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Comment on the πN σ Term

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The sum rule for the on-shell pion-nucleon scattering amplitude at the Cheng-Dashen (CD) point $\nu = \nu_B = 0$ has been recalculated. Instead of the value $1.7\,\mu^{-1}$ as obtained by CD, we find $T(0,0) \simeq 1.1\mu^{-1}$ and $1.3\mu^{-1}$, when the new and old Lovelace phase shifts are used, respectively.

The relevance of the σ term to broken hadron symmetries is well recognized.[1] A noncontroversial determination of the magnitude of this term, however, is yet to be achieved.[2]

In this comment we report a recalculation made on a broad-area subtraction sum rule[3] performed by Cheng and Dashen (CD)[4] some time ago. While we intend not to offer a genuine value for the σ term ourselves, we are sure that the value of CD must be lowered, not by using other arguments,[5] but rather by calculating the same sum rule used by CD.

For the process $\pi(q)+N(p)\to\pi(q')+N(p')$, the on-shell scattering amplitude is given by $\bar{u}(p')\times[-A+i\frac{1}{2}\gamma\cdot(q+q')B]u(p)$.[6] Allowing one or both pions to be off the mass shell, we have at the so-called Adler point[7] ($\nu=0,\ \nu_B=0,\ q^2=-\mu^2,\ q'^2=0$ or $\nu=0,\ \nu_B=0,\ q^2=0,\ q'^2=-\mu^2$), and at the Weinberg point[8-10] ($\nu=0,\ \nu_B=0,\ q^2=0,\ q'^2=0$), the result

$$A^+(0,0,\mu^2,0)=A^+(0,0,0,\mu^2)=K_{NN\pi}(0)\,g^2/m\,, \tag{1}$$

and

$$A^+(0,0,0,0)=K_{NN\pi}{}^2(0)\,g^2/m-\Sigma(0)\,, \tag{2}$$

where

$$\nu=-(p+p')\cdot(q+q')/4m = (s-u)/4m\,, \tag{3}$$

$$\nu_B=q\cdot q'/2m = (t+q^2+q'^2)/4m\,, \tag{4}$$

and $\Sigma(0)$ (Ref. 11) is the σ term at $t=0$. Since the Born term in νB^+ is of the form $K_{NN\pi}(q^2)K_{NN\pi}(q'^2)\times(g^2/m)\nu^2/(\nu_B{}^2-\nu^2)$, CD made the linear combination

$$T(\nu,\nu_B,q^2,q'^2)=A^+(\nu,\nu_B,q^2,q'^2)+\nu B^+(\nu,\nu_B,q^2,q'^2)\,, \tag{5}$$

from which it follows that $T(0,0,0,0)=-\Sigma(0)$ and $T(0,0,\mu^2,0)=T(0,0,0,\mu^2)=0$, if by $\nu=\nu_B=0$ we mean first $\nu_B=0$ and then $\nu=0$. Whether $T(0,0,\mu^2,\mu^2)\equiv T(0,0)$ (Ref. 12) is in reality quite close to $+\Sigma(0)$ or not, it is the amplitude $T(0,0)$ which has been calculated by CD, and is revisited here.

The broad-area subtraction sum rule[3,4] for the on-shell $T(\nu,\nu_B)$ reads as

$$\begin{aligned}T(\nu,\nu_B)={}&\frac{g^2}{m}\,\frac{\nu_B{}^2}{\nu_B{}^2-\nu^2}\,\frac{(\nu_1{}^2-\nu^2)^\beta(\nu_2{}^2-\nu^2)^{1-\beta}}{(\nu_1{}^2-\nu_B{}^2)^\beta(\nu_2{}^2-\nu_B{}^2)^{1-\beta}}+\frac{2}{\pi}(\nu_1{}^2-\nu^2)^\beta(\nu_2{}^2-\nu^2)^{1-\beta}\\&\times\left[\int_{\nu_0}^{\nu_1}d\nu'\,\frac{\nu'}{\nu'^2-\nu^2}\,\frac{\operatorname{Im}T(\nu',\nu_B)}{(\nu_1{}^2-\nu'^2)^\beta(\nu_2{}^2-\nu'^2)^{1-\beta}}+\cos\pi\beta\int_{\nu_1}^{\nu_2}d\nu'\,\frac{\nu'}{\nu'^2-\nu^2}\,\frac{\operatorname{Im}T(\nu',\nu_B)}{(\nu'^2-\nu_1{}^2)^\beta(\nu_2{}^2-\nu'^2)^{1-\beta}}\right.\\&\left.+\sin\pi\beta\int_{\nu_1}^{\nu_2}d\nu'\,\frac{\nu'}{\nu'^2-\nu^2}\,\frac{\operatorname{Re}T(\nu',\nu_B)}{(\nu'^2-\nu_1{}^2)^\beta(\nu_2{}^2-\nu'^2)^{1-\beta}}-\int_{\nu_2}^{\infty}d\nu'\,\frac{\nu'}{\nu'^2-\nu^2}\,\frac{\operatorname{Im}T(\nu',\nu_B)}{(\nu'^2-\nu_1{}^2)^\beta(\nu'^2-\nu_2{}^2)^{1-\beta}}\right],\end{aligned} \tag{6}$$

where $\nu_0 = \mu + t/4m$, $\nu_0 < \nu_1 < \nu_2 < \infty$, and $0 \leqslant \beta \leqslant 1$. Following CD, we put $\nu_B = 0$ (or $t = 2\mu^2$), $\nu = 0$ (or $s = u$), and choose $\nu_1 = 1.52\mu$, $\nu_2 = 2.84\mu$.[13] The input data for (6) include (1) the new[14] and old[15] phase shifts of Lovelace between $\nu_0 = 1.06\mu$ and $\nu_{\max} = 14.9\mu$ and (2) the Regge amplitude of Barger and Phillips[16] beyond $\nu_{\max}$.

The detailed calculation goes as follows. The phase shifts are reconstructed in equal intervals with the aid of the Lagrange interpolation formula (in intervals of 0.02μ between $\nu_0 = 1.06\mu$ and 3.50μ, and in intervals of 0.1μ between 3.5μ and $\nu_{\max} = 14.9\mu$). A Simpson rule is employed for the numerical integration everywhere between ν_0 and $\nu_{\max}$, *except* in the intervals between 1.50μ and 1.54μ, and between 2.82μ and 2.86μ, in which a *linear* approximation to $\mathrm{Im}T(\nu', 0)$ and $\mathrm{Re}T(\nu', 0)$ is used, and an analytic integration is required. The Regge amplitude of Barger and Phillips, which joins smoothly with the phase-shift amplitude at $\nu_{\max}$, is extrapolated to the positive-t region, and is employed for the numerical integration beyond $\nu_{\max}$. The result is shown in Table I.

A few remarks concerning the calculation technique are in order. We have included both the points $\beta = 1.0$ and $\beta = 0.0$, at which (6) becomes singly subtracted at ν_1 and ν_2, respectively. The way the integration is handled is the *same* for all values of β, if the logarithmic singularities at both sides of the singular points ν_1 and ν_2 are precluded when $\beta = 1$ and $\beta = 0$. Certainly we no longer have a broad-area subtraction at these two values of β, but we disagree with the statement that the integration would be easier if they are excluded.

Due to the "singular" factors $(\nu'^2 - \nu_1^2)^\beta$, $(\nu_2^2 - \nu'^2)^{1-\beta}$, etc. in the denominators when $\nu' = \nu_1$ and $\nu' = \nu_2$, it is natural to suspect that the integral is enhanced at these two points. This depends on the value of β. One thing is certain, however: that the integration depends heavily on the numerical values of $\mathrm{Re}T(\nu_1, 0)$ and $\mathrm{Re}T(\nu_2, 0)$. For it can easily be shown that

$$\sin\pi\beta \int_{\nu_1}^{\nu_2} d\nu' \frac{\nu'}{\nu'^2 - \nu^2} \frac{1}{(\nu'^2 - \nu_1^2)^\beta (\nu_2^2 - \nu'^2)^{1-\beta}} = \frac{\pi}{2} \frac{1}{(\nu_1^2 - \nu^2)^\beta (\nu_2^2 - \nu^2)^{1-\beta}} \tag{7}$$

for $\nu^2 < \nu_1^2 < \nu_2^2$ and $0 \leqslant \beta \leqslant 1$, so the term involving the real part of $T(\nu', 0)$ in (6) can be written as

$$\frac{2}{\pi}(\nu_1^2 - \nu^2)^\beta (\nu_2^2 - \nu^2)^{1-\beta} \sin\pi\beta \int_{\nu_1}^{\nu_2} d\nu' \frac{\nu'}{\nu'^2 - \nu^2} \frac{\mathrm{Re}T(\nu', 0) - \mathrm{Re}T(\nu_1, 0) - \mathrm{Re}T(\nu_2, 0)}{(\nu'^2 - \nu_1^2)^\beta (\nu_2^2 - \nu'^2)^{1-\beta}} + \mathrm{Re}T(\nu_1, 0) + \mathrm{Re}T(\nu_2, 0)\,. \tag{8}$$

(At the either end of β one of the two subtraction constraints is redundant.) The ordinary broad-area subtraction[7] (i.e., $\beta = 0.5$) is trying to smear out the dependence of $T(0,0)$ on any particular $\mathrm{Re}T(\nu', 0)$. Thus, with the row $\beta = 0.5$ in Table I, we estimated $T(0,0) \simeq 1.1\mu^{-1}$ or $1.3\mu^{-1}$, depending on whether the new[14] or the old[15] phase-shift analysis of Lovelace is used, in contrast to the value of $T(0,0) \simeq 1.7\mu^{-1}$ obtained by CD.[4]

Knowing that

$$\int_{\nu_{\max}}^{\infty} \frac{d\nu'}{\nu'} \frac{\mathrm{Im}T(\nu', 0)}{(\nu'^2 - \nu_1^2)^\beta (\nu'^2 - \nu_2^2)^{1-\beta}} \equiv \int_{\nu_{\max}}^{\infty} d\nu' I(\nu')$$

is convergent at $\nu' = \infty$, we can change the variable of integration into $(1/\nu')$, obtaining

$$\int_0^{1/\nu_{\max}} d(1/\nu')(1/\nu')^{-2} I(1/\nu')\,.$$

On plotting $(1/\nu')^{-2} I(1/\nu')$ against $(1/\nu')$, this high-energy integral can be roughly estimated in a model-independent way, because we are sure that one end of the plot must be located at the origin. In practice, since the Regge fit of Barger and Phillips[16] has done a pretty job in the physical region $t \leqslant 0$, we have simply adopted their amplitude, even though now t takes on a (small) positive value $2\mu^2$. The high-energy tail $\int_{14.9\mu}^{\infty} d\nu' I(\nu')$ turns out to be of the magnitude of $0.1\mu^{-3}$.

There are other dispersion calculations on the πN σ term. Höhler, Jakob, and Strauss[17] used a forward and a forward-derivative dispersion relation. The reconstruction of the forward-derivative amplitude from the partial-wave series is less

TABLE I. Results.

β	$T(0,0)$ [a]	$T(0,0)$ [b]
1.0	0.52	1.29
0.9	0.71	1.32
0.8	0.86	1.33
0.7	0.97	1.34
0.6	1.04	1.34
0.5	1.09	1.32
0.4	1.11	1.27
0.3	1.11	1.21
0.2	1.10	1.11
0.1	1.07	0.98
0.0	1.05	0.79

[a] New Lovelace phase shifts, Ref. 14.
[b] Old Lovelace phase shifts, Ref. 15.

convergent, however. Jakob[18] later repeated the same kind of calculation, using

$$C^+(\nu,t)=A^+(\nu,t)+\nu(1-t/4m^2)^{-1}B^+(\nu,t)$$

and

$$\frac{\partial}{\partial t}C^+(\nu,t)$$

at $t=0$. In addition, he calculated (6) at $\nu=\nu_B=0$ with $\beta=1.0$ (or 0.0) and for ν_1 (or ν_2) running in the whole interval (ν_0, $\nu_{\max}$). Since $\mathrm{Re}C^+(\nu,2\mu^2)$ at *higher* ν is not guaranteed to be really reliable when reconstructed from a phase-shift analysis (because t is unphysical), his result is not necessarily a more reliable one. Finally, a calculation by Shih and Shepard[19] used the amplitude $A^+(\nu=0, \nu_B=0)$ only. Since the σ term happened to be the difference of two big but nearly equal numbers, the result has been subjected to large errors.

There are also several theoretical papers[9-11,20] on the πN σ term, as well as numerical and theoretical work on other systems.[21] All arguments point to the fact that $T(0,0)$ is much smaller than the value of CD. Although we offer no true solution to a genuine value of the σ term, we have lowered the value of CD by carrying out, independently, the same calculation, in a direction obtained and welcomed by many people.

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$$\lim_{\nu_B\to 0}\left[\lim_{\nu\to 0}\nu B^+(\nu,\nu_B)\right]=0\,,$$

for $\gamma N\to\pi N$ (as also considered by CD, Ref. 4.)

$$\lim_{\nu\to 0}\left[\lim_{\nu_B\to 0}(A+\nu C)\right]=-(eg/2m)(\mu_p/m)\,,$$

but

$$\lim_{\nu_B\to 0}\left[\lim_{\nu\to 0}(A+\nu C)\right]=-\infty\,.$$

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