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PHYSICAL REVIEW D VOLUME 8, NUMBER 1 1 JULY 1973

Dip in High-Energy pp Scattering and the Proton Substructure*

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(Received 17 January 1973)

The recently observed dip in high-energy elastic pp scattering is explained in the framework of models in which the nucleon possesses a layered substructure.

During recent years two types of models for high-energy elastic pp scattering have attracted much interest:

(a) geometrical and Regge versions of the diffraction model[1];

(b) models in which the proton exhibits a layered substructure.[2, 3]

An attractive feature of some of type (a) models is the dip, compatible with recent experimental results from the CERN Intersecting Storage Rings (ISR),[4] which is predicted at high energy for $-2 \lesssim t \lesssim -1$ (GeV/c)^2.

It is quite interesting to see whether a similar dip pattern can be simply explained in the framework of type (b) models. The purpose of this paper is to show that a dip situated in the interval $-2.0 \lesssim t \lesssim -1.2$ (GeV/c)^2 is predicted from a type (b) model; it is also shown that the above result holds even including the single-flip amplitude.

It is well known that a finite sum of Gaussians provides an empirical fit to $d\sigma/dt$.[2] However, such a formula is theoretically unacceptable since it violates the Cerulus-Martin bound.[5] This difficulty was overcome by Fleming, Giovannini, and Predazzi[3] (hereafter FGP) who developed a theoretically consistent model and also obtained an excellent fit for $d\sigma/dt$ over the whole angular region at pre-ISR energies.

From the FGP approach, a picture emerged which visualizes the proton as possessing infinitely many layers. The higher the transverse momentum, the farther in the layer that gets excited in the collision.[6]

Furthermore, it was shown by FGP that at a given x, $x = \beta k \sin\theta \equiv \beta k_\perp$ (where β is the c.m. velocity of the proton, k is its c.m. momentum, and θ is the c.m. scattering angle), we expect an important contribution from the nth layer of the nucleon, where n increases like x^2: $n \simeq x^2/\Delta(x^2)$ [$\Delta(x^2)$ is the increment in x^2 for which a new region of interaction starts to be relevant]. Since a break in $d\sigma/dt$ which indicates a transition from the outermost layer to the second layer is observed at pre-ISR energies around $t \simeq -1.2$ (GeV/c)^2,[7] one expects that for $-1.2 \lesssim t \leq 0$ (GeV/c)^2 only the outermost layer of the nucleon contributes significantly (note that $x^2 \simeq -t$ at high energy and small angle). Thus, $\Delta(x^2) \simeq 1.2$ (GeV/c)^2 and the second layer of the nucleon will give an important contribution for $-2.4 \lesssim t \lesssim -1.2$ (GeV/c)^2.

Let us now estimate the radii of the first two layers of the nucleon. Using Eq. (IV. 27) of FGP[3, 8] and taking 0.9 fm for the radius of the nucleon[9] we find that the second layer of the nucleon is confined between $r_1 = 0.44$ fm and $r_2 = 0.33$ fm.[10]

In the following we show that, assuming the double-helicity-flip amplitudes to be negligible at high energies, the contribution of the second layer of the nucleon, which was found to be important for $-2.4 \lesssim t \lesssim -1.2$ (GeV/c)^2, is considerably reduced for all the other amplitudes for $-2.0 \lesssim t \lesssim -1.2$ (GeV/c)^2, and thus $d\sigma/dt$ is expected to exhibit a dip inside this t interval.

The partial-wave expansion for each one of the five independent helicity amplitudes $F_{\lambda_3\lambda_4;\lambda_1\lambda_2}$ (where λ_1, λ_2 and λ_3, λ_4 stand for the initial and final helicities, respectively) is given by

$$F_{\lambda_3\lambda_4;\lambda_1\lambda_2}(\cos\theta, s) = \frac{1}{k}\sum_0^\infty (2J+1) F^J_{\lambda_3\lambda_4;\lambda_1\lambda_2}(s) \times d^J_{\lambda\mu}(\cos\theta), \qquad (1)$$

where $\lambda = \lambda_1 - \lambda_2$, $\mu = \lambda_3 - \lambda_4$. From the classical relation $J \simeq kr$ (Ref. 11) and the previously stated correspondence between the nucleon layers and the t regions, it follows that different partial sums of Eq. (1) have considerable contributions to the corresponding t regions. One would *a priori* expect that the reduction of the contribution of a specific partial sum in Eq. (1) for its corresponding t region would result from a complicated cancellation mechanism among the different terms in the sum. However, it turns out that a much simpler mechanism is responsible for the reduction of the partial sum, hereafter denoted by Σ_1, which is important for $-2.4 \lesssim t \lesssim -1.2$ (GeV/c)^2 (contribution of the second layer of the nucleon). In Fig. 1, the position of the zeros of d_{00}^J and d_{11}^J is plotted vs J (the zeros of d_{00}^J and d_{11}^J coincide for the energies and angles considered here); one realizes that each term in Σ_1 for the helicity-nonflip amplitudes vanishes inside the interval $-2.0 \lesssim t \lesssim -1.2$ (GeV/c)^2. Furthermore, let us consider the profile function given by FGP:

$$F(b) = \sum_0^\infty A_n e^{-b^2/\langle r^2 \rangle_n}, \tag{2}$$

where A_n are positive parameters fixed by the pre-

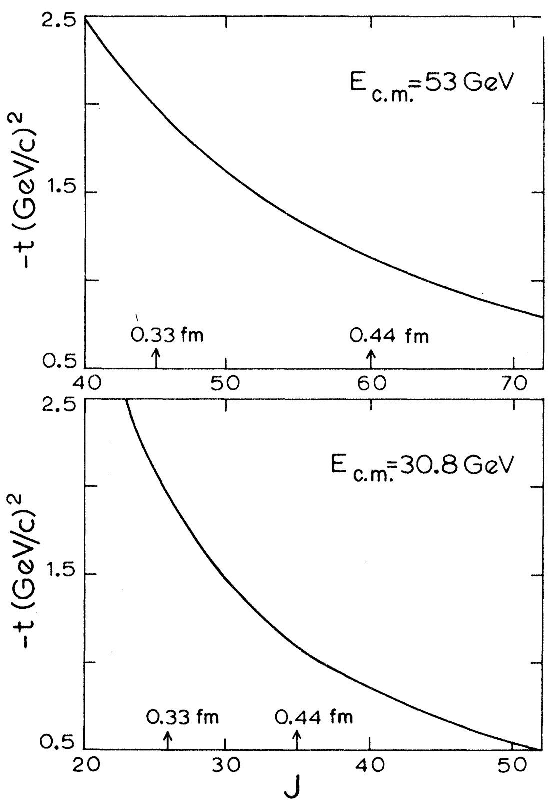

FIG. 1. The position of the zeros of d_{00}^J and d_{11}^J vs J for $E_{\text{c.m.}}=30.8, 53$ GeV. The arrows indicate the radii of the second layer of the nucleon, and thus the lower and upper values of J which appear in Σ_1.

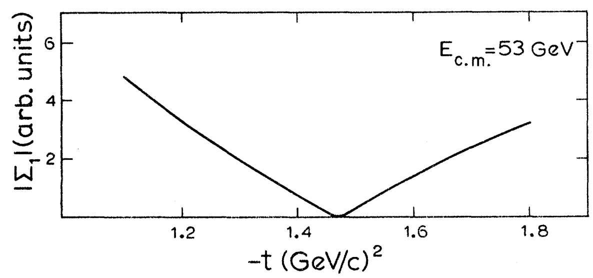

FIG. 2. The partial sum of the helicity-nonflip amplitudes contributed by the second layer of the nucleon, calculated with the profile as given by the second term in Eq. (2).

ISR data[8] and $\langle r^2 \rangle_n$ are the mean-squared radii of the different layers. In Fig. 2 $|\Sigma_1|$, for the helicity-nonflip amplitudes as calculated using the second term in (2), is plotted vs $-t$; very similar results are obtained for the other terms in (2), and for the whole sum. Thus it is clear that the contribution of the second layer of the nucleon by itself exhibits a dip at $t \simeq -1.5$ (GeV/c)^2, and since it gives an important contribution for $-2.4 \lesssim t \lesssim -1.2$ (GeV/c)^2, $d\sigma/dt$ should also exhibit a dip around $t \simeq -1.5$ (GeV/c)^2. Note that it is possible to assign weights to the different layers (which is equivalent to choosing a profile function) in such a way as to fill this dip, as was done by FGP. However, an insignificant change of the profile given in Eq. (2) will easily reveal the above-discussed dip. As an example, we change the second term in (2) in a way suggested by Avni,[12] and in Fig. 3 the new profile (dashed line) is plotted together with the profile given in Eq. (2) (full line). In Fig. 4 the differential cross section as calculated from the new profile at $E_{\text{c.m.}}= 53$ GeV is plotted, together with the experimental results[4]; an excellent agreement is found between the theoretical curve and experiment.[13] It is important

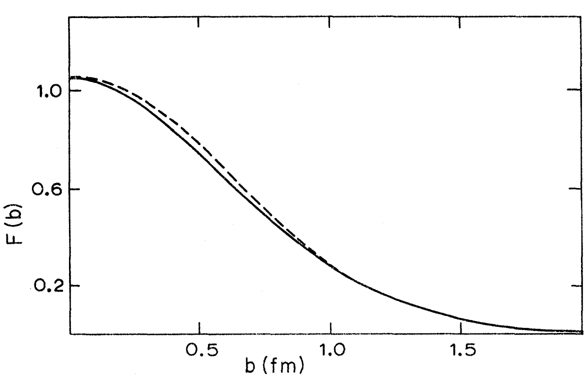

FIG. 3. Full line: the profile function as given by FGP.[8] Dashed line: our new profile function.[12]

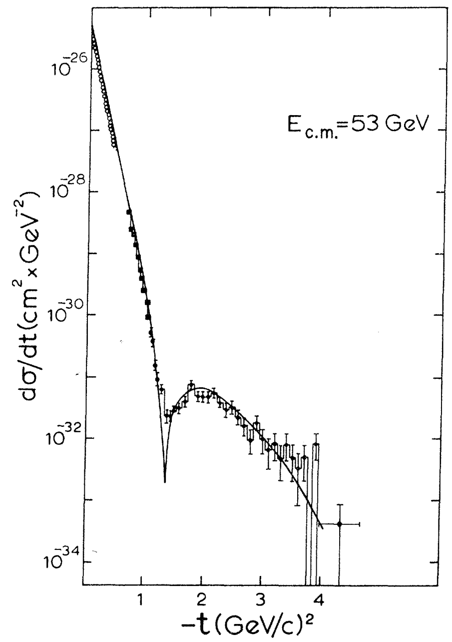

FIG. 4. $d\sigma/dt$ as calculated at $E_{c.m.}=53$ GeV from an imaginary helicity-nonflip amplitude with our new profile function.[12] Experimental data are from Ref. 4.

to note that $|\Sigma_1|$ calculated either with the new second term alone or with the whole new profile will exhibit a zero at $t \simeq -1.5$ $(\mathrm{GeV}/c)^2$; recall that a similar zero was found in $|\Sigma_1|$ when calculated with the second term as a Gaussian or with a sum of Gaussians only.

The above results prove that the weights given to the layers by FGP should be slightly changed, and that the filling of the dip at lower energies is due to contributions from the real part and from the flip amplitudes.

It is usually assumed that the helicity-flip amplitudes may be neglected at high energy, and we have just shown that with such an assumption a dip is predicted at $t \simeq -1.5$ $(\mathrm{GeV}/c)^2$. Since d^J_{10}, which appear in Σ_1, do not vanish for $-2.0 \lesssim t \lesssim -1.2$ $(\mathrm{GeV}/c)^2$, one would *a priori* expect that the helicity single-flip amplitudes tend to fill the dip; however, let us prove now that the reduction of the helicity-nonflip amplitudes together with coplanar $\mathrm{U}(3)\times\mathrm{U}(3)$-symmetry considerations and the assumption of no helicity double-flip contribution results in the reduction of the single-flip amplitude for the same t interval at high energy. Let us briefly review some facts concerning the application of coplanar $\mathrm{U}(3)\times\mathrm{U}(3)$ to high-energy pp elastic scattering. It was shown by Dashen and Gell-Mann[14] that symmetry with respect to coplanar $\mathrm{U}(3)\times\mathrm{U}(3)$ defined through the generators $(1\pm\gamma_0\vec{\sigma}\cdot\vec{n})\lambda_\alpha$ [$\vec{n}=\vec{k}\times\vec{k}'$, $\vec{k}, \vec{k}'$ are the c.m. initial and final momenta of the proton, $\vec{\sigma}$ are the Pauli matrices, γ_0 is a Dirac matrix and λ_α are Gell-Mann's SU(3) matrices] implies conservation of transversity (the total spin perpendicular to the reaction plane) for pp elastic scattering. This result can be tested experimentally since it implies certain relations for the Wolfenstein parameters.[15] Due to the experimental difficulties involved in testing these relations, the only experimental support comes from a rather low-energy experiment.[16] Since it was pointed out[17] that the above symmetry should be well satisfied at energies in which the mass splitting among members of the $\underline{36}$ SU(6) meson multiplet may be neglected, it is safe to use transversity conservation in the high-energy domain considered here. Therefore, one may write the following constraint for the transversity amplitudes[18]:

$$T_{t_3t_4;t_1t_2}=0 \text{ unless } t_1+t_2=t_3+t_4\,, \tag{3}$$

where t_1, t_2 and t_3, t_4 stand for the initial and final transversities, respectively. The relations between the transversity amplitudes and the helicity amplitudes are:

$$T_{t_3t_4;t_1t_2}=\sum_{\lambda_1\lambda_2\lambda_3\lambda_4} U^*(s_3)_{t_3\lambda_3}U^*(s_4)_{t_4\lambda_4} \times F_{\lambda_3\lambda_4;\lambda_1\lambda_2}U(s_1)_{\lambda_1t_1}U(s_2)_{\lambda_2t_2}\,, \tag{4}$$

where s_i is the spin of the ith particle, and $U(s_i)$ are given (for spin $\frac{1}{2}$) by:

$$U(\tfrac{1}{2})=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}.$$

From Eq. (3) it follows that $T_{\frac{1}{2}\frac{1}{2};-\frac{1}{2}-\frac{1}{2}}=0$, therefore, wherever the even-flip amplitudes are reduced, it follows from Eq. (4) that the single-flip amplitudes will also be reduced in the same t interval, thus proving our statement that at high energy the single-flip amplitudes are also reduced in the interval $-2.0 \lesssim t \lesssim -1.2$ $(\mathrm{GeV}/c)^2$.

To summarize: We have shown that a dip at $t\simeq -1.5$ $(\mathrm{GeV}/c)^2$ is predicted in the high-energy angular distribution of pp elastic scattering in the framework of a model in which the nucleon posses-ses a layered structure.

We expect that the filling of the dip at lower energies is due to the following:

(1) a substantial real part which was neglected in our treatment;

(2) contribution of the helicity-flip amplitudes. However, due to the present uncertainty concerning these two points it is difficult to predict the exact filling of the dip. It is important to note

that the mechanism responsible for the predicted structure is the simplest possible, namely the separate vanishing of the relevant partial waves at t values around $t \simeq -1.5\ (\mathrm{GeV}/c)^2$. Though one should look at our numerical values as an approximation only, qualitatively it seems that our analysis is an additional indication for a substructure of the nucleon.

The authors would like to thank Dr. B. Margolis, Dr. R. Henzi, and Dr. W. J. Meggs for helpful discussions.

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*Work supported in part by the National Research Council of Canada and the Department of Education of the Province of Quebec.

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[8]In the fit presented by FGP to $d\sigma/dt$ at pre-ISR energies there are four parameters: $a=5$, $\bar{b}=10.4\ (\mathrm{GeV}/c)^{-2}$ (Ref. 9), $c=2$, $\nu=3.5$. Starting with these values for $a, \bar{b}, c, \nu$ we pass, assuming an imaginary amplitude, to the equivalent set $\alpha=4.2$, $\bar{\beta}=5.2\ (\mathrm{GeV}/c)^{-2}$, $\gamma=3.6$, $\mu=2$, where the profile is given now by

$$F(b)=\sum_{0}^{\infty}\frac{\alpha^{-n}}{(\gamma n+1)^{\mu-1}}\exp\left(-\frac{\gamma n+1}{4\bar{\beta}}b^2\right)$$

[see Eq. (IV.13) of FGP, where $A_n=\alpha^{-n}/(\gamma n+1)^{\mu-1}$ and b is the impact parameter].

[9]This is in accordance with $\bar{b}=10.4\ (\mathrm{GeV}/c)^{-2}$. See F. T. Dao *et al.*, paper presented at the Sixteenth International Conference on High Energy Physics, National Accelerator Laboratory, Batavia, Ill., 1972 (unpublished).

[10]It is encouraging to compare our estimate of r_1 to the values obtained by Islam and Rosen ($r_1=0.44$ fm) and by Krisch ($r_1=0.48$ fm). Krisch also obtained $r_2=0.32$ fm, which is very close to our value, but the value obtained by Islam and Rosen ($r_2=0.2$ fm) is different from ours due to the difference in the fitting procedure for inner layers.

[11]This is a high-energy result, since only at high energy can one identify, due to Lorentz contraction, impact parameters with radii. See T. T. Wu and C. N. Yang, Phys. Rev. 137, B708 (1965).

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$$e^{-b^2/\langle r^2\rangle_1} \to e^{-ub^2}I_0(vb)\ ,$$

where I_0 is the modified Bessel function of order zero. Since the dip in $d\sigma/dt$ is predicted from the model to occur at $t\simeq -1.5\ (\mathrm{GeV}/c)^2$, and since the profile should change only slightly, we choose the values of the two new parameters to be: $u=0.24\ (\mathrm{GeV}/c)^2$, $v=1.12\ \mathrm{GeV}/c$. To summarize, our profile now reads:

$$F(b)=\sum_{\substack{n=0\\ n\neq 1}}^{\infty}A_n e^{-b^2/\langle r^2\rangle_n}+A_1 e^{-ub^2}I_0(vb)\ .$$

[13]The above change in profile is by no means unique, and it is also possible to change the second term in the profile in such a way as to cause only a slight change even in the second term itself; again, a zero appears in $|\Sigma_1|$ at $t\simeq -1.5\ (\mathrm{GeV}/c)^2$, and excellent agreement with the experimental data results.

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