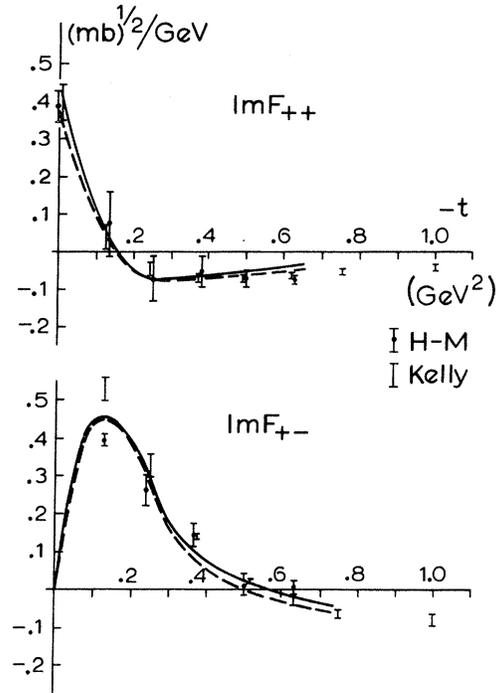


FIG. 1. Imaginary parts of helicity-nonflip ( $F_{++}$ ) and -flip ( $F_{+-}$ ) amplitudes at 6 GeV used as input. Continuous lines: input in the calculation with phase shifts; dashed lines: input with  $\pi N$  resonances. Data: H-M as in Ref. 1(b); Kelly as in Ref. 2.

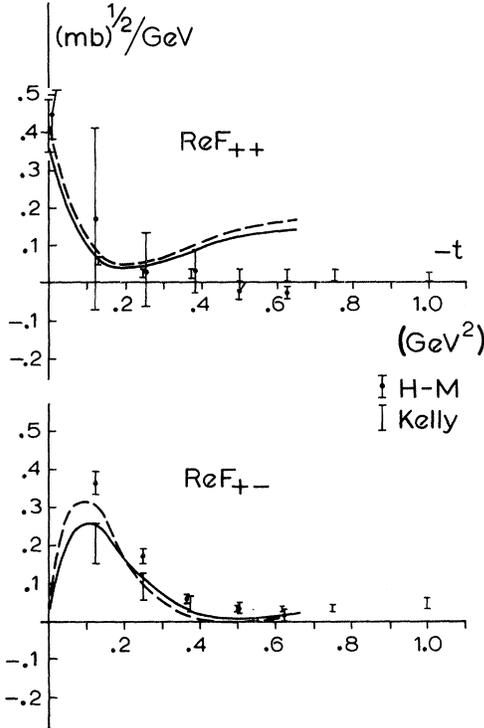


FIG. 2. Calculated real parts of helicity-nonflip and -flip amplitudes at 6 GeV. Continuous and dashed lines and data as in Fig. 1.

Next, our results for  $\text{Re}F_{+-}$  throughout  $0 \leq -t \leq 0.625$  can be considered as very satisfactory. Clearly, the calculated  $\text{Re}F_{+-}$  exhibits a “double-zero” structure at  $t \approx -0.5 \text{ GeV}^2$ . This is in accord with dominance of  $F_{+-}$  by unabsorbed  $\rho$  Regge exchange (with NWSZ) and is clearly exhibited in Kelly’s analysis. In fact, throughout  $0 \leq -t \leq 0.625$  the resulting  $\text{Re}F_{+-}$  is related to the input  $\text{Im}F_{+-}$  by, roughly, the ratio  $\tan\pi\alpha(t)/2$ .

Up to  $-t \approx 0.4$  the calculated  $\text{Re}F_{++}$  are very satisfactory, as well. For  $-t > 0.5$  they exceed Kelly’s by factors of 2 to 4. As a result, at large  $|t|$  our differential cross sections also exceed the experimental by factors of 2 to 4 and our polarizations drop somewhat too fast. Nevertheless, on the whole, our approach does reproduce the basic features of  $\text{Re}F_{++}$  (lack of peripherality) as well as of  $d\sigma/dt$  and  $P$ . Notice that as  $|t|$  increases certain theoretical difficulties (like rapidity of convergence of the partial-wave expansions) may significantly affect our method.

Our results are in qualitative agreement with Ref. 5. However, there are important *quantitative* differences, e.g., in the magnitude of the predicted polarizations. This is due to using, in the dispersion relation (2.14), the expression (2.15) down to threshold (a procedure justified on grounds of

duality). Notice that our work as well as Ref. 5 use as input imaginary parts of amplitude analyses performed on the basis of the *same* polarization data (the 6- and 8-GeV data of Fig. 4); it is thus of interest that our approach predicts polarizations in better agreement with these data.<sup>14</sup>

We conclude with a more general remark. Harari and Schwimmer<sup>15</sup> have shown that if at all  $s$  the imaginary part of the  $s$ -channel helicity amplitude  $F_{\pm}(s, t)$  shrinks indefinitely as  $s \rightarrow \infty$ , then the strong-interaction radius  $R$  must increase with  $s$ . In our approach the energy dependence of  $\text{Im}F_{\pm}(s, t)$  is controlled by the  $\rho$  Regge trajectory [of nonzero slope, Eq. (2.13)]; hence,  $\text{Im}F_{\pm}(s, t)$  does shrink indefinitely. On the other hand  $R = \text{const}$  ( $\approx 1$  fermi), as follows from the fact that we take the functions  $\beta_{\pm}(t)$  of (2.11) and (2.12) to be independent of energy. Thus, strictly speaking, our approach is in contradiction with Ref. 15. Nevertheless for the particular type of calculation we are interested in, we can easily satisfy the demands of Ref. 15 by assuming that our expressions for  $\text{Im}F_{\pm}(s, t)$  hold up to a certain very high-en-

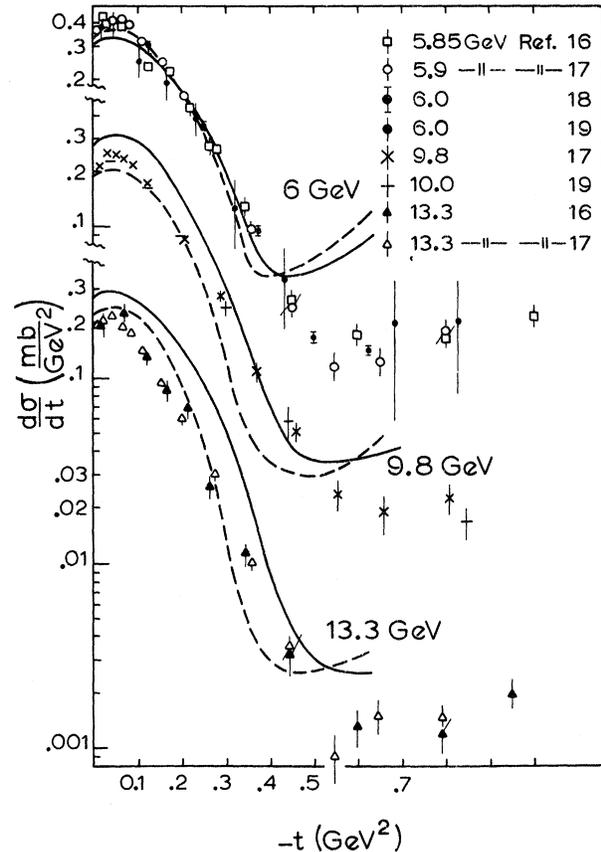


FIG. 3. Predicted differential cross sections at 6, 9.8, and 13.3 GeV.

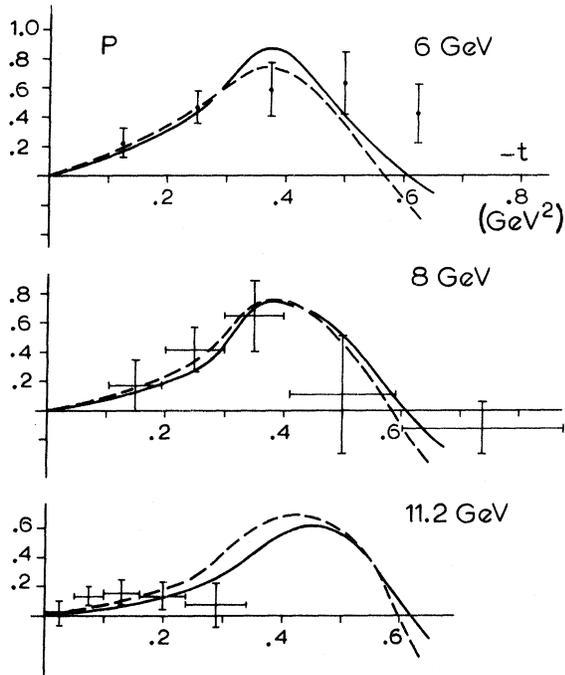


FIG. 4. Predicted polarizations at 6, 8, and 11.2 GeV. Data as in Ref. 20.

ergy  $\bar{\nu}$  (say  $\bar{\nu} \sim 500$  GeV), whereas for  $\nu > \bar{\nu}$ ,  $\text{Im}F_{+\pm}$  are somewhat modified [e.g., allowing  $R$  to vary like  $(\ln\nu)^{1/2}$ ]. Since the contributions to the dispersion integrals of Eqs. (2.18) from  $\nu > \bar{\nu}$  are completely negligible, our numerical results will be unaffected.

Another important question is why we fail to reproduce the experimental  $\text{Re}F_{++}$  for  $|t| \geq 0.4$ . The simplest way to improve our results would be to use  $\alpha_+(t) \neq \alpha_-(t) = \alpha_p(t)$ ; this is suggested by the fact that, in contrast to  $F_{+-}$ , the  $t$  structure of  $\text{Im}F_{++}$  cannot be accounted for by a simple  $\rho$  Regge pole. Indeed, we have found that, e.g., at  $t = -0.5$ , by keeping  $\alpha_-(t)$  as in Eq. (2.13), but taking  $\alpha_+(t) = 0.25$  (i.e., somewhat higher than before) we decrease  $\text{Re}F_{++}$  ( $t = -0.5$ ) by about 20%, while  $\text{Im}F_{++}$  ( $t = -0.5$ ) remains practically unaffected. It is perhaps of interest to remark that if, in addition to the  $\rho$  Regge pole,  $F_{++}$  receives contribution from a  $\rho$ -Pomeranchukon cut, then the effective  $\alpha_+(t)$  will be higher than  $\alpha_-(t)$  for  $t < 0$ .

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<sup>1</sup>(a) F. Halzen and C. Michael, *Phys. Lett.* **36B**, 367 (1971). (b) C. Michael, CERN Report No. CERN-TH 1480 (unpublished) (Review talk at the Fourth International Conference on High Energy Collisions, Oxford, 1972); this contains further references.

<sup>2</sup>R. L. Kelly, *Phys. Lett.* **39B**, 635 (1972).

<sup>3</sup>H. Harari, *Phys. Rev. Lett.* **26**, 1400 (1971).

<sup>4</sup>E. N. Argyres and A. P. Contogouris, *Phys. Rev. D* **6**, 2018 (1972).

<sup>5</sup>M. Coirier, J. Guillaume, Y. Leroyer, and P. Salin, *Nucl. Phys.* **B44**, 157 (1972); G. I. Ghandour and R. G. Moorhouse, *Phys. Rev. D* **6**, 856 (1972). These papers appeared when most of our work was completed.

<sup>6</sup>A similar approach ("hybrid" calculation) is also mentioned by R. Field, Jr. and J. D. Jackson [*Phys. Rev. D* **4**, 693 (1971)] in connection with hypercharge-exchange reactions.

<sup>7</sup>R. J. Eden, *High Energy Collisions of Elementary Particles* (Cambridge Univ. Press, New York, 1967), Chap. 3.6.

<sup>8</sup>G. Chew, M. Goldberger, F. Low, and Y. Nambu, *Phys. Rev.* **106**, 1337 (1957).

<sup>9</sup>The invariant amplitudes  $A$  and  $B$  of Eqs. (2.1)–(2.6) are related to the invariants  $A^{(-)}$  and  $B^{(-)}$  of Ref. 8 by  $A = -\sqrt{2}A^{(-)}$  and  $B = -\sqrt{2}B^{(-)}$ .

<sup>10</sup>The quantity  $\nu - t/4M$  is the pion lab energy, so that

our expression (2.11) is in conformity with Regge expressions usually employed in extrapolations to the resonance region [e.g., R. Dolen, D. Horn, and C. Schmid, *Phys. Rev.* **166**, 1768 (1968)].

<sup>11</sup>In Ref. 4 we have varied  $\alpha_+(t)$  at  $t \approx -0.5$  GeV<sup>2</sup> in the range  $-0.2 \leq \alpha_+ \leq 0.2$ ; our results were not significantly affected.

<sup>12</sup>D. Herndon, A. Barbaro-Galtieri, and A. H. Rosenfeld, LBL Report No. UCRL-20030  $\pi N$ , 1970 (unpublished); also CERN experimental solution (unpublished).

<sup>13</sup>We have carried out calculations using the Saclay phase shifts of Ref. 12, as well. The difference of the results obtained via the CERN and the Saclay phase shifts exceeds the difference of those obtained via resonances and the CERN phase shifts.

<sup>14</sup>A recent Argonne experiment at 5 GeV [D. Hill *et al.*, in Proceedings of the Sixteenth International Conference on High Energy Physics, Chicago-Batavia, 1972 (unpublished)], gives significantly smaller  $\pi^- p \rightarrow \pi^0 n$  polarizations. An amplitude analysis on the basis of the Argonne data (A. Yokosawa, private communication) leads to  $\text{Im}F_{+\pm}$  which, although in accord with DAM, differ in magnitude from those of Refs. 1 and 2. With the  $\text{Im}F_{+\pm}$  of Argonne as inputs our approach leads to different results.

<sup>15</sup>H. Harari and A. Schwimmer, *Phys. Rev. D* **5**, 2780 (1972). We thank H. Harari for raising this question.

<sup>16</sup>P. Sonderegger *et al.*, *Phys. Lett.* **20**, 75 (1966).

<sup>17</sup>A. V. Stirling *et al.*, *Phys. Rev. Lett.* **14**, 763 (1965).

<sup>18</sup>G. Giacomelli, P. Pini, and S. Stagni, CERN Report No. CERN-HERA 69-1, 1969 (unpublished); M. Yvert

(unpublished); O. Guisan (unpublished).

<sup>19</sup>M. A. Wahlig *et al.*, Phys. Rev. **168**, 1515 (1968).

<sup>20</sup>Polarization data. 6 GeV: P. Bonamy *et al.*, in *Proceedings of the Amsterdam International Conference on Elementary Particles, 1971*, edited by A. G. Tenner

and M. Veltman (North-Holland, Amsterdam, 1972); 8 GeV: O. Guisan *et al.*, Saclay report, 1971 (unpublished); 11.2 GeV: P. Bonamy *et al.*, Phys. Lett. **23**, 501 (1966).

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## Quark-Fragmentation Models for Very-High-Energy Production Processes

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The qualitative features of models for high-energy production processes based on the fragmentation of the dressed quarks assumed to make up hadrons are discussed. It appears that, if quarks are very massive (i.e., quark mass  $\approx 10 \text{ GeV}/c^2$ ), such models could explain in a simple way the experimental elusiveness of quarks, the limitations on transverse momenta observed in production reactions, the average inelasticity observed in cosmic-ray proton interactions, the abundance of pions produced in high-energy collisions, and the large transverse momenta observed in air showers for primary particle energies above  $10^6 \text{ GeV}$ .

### INTRODUCTION

There are indications that known hadrons are composites of more fundamental particles. If a theory of strong interactions is to be developed in terms of these particles, called quarks for convenience, all strong-interaction processes must be described in terms of quarks, and ordinary hadrons should appear only as quark bound states.

We present a qualitative discussion of a class of models for very-high-energy production reactions in which baryons are composed of three quarks, mesons are composed of a quark ( $Q$ ) and an anti-quark ( $\bar{Q}$ ), and the only processes which can take place are the production and absorption of  $Q\bar{Q}$  pairs. The interactions between quarks are due to a direct self-interaction and  $Q\bar{Q}$  pair (meson) exchange. We assume that quarks are very massive and we arbitrarily set the quark mass  $M$  at  $10 \text{ GeV}/c^2$  in this paper. It does not matter if there are three or nine quarks in these models, as long as the baryons are tightly bound states of three massive particles and mesons are tightly bound states of a massive particle and antiparticle.

### QUARK-FRAGMENTATION MODELS

The class of production models we consider, which we call quark-fragmentation models, also

make the following set of assumptions:

(1) Hadrons are composed of "dressed" quarks, so each quark is surrounded by a virtual  $Q\bar{Q}$  cloud. It is assumed that virtual  $Q\bar{Q}$  pairs are produced isotropically with an average momentum of about  $300 \text{ MeV}/c$  in the rest frame of the quark. By the uncertainty principle this leads to a characteristic radius of the virtual  $Q\bar{Q}$  cloud of a dressed quark of around  $\frac{2}{3} F$ .

(2) At very high energies, hadronic collisions usually consist of a single scattering of a quark in one hadron and a quark in the other.<sup>1,2</sup> Thus, the hadronic scattering amplitude is the sum of contributions from all diagrams in which a (dressed) quark in one colliding hadron scatters off a dressed quark in the other colliding hadron, while the other quarks making up the hadrons continue undisturbed as spectators.<sup>3</sup>

(3) The only production mechanism is the isotropic production of virtual  $Q\bar{Q}$  pairs with low momentum ( $\approx 0.3 \text{ GeV}/c$ ) in the quark rest frame.

(4) Quark-quark scattering is predominantly forward ( $p_T \approx 0$ ) and results in the fragmentation of the colliding dressed quarks. In this fragmentation, the dressed quarks share their momentum with the constituents of their virtual  $Q\bar{Q}$  clouds. Thus, the mechanism by which a hadron loses momentum in a high-energy collision is through the sharing of the momentum of one dressed quark in