Fixed-Pole Analysis of the Deuteron Compton Amplitude*

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Using standard finite-energy sum rules we evaluate the $\alpha = 0$, energy-independent real part of Compton deuteron scattering. In contrast with the proton and neutron cases, the result is not consistent with the Thomson limit.

I. INTRODUCTION

The high-energy behavior of Compton scattering has been of particular interest in atomic, nuclear, and particle physics alike. Gell-Mann, Goldberger, and Thirring[1] considered the problem for atomic and nuclear systems. They conjectured that at high enough energies, the coherent Compton scattering amplitude on a bound electron, $f_B(\nu)$, would approach $f_{\text{free}} = -\alpha/m_e$.[2] Years later, Goldberger and Low (GL)[3] actually performed the calculation for an electron bound to an infinitely massive force center, and found that, in the high-energy regime, the scattering amplitude is real and energy-independent but that its value may differ considerably from the free-electron value, particularly in the strong-coupling limit.

A general form, for composite systems of pointlike constituents, which incorporates the possible subtleties of binding effects, has recently been discussed by Brodsky, Close, and Gunion (BCG).[4] They find that the forward Compton amplitude on an atom has the high-energy limit

$$f_{\text{at}}(\nu)\xrightarrow[\nu\gg\text{B.E.}]{} -\frac{Z\alpha}{M_{\text{at}}}\int_0^1 \frac{f_e(x)}{x}\,dx = \frac{-Z\alpha}{m_{\text{eff}}}, \tag{1}$$

where $f_e(x)$ is the normalized, $\int_0^1 f_e(x)\,dx = 1$, probability distribution for finding an electron with momentum $x\vec{P}$ in an atom moving with momentum $\vec{P}$ in the limit $|\vec{P}|\to\infty$, $\langle 1/xM_{\text{at}}\rangle \equiv 1/m_{\text{eff}}$ plays the role of an effective electron mass. The high-energy limit arises from the coherent sum of the "seagull" terms for the individual constituents.[5]

These same qualitative features apply to the high-energy behavior of Compton scattering on composite hadrons despite the additional complications of Regge behavior. That is, the constituent "seagulls" still give rise to a constant real part.[4]

A direct evaluation of the $\alpha = 0$ contribution to the real part of on-shell proton Compton scattering[6] yielded a result consistent in magnitude and sign with the Thomson limit:

$$\begin{aligned} f_1^{\alpha=0}(\infty) &= f_1(0) \\ &= -\alpha/M_p\,, \end{aligned} \tag{2}$$

i.e., the proton behaves like a pointlike object as far as the $\alpha=0$ fixed-pole behavior is concerned.

More recently, it has been shown[7] that a careful extraction of the total neutron photoabsorption cross section, in combination with the standard finite-energy-sum-rule (FESR) techniques, leads us to believe that an $\alpha=0$ term may not be required in neutron Compton scattering (in agreement with the Thomson limit for the neutron).

These results are particularly curious in light of the fact that complications due to Regge behavior have to be considered. The correct prescription for evaluating the $\alpha=0$ contribution is obtained after the leading $\alpha>0$ Regge behavior has been subtracted. The $\alpha=0$ term is given by

$$\begin{aligned} C &= f_1^{\alpha=0} \\ &= -\frac{e^2}{4\pi M}\sum_i \lambda_i{}^2\int_0^\infty\left[\frac{f_i(x)}{x}\,\theta(1-x) - \sum_{\alpha>0}\frac{\gamma_\alpha{}^i}{x^{\alpha+1}}\right]dx, \end{aligned} \tag{3}$$

where f_i is the ith parton's infinite-momentum-frame distribution function, which is related to the deep-inelastic structure function, using the conventional normalization, by

$$\nu W_2(\omega = -q^2/2M\nu) = e^2\sum_i \lambda_i{}^2\omega f_i(\omega).$$

Due to the Regge behavior of hadronic amplitudes $f_i(x)\sim\gamma_\alpha{}^i/x^\alpha$ for small x.

An underlying reason as to why this expression should reduce to the Thomson limit is not apparent.

Given this state of affairs it becomes of interest to see how a composite system of nucleons behaves in regard to the $\alpha=0$ energy-independent piece in its Compton amplitude. The deuteron is, of course, the only such system for which adequate data are available. In the weak-binding limit, $f_{p,n}(x)\sim\delta(x-\frac{1}{2})$, one expects (given the phenomenological fact that nucleons behave as if pointlike in their $\alpha=0$ behavior) that

TABLE I. Regge fits, with $\alpha(0)$ variable and $\alpha(0)$ fixed at $\frac{1}{2}$. Errors are only quoted for the last case.

No.	Data from	Cutoff C_0 (GeV)	Functional form =fit	N_p	A	B	χ^2
1	DESY (D), NINA (N), Santa Barbara (SB)	2.0 GeV	$A+B/\sqrt{\nu}$	139	191.38± 3.18	106.99± 7.17	101.25
2	SB, D	2.0 GeV	$A+B/\sqrt{\nu}$	50	190.04± 3.93	117.06± 11.14	51.87
3	D, N, SB	3.0 GeV	$A+B/\sqrt{\nu}$	95	194.41± 3.95	98.03± 9.90	78.55
4	SB, D, N	4.0 GeV	$A+B/\sqrt{\nu}$	50	196.79± 5.19	90.66± 14.58	53.22
5	SB, D	4.0 GeV	$A+B/\sqrt{\nu}$	41	193.29± 5.48	102.13± 15.67	45.63
6	D, N, SB	2.0 GeV	$A+B\nu^{\alpha-1}$, $\alpha=0.280$	139	204.83	106.59	100.68
7	D, SB	2.0 GeV	$A+B\nu^{\alpha-1}$, $\alpha=0.26$	50	204.72	113.88	51.72
8	D, SB, N	3.0 GeV	$A+B\nu^{\alpha-1}$, $\alpha=0.64$	95	179.9	104.6	78.73
9	D, SB, N	4.0 GeV	$A+B\nu^{\alpha-1}$, $\alpha=0.498$	50	194.74	97.03	53.46

$$C=f_{1(\text{deut})}^{\alpha=0} = -\frac{\alpha}{M_d}\,\frac{\lambda_p^2+\lambda_n^2}{\frac{1}{2}} = -3\ \mu\text{b GeV}. \tag{4}$$

On the other hand, it is possible that binding effects would be sufficient to give $C=-\alpha/M_d = -1.5\ \mu$b GeV, the deuteron's Thomson limit. The important binding corrections arise primarily from two sources: (a) photodisintegration of the deuteron and (b) shadow and Fermi-motion effects.

Even at high energies, $\sigma_d \neq \sigma_p + \sigma_n$ as a result of the Glauber and smearing effects.[7,8] It is the purpose of this paper to evaluate the deuteron Compton amplitude's constant real piece ($\alpha=0$ fixed pole) including the above-mentioned effects in order to see if a qualitative picture emerges. We shall see that the weak-binding result, i.e., $C=-3\ \mu$b GeV, appears to be consistent with the data, while the Thomson-limit value is not.

II. ANALYSIS OF THE DATA

The procedure for obtaining the $\alpha=0$ contribution is by now familiar. For the deuteron, care has to be taken that contributions below normal nucleon threshold are included. One has

$$C=f_1^d(\nu=0)-\frac{1}{2\pi^2}\left[\int_0^N \sigma_T^d(\nu)\,d\nu-\left(NA+\frac{N^\alpha}{\alpha}B\right)\right], \tag{5}$$

where a simple power-law behavior has been assumed for $\nu > N$:

$$\sigma_0(\nu)\sim A+B\,\nu^{\alpha(0)-1}\,. \tag{6}$$

A and B are the Pomeranchuk and P' trajectory residues, respectively. In contrast to the analysis on nucleons we will not assume *a priori* $\alpha(0)=\frac{1}{2}$. The energy-dependent nuclear physics corrections could possibly change $\alpha(0)$ from the value observed on free nucleons. We performed fits to the available data[9] using the form given in Eq. (6). The results for A, B, and $\alpha(0)$ for a variety of cutoffs and combinations of data groups are presented in Table I. The errors assigned to the deuteron total cross section in the fitting program included both statistical and averaged systematic errors. For comparison we give in the same table results for fits of the form $a+b/\sqrt{\nu}$.

One should notice that as the low-energy cutoff increases, the preferred value of $\alpha(0)$ approaches $\frac{1}{2}$. This is a reflection of the fact that above 4 GeV the shadow and smearing corrections become almost energy-independent,[7] thus affecting only the Pomeranchuk piece. On the other hand, in the region between 2 and 4 GeV, both corrections show a substantial energy dependence and deviations from the simple form $\sigma\sim a+b/\sqrt{\nu}$ seem to be present, as the fitted values of $\alpha(0)$ indicate. The χ^2 minimum is, however, extremely shallow, so the above deviations may not be statistically meaningful. In any case, despite the inherently larger errors in the Regge parameters, the most trustworthy results for the $\alpha=0$ residue will be those for which the data are directly integrated up to 4 GeV and a normal Regge form is used thereafter.

In Table II we present the low-energy integrals,

TABLE II. Low-energy integrals for different cutoffs; we tabulate the integral $\int_0^N \sigma_d(\nu)d\nu = I_0$. The values of I_0 tabulated correspond to the choice $\sigma_d(\nu) = 2\sigma_p(\nu)$ for $0.150 < \nu < 0.265$ GeV. The corresponding values for the choice $\sigma_d(\nu) = 2\sigma_p^{\text{smeared}}(\nu)$ for $0.150 < \nu < 0.265$ GeV are 5 μb GeV smaller when a Hamada-Johnston wave function is used.

N (GeV)	I_0 (μb GeV)
2.015	715.4
3.015	975.08
4.015	1223.3

over the deuteron data, required in Eq. (5); $\int_0^N \sigma_T^d(\nu)d\nu$ was evaluated using the $\gamma + d \to p + n$ data up to 0.150 GeV and the Daresbury data from 0.265 GeV to N. Between 0.150 GeV and 0.265 GeV, two alternatives were used to interpolate the two sets of data:

(a) $\sigma_d(\nu) = 2\sigma_p(\nu)$, i.e., no smearing corrections, and

(b) $\sigma_d(\nu) = 2\sigma_p^{\text{smeared}}(\nu)$,

where $\sigma_p^{\text{smeared}}$ was calculated using a standard Hamada-Johnston deuteron wave function. The true deuteron cross sections may lie between these two limits, the s-wave contribution in that region being somewhat larger for neutrons than protons.

It should be pointed out that the $\gamma + d \to p + n$ channel gives $\int_0^{0.150} \sigma_T^d(\nu)\,d\nu \sim 18.9$ μb GeV, a contribution to

$$f_1^{\alpha=0}(\text{deut}) = -0.95\ \mu\text{b GeV}.$$

Excluding the low-cutoff fits with α not $\approx \frac{1}{2}$ we obtain a range of fixed-pole values that lie between -2.2 μb GeV and -6.6 μb GeV. Table III presents a complete compilation. The quoted errors, for the fixed $\alpha = \frac{1}{2}$ fits, include those arising from the Regge-parameter error matrices; small additional uncertainties are present due to error in the low-energy intergrals. For the nonlinear three-parameter fits, the error matrices were not judged to be meaningful. Although the results are anything but conclusive they suggest a value for C that is larger than the low-energy theorem value and perhaps consistent with the proton Thomson limit. Thus the various nuclear binding effects appear to at least partially cancel, leaving a result consistent with the "weak-binding limit."[10] A direct measurement of the real part of deuteron Compton scattering would, of course, be preferable to the above analysis and could provide a clue as to possible anomalous behavior of the total photoabsorption cross section at energies above those currently measured.

TABLE III. Fixed-pole results; the numbers in the first column correspond to the labels in Table I. The errors quoted for the two-parameter fits take into consideration only the uncertainties due to the Regge parameters.

Fit number	Fixed-pole, C (μb GeV)	Error (μb GeV)
1	-2.82	± 0.73
3	-3.96	± 1.68
4	-4.77	± 1.86
5	-2.22	± 2.18
6	$+6.63$	
7	$+9.77$	
8	-6.64	
9	-4.14	

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[2]Throughout the paper, we will only consider the spin-averaged forward scattering amplitude, to lowest order in α; to that order the amplitude on a free electron is equal to $-\alpha/m_e$, the Thomson limit.

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[5]This is strictly true only in the scalar case; for partons with spin $\frac{1}{2}$ the so-called z graph is the one that survives at high energies, but this diagram effectively behaves as a seagull when seen in the above-described infinite-momentum frame. (See Brodsky, Close, and Gunion, Ref. 4.)

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[10]It is amusing to notice that this result is again consistent with Harari's argument, in which one can switch off the strong interactions (leaving behind an elementary neutron and proton), since the deuteron becomes unbound in this limit. In this case, the proton and neutron $\alpha = 0$ terms are given by their respective Thomson limits, while that of the deuteron is the sum of the two. This picture becomes suspect when one considers the nucleons to be composite systems.

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Differential Cross Sections for Pair Production by Photons on Electrons

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The cross section for pair production by unpolarized photons on free, unpolarized electrons is differential in four nontrivial variables. This cross section is integrated numerically over two or three of the variables, and various energy spectra and angular distributions are obtained. The calculations are restricted to low photon energies, below 5 MeV, where effects of recoil and exchange are important and may be observed.

INTRODUCTION

The differential cross section for pair production by unpolarized photons on free, unpolarized electrons has been calculated earlier,[1] and the total cross section has been obtained by Mork.[2] It has been shown that for high photon energies, above a few hundred MeV, the difference between the cross sections for production of pairs on a heavy target and on a light target vanishes, since almost all pairs are created at low-momentum transfers and the recoil of the target particle is negligible. Also exchange effects are negligible at high energies. According to Ref. 2, exchange effects may also be neglected for lower photon energies (down to about 6 MeV), and the triplet cross section is well represented by the Borsellino formula[3] which only includes recoil effects. This has been shown for the total cross section, and it must also be true for the differential cross sections except for some special geometries.

For photon energies below about 6 MeV, both exchange and γ-e diagrams are important, and these effects reduce the triplet cross section considerably compared to the Borsellino cross section. In order to verify these effects, it is therefore of interest to study the triplet cross section for energies between threshold ($4mc^2$) and 6 MeV. This energy region also has the advantage in that screening, binding, and Coulomb corrections should be small.

At present, a group at the University of Clermont is making detailed investigations of the triplet process for low photon energies by using a streamer-chamber technique,[4] and motivated by this experiment, we have calculated various cross sections differential in one or two variables. As has been shown by the experiment, it is imperative to make use of the kinematical relations for the triplet process in order to separate real triplets from false ones. For some cases the kinematical limits of the triplet variables can be given by simple analytic expressions (cf. Ref. 1). Some other more complicated cases are discussed below.

We use units in which $\hbar = 1$, $c = 1$, $m = 1$.

I. THE DIFFERENTIAL CROSS SECTION

The differential cross section for unpolarized particles is given by

$$d\sigma = \frac{\alpha r_0^2}{|p\cdot k|}\,\frac{d^3p_1 d^3p_2 d^3p_3}{4\pi^2\epsilon_1\epsilon_2\epsilon_3}\,\delta^4(p+k-p_1-p_2-p_3)X, \tag{1.1}$$

where X is a function of invariant products and it is given in Ref. 1. The four-momenta of the incoming photon and electron are k and p, and the four-momenta of the outgoing electrons and positron are p_1, p_2, and p_3, respectively. Electron energies and momenta are ϵ, ϵ_1, ϵ_2 and $\vec{p}$, $\vec{p}_1$, $\vec{p}_2$; positron and photon energies and momenta are ϵ_3, ω and $\vec{p}_3$, $\vec{k}$, respectively.