for finding the Lorentz-covariant equations of structure and motion of particles in Einstein's gravitational theory appears to be that of P. Havas and J. N. Goldberg [Phys. Rev. 128, 398 (1962)]. For a discussion of why the author considers his method superior to that of previous authors see Papers I, V, and VI.

[2]In the earlier papers the author discusses two versions of his approximation method—an earlier version which he calls the older method and a later version which he calls the new method. Since the two methods are equivalent in the sense that they are just different ways of computing the same quantities under the same assumptions (the two methods can be derived from each other), we regard them here as two versions of the same method.

[3]By a neutral particle we mean a particle possessing no charge, subcharge, or higher electromagnetic moments. The definition of subcharge is given in Paper VI.

[4]See the beginning of Paper II for a discussion of this.

[5]For a discussion of why this is so see Appendixes C and D of Paper II. See also Paper E and the Introduction to Paper I. The author regards it as an open question whether the complex interaction one expects to find among charged elementary particles interacting over atomic and molecular distances in Einstein's theory will, when treated statistically, give rise to a deeper understanding of quantum mechanics.

[5a]Paper II of this series (Ref. 1) contains an important misprint. In the first equation in (1.133) the subscript B on l should be deleted. The equation should read

$$e^D = 2c^2 l^2 c_B .$$

[6]These relations are discussed in A. Einstein, Can. J. Math. 2, 120 (1950).

[7]The quantity $t_{\mu\nu}$ is not a tensor with respect to general coordinate transformations but it is a tensor with respect to Lorentz transformations. The physical meanings of s_μ and $t_{\mu\nu}$ are discussed in the author's earlier papers. See Ref. 1.

[8]By definition we raise and lower the indices on $\gamma_{\mu\nu}$ by means of the Minkowski metric $\eta^{\mu\nu}$ or $\eta_{\mu\nu}$. See Paper I, Sec. IV B.

[9]The world line, mass, spin, and higher mass moments associated with a particle will be understood as expanded in a power series in κ containing only even powers of κ. The subcharge, charge, and higher moments associated with subcharge and charge will be understood as expanded in a power series containing only odd powers of κ.

[10]This confirms the statement made in Appendix C of Paper II that "the higher-order terms in the force acting on a generalized Maxwellian particle which would have been proportional to ${}^{(p)}e^{E}\,{}^{(p')}e^{E}\,{}^{(p'')}e^{E}$ vanish identically."

[11]We raise and lower the indices on $h_{\mu\nu}$ by means of the Minkowski metric.

[12]See for example Paper II, Eqs. (1.51)–(1.58).

[13]Reasons for believing this are many. See for example L. Infeld and A. Schild, Rev. Mod. Phys. 21, 408 (1949).

[14]See for example Paper II, Eqs. (1.68)–(1.73).

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Analytic Neutron-Star Models

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We give a fully relativistic and analytic solution representing a slowly rotating neutron-star model.

I. INTRODUCTION

As shown previously,[1-4] the problem of finding the interior and exterior metric associated with a slowly rotating body splits into two parts: (a) One first solves the problem of a spherically symmetric nonrotating body [for normal neutron-star models this consists of solving the Tolman-Oppenheimer-Volkoff (TOV) equations[5]]; (b) the effect of rotation is found by solving one ordinary linear differential equation. The solution of this "rotation equation" gives the single off-diagonal term in the metric—, the term which gives rise to the dragging of inertial frames.[1-4] We use this method to find a new analytic solution exhibiting rotation.

Here we treat an idealized neutron star consisting of a gas with equation of state $p = \alpha\rho$ surrounded by a thin shell. Such a model is reasonable since realistic neutron-star models (obtained via computer) consist of a hyperon gas surrounded by a crystalline crust.[5]

II. NONROTATING BODY

For a nonrotating spherical body with equation of state $p = \alpha\rho$, the gravitational mass, pressure p, and density ρ are given by

$$m = 2\alpha r D^{-1},$$

$$\rho=\alpha(2\pi Dr^2)^{-1},\qquad (1)$$
$$p=\alpha^2(2\pi Dr^2)^{-1},$$

where m is the gravitational mass, r the radius parameter, and the denominator $D=(1+\alpha)^2+4\alpha$. This solution can be obtained directly from Einstein's equations or via the TOV equations. The standard procedure is to cut off this solution at some finite radius and to continue exterior to the star, using the Schwarzschild solution. All this is well known, but unfortunately incorrect. This is because the pressure does not become zero at any finite radius r and thus cannot match continuously to a free-space solution (with zero pressure).

This difficulty can be circumvented by surrounding the $p=\alpha\rho$ gas with a thin shell over which the pressure drops to zero. In the limit of a zero-thickness shell, we find that the shell mass m_s is given by

$$m_s=m_0[2^{1/2}-1+(3.2^{1/2}-4)\alpha],\qquad (2)$$

where $m_0=2\alpha r_0 D^{-1}$ is the mass of the gas surrounded by the shell of radius r_0 and $m=m_0+m_s$ is the total stellar mass.

Note that in all cases the *shell mass* is at least 40% of the mass of the gas and *can never be neglected.*

The spherically symmetric metric of the star can be written in the form[6]

$$ds^2=-A^2dt^2+B^2dr^2+r^2d\theta^2+r^2\sin^2\theta\, d\varphi^2,\qquad (3)$$

where inside the star

$$A^2=[1-2(m_0+m_s)r_0^{-1}](r/r_0)^{4\alpha(1+\alpha)^{-1}},\qquad (4)$$
$$B^2=D(1+\alpha)^{-2},$$

and outside we have the usual Schwarzschild metric

$$A^2=B^{-2}=1-2(m_0+m_s)r^{-1}.\qquad (5)$$

III. ROTATING BODY

Once the nonrotating solution has been obtained, the rotational effects can be found using the linear rotation equation[1-4]

$$[r^4(BA)^{-1}\bar{\Omega}']'=16\pi BA^{-1}r^4(\rho+p)\bar{\Omega},\qquad (6)$$

where $\bar{\Omega}=\Omega-\omega$, ω is the star's angular velocity, and Ω is the angular velocity of inertial frames along the rotation axis.[1,4,7] Prime denotes differentiation with respect to r. Equivalently, the rotation equation may be expressed entirely in terms of p, ρ, and m,

$$[1-2m(r)r^{-1}](r^4\bar{\Omega}')'=4\pi(\rho+p)r(r^4\bar{\Omega})'.\qquad (7)$$

In free space exterior to the star, where $p=0=\rho$, Eq. (6) gives[1-4]

$$\Omega=2Jr^{-3},\qquad (8)$$

where J is the star's total angular momentum.[8]

Within the star, the rotation equation takes the form

$$(r^4\bar{\Omega}')'=2\alpha_N r^{-1}(r^4\bar{\Omega})',\qquad (9)$$

where $\alpha_N=\alpha(1+\alpha)^{-1}$. This has the simple solution

$$\Omega=\omega-C_1r^n,\qquad (10)$$
$$2n=(2\alpha_N-3)+[(3-2\alpha_N)^2+32\alpha_N]^{1/2}.$$

We have chosen the solution which is finite at the origin. Note the occurrence of perfect dragging of inertial frames at the origin ($\Omega=\omega$ at $r\to 0$) since $n>0$. This means that the inertial frames at the origin rotate at the same rate as the body. Similar results have been noted for hollow massive shells[1,2] (analytically) and incompressible fluids[3] (via computer).

To complete the solution we match the interior (10) and exterior (8) solutions. This is done[1] by requiring continuity of Ω across the shell and the condition obtainable by integrating Eq. (7) across the shell.

These conditions allow us to relate the constants J and C_1 to the basic physical parameters describing the body α and r_0,

$$J=\tfrac{1}{2}\omega r_0^3\,\frac{(n+4)B_-^{-1}-4B_+^{-1}}{(n+4)B_-^{-1}-B_+^{-1}},\qquad (11)$$

$$\Omega=\omega\{1-3(r/r_0)^nB_+^{-1}[(n+4)B_-^{-1}-B_+^{-1}]^{-1}\},\quad r<r_0.\qquad (12)$$

Here + (−) denotes the limit $r\to r_0$ from outside (inside) the star from Eqs. (4) and (5). In the small-α limit, we obtain the classical result

$$J=\tfrac{2}{3}(m_0+m_s)r_0^2\omega[1-(2^{1/2}/3)].\qquad (13)$$

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[3]J. M. Cohen and D. R. Brill, Nuovo Cimento 56B, 209 (1968).

[4]For a review of the method for treating rotating bodies,

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Dynamics of Fluid Spheres of Uniform Density

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The adiabatic collapse of spheres of uniform density made up of perfect fluids either electrically neutral or charged has been studied. A set of field equations equivalent to Einstein's field equations are derived in terms of the energy-momentum tensor. With the help of these equations, the following theorems for perfect fluid spheres have been established: (1) For the gravitational collapse or bounce of a homogeneous sphere of perfect fluid the only motions possible are shear-free, uniform contraction or expansion. (2) For a charged fluid sphere embedded in empty space the conditions $\rho = \rho(t)$ and $\dot\lambda = \dot\mu$ are inconsistent in the sense that either the solutions are static or matter is uncharged.

I. INTRODUCTION

Recently, there have appeared a large number of papers in which the motion and stability of the spherically symmetric systems have been discussed with a view to explain the nature of peculiar astrophysical bodies.[1] With few exceptions, these articles deal with spheres composed of neutral or charged perfect fluids.[2-11] In this note too, we will discuss the adiabatic collapse of spheres made of perfect fluids which are electrically neutral or charged.

The analysis in this case is complex; first, because of the nonlinear character of the field equations, and second because of the lack of a sufficient number of equations to determine the unknown physical variables of the system. In these circumstances, it is natural to make some simplifying assumptions either in the nature of symmetries introduced in the space-time or idealizations of the physical system or both. One such simple system is the homogeneous sphere in which the matter density is a function of time only. Even then the dynamics of such simple systems are complicated when considered in their generality, and additional conditions are imposed on the metric tensor in order to solve the field equations. Besides, usual regularity conditions at the boundary and center are required to be satisfied. In this note we also investigate the motion of homogeneous spheres and show that the assumption of uniform density and the requirement of regularity at the center are sufficient to yield a solution of the field equations. It is not necessary to impose any additional restriction on the metric coefficients. We have been able to integrate the field equations without the assumption of spatial isotropy. However, it is shown that if one wants to obtain a singularity-free solution with a perfect fluid sphere of uniform density as a source, one is led to the requirement of spatial isotropy.

In Sec. II we set up the field equations in a form suitable for further application. Section III deals with perfect fluid spheres, whereas in Sec. IV we discuss the motion of charged perfect fluid spheres.

II. THE FIELD EQUATIONS

We choose the line element

$$ds^2 = e^{\lambda} dr^2 + e^{\mu}(d\theta^2 + \sin^2\theta\, d\phi^2) - e^{\nu} dt^2 \qquad (2.1)$$

and make use of comoving coordinates (r, θ, ϕ, t). The functions λ, μ, and ν are functions of r and t only. The field equations as considered here are

$$G^i{}_j \equiv R^i{}_j - \tfrac{1}{2} R \delta^i{}_j = -8\pi T^i{}_j , \qquad (2.2)$$

where $T^i{}_j$ is the energy-momentum tensor for the system. We choose the sign and other conventions as in Synge.[12]

The nonvanishing components of the Einstein tensor can be easily calculated and the field equations (2.2) are obtained as: