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Intersecting-Storage-Rings Inclusive Data and the Charge Ratio of Cosmic-Ray Muons*

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The (μ^+/μ^-) ratio at sea level has been calculated by Frazer *et al.*, using the hypothesis of limiting fragmentation together with the inclusive data below 30 GeV/c. They obtained a value of $\mu^+/\mu^- \simeq 1.56$, to be compared with experimental value of 1.2 to 1.4. We have calculated the ratio using the recent ISR (CERN Intersecting Storage Rings) data, and obtained a value of $\mu^+/\mu^- \sim 1.40$ in good agreement with the experimental result.

Recently, Frazer *et al.*[1] have calculated the μ^+/μ^- ratio at sea level from the known cosmic-ray primary spectrum, using the hypothesis of limiting fragmentation[2] (or scaling[3]) together with the inclusive data below 30 GeV/c. They obtained a value of 1.56 for μ^+/μ^-, to be compared with the experimental value of 1.2–1.4.[4] They also suggested two possible causes for this discrepancy: the nonscaling behavior of 30-GeV/c data near $x \simeq 0$, and the possible inaccuracy of the factorization hypothesis.

In this paper we report a calculation of the μ^+/μ^- ratio at sea level, using the formalism developed by Frazer *et al.*[1] and recent ISR inclusive data.[5]

The most sensitive quantity in determining the μ^+/μ^- ratio is $Z^+_{p\pi}/Z^-_{p\pi}$ which is given by

$$Z^{\pm}_{p\pi} = \int_0^1 x^{\gamma-1} \tilde{f}^{\pm}_{p\pi}(x)\, dx\,,$$

where x is the usual Feynman scaled variable,

$$\tilde{f}^{\pm}_{p\pi}(x) = \lim_{E\to\infty} \frac{E_\pi}{\sigma^{\text{inel}}_{pp}} \frac{d\sigma_{p\to\pi^\pm}}{dE_\pi}\,,$$

and γ is the exponent in the primary cosmic-ray energy spectrum given by

$$\frac{dN}{dE} = N_0 E^{-(1+\gamma)}\,,$$

with $\gamma \simeq 1.7$.

Due to the fast falloff [roughly exponential in $\tilde{f}^{\pm}_{p\pi}(x)$] with increasing x, it is clear that the result of μ^+/μ^- is relatively insensitive to the limiting behavior at the large-x region. This point can be seen from the fact that

$$\int_{0.35}^1 x^{\gamma-1}\tilde{f}^{\pm}_{p\pi}(x)\, dx \lesssim \tfrac{1}{3}\int_0^{0.35} x^{\gamma-1}\tilde{f}^{\pm}_{p\pi}(x)\, dx\,.$$

From the 28.5-GeV/c and ISR data, it can be seen that from 28.5 GeV/c to 500 GeV/c, $x\,d\sigma/dx$ for π^- around $x \sim 0.2$ increases by approximately 30%, while $x\,d\sigma/dx$ for π^+ is essentially unchanged. Therefore, by using the ISR data we expect to obtain a smaller $Z^+_{p\pi}/Z^-_{p\pi}$ and μ^+/μ^- than did Frazer *et al.*[1]

We have calculated $Z^+_{p\pi}/Z^-_{p\pi}$ and μ^+/μ^-, using the ISR data for $(E_\pi/\sigma^{\text{inel}}_{pp})\, d\sigma/dE_\pi(p\to\pi\pm)$ as $\tilde{f}^{\pm}_{p\pi}(x)$ for $x<0.4$. Since there are no ISR data yet on $(E_\pi/\sigma^{\text{inel}}_{pp})\, d\sigma/dE_\pi(p\to\pi\pm)$ for $x>0.4$, we have used the 28.5-GeV/c data of Panvini *et al.*[6] We believe that the use of 28.5-GeV/c data for the large x region will have very little effect on our result for μ^+/μ^- because: (i) We expect the approach to the limit in the large-x region to be fast compared with that in the small-x region[7]; (ii) the result is rather insensitive to the large-x region. We obtained $Z^+_{p\pi}/Z^-_{p\pi} \simeq 1.70$.

Using the equation

$$\frac{\mu^+}{\mu^-} = \left(\frac{Z^+_{p\pi}}{Z^-_{p\pi}} + \frac{S^-_\pi}{S^+_\pi}\right) \Big/ \left(1 + \frac{S^-_\pi}{S^+_\pi}\frac{Z^+_{p\pi}}{Z^-_{p\pi}}\right)$$

and the parameters given in Ref. 1, we found that

$$S^-_\pi / S^+_\pi = 0.22^8 \text{ and } \mu^+/\mu^- = 1.40 \, .$$

We shall conclude with the following remarks:

(i) The nonscaling behavior in the region of small but finite x and *the different rate of approaching the limit for* π^+ *and* π^- are indeed the main causes for the discrepancy between the calculation of Frazer *et al.*[1] and the experimental value.

(ii) The calculation gives indirect support for the hypothesis of limiting fragmentation (or scaling) scaling) above the ISR energies.

(iii) The hypothesis of limiting fragmentation (or scaling) is indeed useful in correlating data at machine-energy and cosmic-ray phenomena.

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[8]The value for μ^+/μ^- is not very sensitive to the parameters used in determining S^-_π/S^+_π. Varying S^-_π/S^+_π between 0.18 and 0.26 (i.e., 20% variation), we found μ^+/μ^- to be between 1.44 and 1.36.