Non-Hermitian Interactions and the Evidence for Violation of T Invariance*

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It is shown that the usual argument based on the Bell-Steinberger sum rule to the effect that the data on neutral-K-meson decay provide evidence that T invariance *must* be violated contains a logical weakness because T-invariant, CPT-violating interactions that are non-Hermitian would abrogate the sum rule, which is a unitarity condition. The required modification of the sum rule for such an interaction is given explicitly but it introduces several quantities that cannot be measured directly; therefore the modified sum rule does not provide useful general information concerning the validity of T invariance.

The well-established existence of violation of CP invariance in the neutral-K system leads one naturally to assume the coexistence of violation of T invariance in that system (and possibly elsewhere) on the basis of the CPT theorem. However, the existence of violation of T invariance has not been *directly* established in the neutral-kaon system nor in any other decay process or interaction at the present time.

Several authors[1] have addressed themselves to the question of whether or not violation of T invariance could be inferred from data on the neutral-kaon system without assuming the validity of the CPT theorem. The conclusion of these authors has been that indeed T is violated.

The standard argument is made by assuming the validity of T invariance and showing that the Bell-Steinberger sum rule[2] in the particular form resulting from this assumption is not in agreement with the available experimental data on K^0 decay.

It is the purpose of this note to point out that there is a logical weakness in that line of argument if it is used to conclude from these data alone that T invariance *must* be violated.

The assumption of T invariance in making this argument is equivalent to the assumption of CPT violation, although it does not specify the origin of the violation. The origin must be in the failure of some one (or more) of the axioms underlying the proof of the CPT theorem or some aspect of the structure of the theory. The failure might occur in such a way as to invalidate the Bell-Steinberger sum rule, thereby introducing a flaw in the argument cited above. In particular, one way in which CPT could fail would be by the existence of a non-Hermitian term in the Hamiltonian, but it is well known that such a term will also lead to a violation of unitarity and therefore of the sum rule, which is an expression of unitarity.

Any attempt to write down a local, CPT-violating interaction that is consistent with Lorentz invariance will lead to a non-Hermitian term in the Hamiltonian.[3] Since several authors[4] have found reason to introduce models invoking just such non-Hermitian weak or superweak interactions, a case can be made for examining the consequences of this particular way of violating CPT. However, in doing so, we do not wish to create the impression that there is any substantial evidence for a non-Hermitian term in the weak interaction. We would take the view that unitarity should be abandoned only as a last resort for the explanation of any phenomenon.

In order to show explicitly how the Bell-Steinberger sum rule is modified by the introduction of a non-Hermitian interaction, it is necessary to be specific about our assumptions. We assume that the time development of the neutral-K-meson system together with its decay channels can be described by a state vector satisfying the usual differential equation

$$i\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle\ . \tag{1}$$

We also assume that this state vector can be expanded in terms of the eigenvectors of a Hermitian strong-interaction Hamiltonian H_0 spanning a unitary Hilbert space and that these eigenvectors are given the usual interpretation of quantum mechanics. These eigenvectors are denoted by $|K^0\rangle$, $|\bar{K}^0\rangle$ for the neutral K system and $|c\rangle$ for each of the various decay channels.

The total Hamiltonian is

$$H = H_0 + H'\ , \tag{2}$$

where H' includes the weak interactions responsible for decay and would also include superweak interactions,[5] if they exist. For our present purpose it is assumed that H' may include non-Hermitian terms.

Since the terms in H' responsible for the decay characteristics of the K^0 and $\bar{K}^0$ are very weak, it is appropriate to use the Weisskopf-Wigner perturbation method[6] to describe the time dependence of the system. As is well known, this leads to a complex 2×2 mass matrix $\underline{M}-\frac{1}{2}i\underline{\Gamma}$ connecting the states $|K^0\rangle$ and $|\bar{K}^0\rangle$, and this matrix serves to determine the time evolution of the neutral-K system.

By following the usual method[7] modified to take into account the possibility of non-Hermitian terms, we find that

$$\langle j|\underline{M}|k\rangle = m_K\delta_{jk}+\langle j|H'|k\rangle + \mathrm{P}\int\frac{dE}{m_K-E}\sum_c \rho_c(E)\tilde{H}'_{cj}(E)H'_{ck}(E) \tag{3}$$

and

$$\langle j|\underline{\Gamma}|k\rangle = 2\pi\sum_c \rho_c(m_K)\tilde{H}'_{cj}(m_K)H'_{ck}(m_K)\;, \tag{4}$$

where

$$H'_{ck}(E)=\langle c|H'|k\rangle,\quad \tilde{H}'_{ck}(E)=\langle k|H'|c\rangle\;, \tag{5}$$

ρ_c is the density of states, and E is the energy of channel c, while the states $|j\rangle$, $|k\rangle$ denote either $|K^0\rangle$ or $|\bar{K}^0\rangle$.

We note that both $\underline{M}$ and $\underline{\Gamma}$ are Hermitian when H' is Hermitian, but when the submatrices of H' whose matrix elements appear in Eqs. (3) and (4) are not Hermitian, there are non-Hermitian terms in $\underline{M}$ and $\underline{\Gamma}$. In either case the time evolution of the K^0 states may be obtained by the usual procedure, which makes use of the eigenstates $|S\rangle$ and $|L\rangle$ of $\underline{M}-\frac{1}{2}i\underline{\Gamma}$. The corresponding eigenvalues are $\lambda_S=m_S-\frac{1}{2}i\Gamma_S$, $\lambda_L=m_L-\frac{1}{2}i\Gamma_L$. The states $|S\rangle$ and $|L\rangle$ are linear combinations of $|K^0\rangle$ and $|\bar{K}^0\rangle$, and they decay exponentially at the rates Γ_S and Γ_L, respectively.

In order to obtain sum rules which are generalizations of the unitarity conditions, it is convenient to write

$$\underline{M}-\tfrac{1}{2}i\underline{\Gamma}=\underline{M}_h-\tfrac{1}{2}i\underline{\Gamma}_h, \tag{6}$$

where $\underline{M}_h$ and $\underline{\Gamma}_h$ are Hermitian. Thus

$$\underline{M}_h=\tfrac{1}{2}(\underline{M}+\underline{M}^*)-\tfrac{1}{4}i(\underline{\Gamma}-\underline{\Gamma}^*) \tag{7}$$

and

$$\underline{\Gamma}_h=\tfrac{1}{2}(\underline{\Gamma}+\underline{\Gamma}^*)+i(\underline{M}-\underline{M}^*)\;. \tag{8}$$

If the states $|L\rangle$, $|S\rangle$ are denoted by $|\alpha\rangle$ or $|\beta\rangle$, they can be written as solutions of the eigenvalue problem

$$(\underline{M}_h-\tfrac{1}{2}i\underline{\Gamma}_h)|\alpha\rangle=\lambda_\alpha|\alpha\rangle \tag{9}$$

or, by taking the Hermitian conjugate,

$$\langle\beta|(\underline{M}_h+\tfrac{1}{2}i\underline{\Gamma}_h)=\langle\beta|\lambda_\beta^*\;. \tag{10}$$

Therefore

$$\langle\beta|(\underline{M}_h-\tfrac{1}{2}i\underline{\Gamma}_h)|\alpha\rangle=\lambda_\alpha\langle\beta|\alpha\rangle \tag{11}$$

and

$$\langle\beta|\underline{M}_h+\tfrac{1}{2}i\underline{\Gamma}_h|\alpha\rangle=\lambda_\beta^*\langle\beta|\alpha\rangle\;. \tag{12}$$

Hence, by subtracting Eq. (11) from Eq. (12)

$$i\langle\beta|\underline{\Gamma}_h|\alpha\rangle=(\lambda_\beta^*-\lambda_\alpha)\langle\beta|\alpha\rangle\;. \tag{13}$$

Thus, since $\lambda_\alpha=m_\alpha-\frac{1}{2}i\Gamma_\alpha$ and the states $|\alpha\rangle$ may be normalized so that $\langle\alpha|\alpha\rangle=1$,

$$\langle\alpha|\underline{\Gamma}_h|\alpha\rangle=\Gamma_\alpha\;. \tag{14}$$

Also

$$i\langle S|\underline{\Gamma}_h|L\rangle=[(m_S-m_L)+\tfrac{1}{2}i(\Gamma_S+\Gamma_L)]\langle S|L\rangle\;. \tag{15}$$

We have noted that when H' is Hermitian, so are $\underline{M}$ and $\underline{\Gamma}$. Then $\underline{\Gamma}_h=\underline{\Gamma}$ which, in turn, is given by Eq. (4), and Eq. (14) becomes the usual expression for the decay rate:

$$\Gamma_\alpha=\Sigma_c|A_c^\alpha|^2\;, \tag{16}$$

where

$$A_c^\alpha=[2\pi\rho_c(m_K)]^{1/2}\sum_k H'_{ck}(m_K)\langle k|\alpha\rangle \tag{17}$$

is the decay amplitude of the state $|\alpha\rangle$ into the channel c, while Eq. (15) is the usual Bell-Steinberger sum rule. The significance of these results follows from the fact that $\underline{\Gamma}_h$ contains only absorptive (on the mass shell) terms when H' is Hermitian; therefore only the directly measurable parts of the mass matrix are involved.

When H' is not Hermitian, both Eqs. (14) and (15) reflect the breakdown of unitarity in such a way as to involve contributions that are not directly measurable. The terms in $\underline{\Gamma}_h$ of the form $\underline{M}-\underline{M}^*$ contain either dispersive or superweak parts, or both, and neither of them can be determined by direct measurements. Furthermore, in addition to the measurable decay amplitudes A_c^α, the equations involve the nonmeasurable transposed amplitudes

$$\tilde{A}_c^\alpha=[2\pi\rho_c(m_K)]^{1/2}\,\Sigma_k\langle\alpha|k\rangle\tilde{H}'_{ck}(m_K)\;. \tag{18}$$

As a result of these changes introduced into Eq. (15) by a non-Hermitian term in the weak interaction, the general validity of the Bell-Steinberger sum rule cannot be assumed in the context of an argument permitting a completely unspecified form of CPT violation. That is the weak link in the logic of the argument for T violation based on the sum rule.[8] In fact we see that the modified form of the sum rule provided by Eq. (15) when there is a contribution to Eq. (3) from non-Hermitian terms can yield no information concerning the validity of T invariance.

The most direct indication of the failure of unitarity is that the decay rates Γ_S and Γ_L, given by

Eq. (14), are no longer the sums of partial transition rates, as in Eq. (16). The rates of creation of the decay channels (partial transition rates) are proportional to $|A_c^\alpha|^2$, as in the conventional case, even when H' is not Hermitian.[9] Since Γ_α gives the rate of decay, the balance between decay and creation would be destroyed by a non-Hermitian interaction H'.

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Amplitude Structure in Predominantly Pomeranchukon-Exchange Reactions and *s*-Channel Helicity Nonconservation*

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The unit-helicity-flip and nonflip amplitudes of $\gamma p \to \rho^0 p$ and $\pi N \to \pi N$ for $I = 0$ exchange are shown to be similar in magnitude when scaled by the vector-dominance-model constant. We separate the ρ^0 amplitudes for Pomeranchukon and f exchange using the dual absorption model. Possible sources of the helicity-flip term are discussed.

The experimental observation that ρ^0 photoproduction is diffractive in character[1] and is predominantly s-channel helicity conserving (SHC) at the γ-ρ^0 vertex[2] has led to a postulate[3] that Pomeranchukon (P) exchange in general possesses SHC as one of its characteristics. An analysis of πN polarization correlations at 6 GeV/c (Ref. 4) confirms this speculation, but shows helicity-flip amplitudes to be $\approx$15% of the SHC term. In an extension of some earlier[2] experiments to higher energy[5] (9.3 GeV) and with increased statistics the existence of a small but significant s-channel helicity-nonconserving amplitude in ρ^0 photoproduction has been suggested.

In this paper we compare in detail the amplitude for unit helicity flip at the nucleon vertex for isospin-zero exchange in πp scattering with that for the γ-ρ vertex in photoproduction. Good agreement is obtained when the photoproduction amplitude is scaled by the VDM (vector-dominance-model) constant. This observation has led us to compare the dominant SHC amplitudes as well, using the same dual absorptive model[6] (DAM) already applied to $\pi^\pm N$ scattering,[7] in order to separate the P and f exchanges. Again good agreement is found in the parametrizations of the model.

The application of the DAM approach leads to a P trajectory with nonzero slope in t (i.e., a shrinking forward peak). Recent evidence for nonshrinkage in ϕ photoproduction[8] has made the DAM assumptions controversial.[9] Although the