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Comparison of the Parton and Generalized Vector-Dominance Analyses of Deep-Inelastic Electron-Nucleon Scattering

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It is shown that, while generalized vector dominance is strictly applicable in the diffractive limit ($\nu \to \infty$, any $Q^2 \geq 0$ such that $x \equiv Q^2/2M\nu \to 0$) of electron-nucleon scattering and the parton model is applicable in the scaling limit ($Q^2 \to \infty$, x fixed), these two models are identical for describing the deep diffractive limit ($Q^2 \to \infty$, $x \to 0$). This equivalence is then used to interpret the different contributions of vector hadronic states which couple to the photon. A new scaling variable $\tilde{x}$ comes out naturally from our analysis.

The generalized vector-dominance model[1] has been made consistent with the present experimental data[2] of electron-nucleon scattering in the deep-inelastic region and can appear as an alternative for the parton model,[3] where the incident electron scatters elastically and incoherently off the pointlike constituents of the target nucleon (Fig. 1). In the vector-dominance model, the collision between a (real or virtual) photon and a nucleon proceeds in two steps: (i) the photon (q) transforms into a vector hadronic state V, and then (ii) V collides with the nucleon (P). The total (virtual) photoabsorption cross section, which is proportional to the imaginary part of the (virtual) forward Compton scattering amplitude, is represented in Fig. 2:

$$[\gamma(q) \to V] + N(P) \to [V' \to \gamma(q)] + N(P)\,.$$

Only the three low-lying vector-meson contributions, $V = \rho, \omega, \phi$, were taken into account in the earlier versions of the vector-dominance model. However, it is clear now that these earlier versions are unsatisfactory at the phenomenological level for at least three reasons: (i) their failure to explain the total photoabsorption cross-section data, (ii) their prediction $\sigma_L \gg \sigma_T$ for deep-inelastic electroproduction which is contrary to the experimental data, and (iii) the big e^+e^- annihilation hadronic cross sections at c.m. energy higher than 1.3 GeV. The generalized vector dominance has been proposed precisely to overcome these difficulties. Here V or V' are interpreted as any hadronic vacuum fluctuation of the photon, and not only as the low-lying mesons ρ, ω, ϕ but also the higher-mass vector mesons such as ρ' ($m_{\rho'} \sim 1.6$ GeV) *and* the nonresonating vector hadronic-state contributions with $J^c = 1^-$, which are very possibly not negligible at all. But this is not enough to fix the present form of the generalized vector-dominance model. For that, we have to introduce the further assumption, which can only be justified in the case of *diffraction*, that, if the ingoing photon in Compton scattering is coupled to a given hadronic vector state V, then the outgoing photon has to be coupled to the *same* hadronic state V. Therefore we take $V = V'$ in Fig. 2. This is analogous to the assumption of parton models, that in the scaling region, only elastic photon-parton scattering contributes (Fig. 1).

Before we begin our discussion, we notice that, in contrast to the generalized vector-dominance model, Brodsky, Close, and Gunion[4] propose that the discrepancy between the naive vector-dominance model (i.e., $V=\rho,\omega,\phi$) and the photoproduction data indicates that we are seeing, besides the ρ,ω,ϕ contributions, the effect of parton scattering, just as in the scaling region (Fig. 1). For their analysis, they use a parton model due to Landshoff, Polkinghorne, and Short,[5] where the photons interact with the proton, as far as the absorptive part of the (virtual) Compton scattering amplitude is concerned, in two different ways: (i) through connected off-shell (parton-parton-proton)^2 six-point functions [Fig. 1(a) of Ref. 4], which are assumed to vanish in the Bjorken scaling limit, and (ii) through (parton-proton)^2 four-point functions in which the parton propagates freely between the ingoing and outgoing photons [Fig. 1(b) and Fig. 1(c) of Ref. 4; our Fig. 1] and which are the only nonvanishing terms in the Bjorken scaling limit. Furthermore they make the hypothesis that the above-mentioned six-point functions can be evaluated by the ρ,ω,ϕ contributions as is done in the naive vector-dominance model [Fig. 3 of Ref. 4]. Because our analysis will show that, at least when $x\to 0$, the contributions of the vector hadronic-state continuum in the generalized vector-dominance model have an analytical structure identical to the above-mentioned four-point functions of the parton model, we would expect an intimate connection between the two models.

We start with the usual tensor $W_{\mu\nu}$, which includes all strong-interaction effects on inelastic electron-nucleon scattering, assuming one-photon exchange:

$$W_{\mu\nu}=4\pi^2\frac{E}{M}\int d^4y\, e^{iq\cdot y}\langle p|[J_\mu(y),J_\nu(0)]|P\rangle$$
$$=-\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)W_1(q^2,\nu)+\frac{1}{M^2}\left(P_\mu-\frac{P\cdot q}{q^2}q_\mu\right)\left(P_\nu-\frac{P\cdot q}{q^2}\right)W_2(q^2,\nu)$$
$$=-\frac{1}{M}\{(g_{\mu\nu}q^2-q_\mu q_\nu)C_1(q^2,\nu)+[q^2P_\mu P_\nu-M\nu(P_\mu q_\nu+P_\nu q_\mu)+(M\nu)^2g_{\mu\nu}]C_2(q^2,\nu)\}. \quad (1)$$

We have used the following definitions: $P\cdot q\equiv M\nu$, $q^2\equiv -Q^2\le 0$, $x\equiv Q^2/2M\nu$ $(-1\le x\le +1)$ and relations:

$$C_1=\frac{1}{4M\nu x^2}(\nu W_2-2xMW_1), \quad (2a)$$

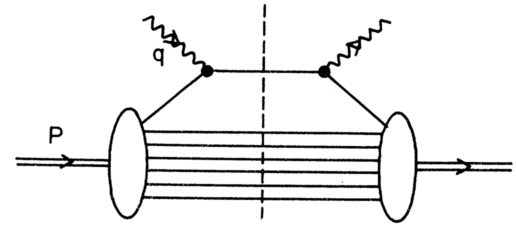

FIG. 1. Parton-model diagram for highly virtual-photon–hadron collision.

$$C_2=\frac{1}{Q^2M}W_2. \quad (2b)$$

Defining the Fourier transforms

$$C_i(q^2,\nu)=\int d^4y\, e^{iq\cdot y}C_i(y^2,y\cdot P), \quad (3)$$

we get the space-time representation of $W_{\mu\nu}$:

$$4\pi^2E\langle P|[J_\mu(y),J_\nu(0)]|P\rangle=(g_{\mu\nu}\Box-\partial_\mu\partial_\nu)C_1(y^2,y\cdot P)+[P_\mu P_\nu\Box-(P\cdot\partial)(P_\mu\partial_\nu+P_\nu\partial_\mu)+g_{\mu\nu}(P\cdot\partial)^2]C_2(y^2,y\cdot P), \quad (4)$$

with $C_i(y^2,y\cdot P)=-C_i(y^2,-y\cdot P)$ and $C_i(y^2,y\cdot P)=0$ for $y^2<0$.

Now, it has been shown[6,7] that (i) if MW_1 and νW_2 scale in the Bjorken limit:

$$\lim_{\substack{Q^2\to\infty\\ x\text{ fixed}}} MW_1(q^2,\nu)=F_1(x), \quad (5a)$$

$$\lim_{\substack{Q^2\to\infty\\ x\text{ fixed}}} \nu W_2(q^2,\nu)=F_2(x), \quad (5b)$$

and (ii) if the dominant singularities of the current commutator are on the light cone rather than inside it in coordinate space, then the matrix element of the current commutator, *up to terms whose Fourier transforms vanish in the scaling limit,* is given

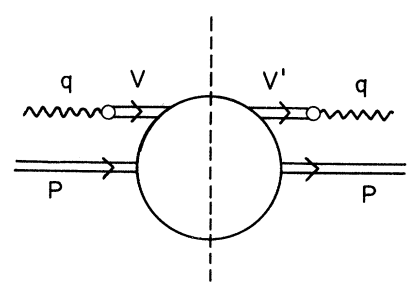

FIG. 2. Vector-dominance-model diagram for the imaginary part of the (virtual) forward Compton scattering amplitude.

by Eq. (4), with

$$C_1(y^2,y\cdot P)=\frac{1}{2\pi i}\,\Delta(y,m^2)\times\int_0^1\frac{d\eta}{\eta^2}\,[F_2(\eta)-2\eta F_1(\eta)]\cos\eta(P\cdot y),\quad (6a)$$

$$C_2(y^2,y\cdot P)=\frac{1}{2\pi i}\,4\Delta'(y,m^2)\int_0^1\frac{d\eta}{\eta(P\cdot y)}\sin\eta(P\cdot y)F_2(\eta)\,.\quad (6b)$$

Furthermore, Jaffe[6] shows that Eq. (4), Eq. (6a), and Eq. (6b) provide *in the scaling limit* a space-time representation for the parton model—assuming a common mass m for all charged partons—in a form which satisfies current conservation. The explicit forms of the singular functions Δ and Δ' are

$$\Delta(y,m^2)=\frac{1}{2\pi}\epsilon(y_0)\left\{\delta(y^2)-\theta(y^2)\frac{m\,J_1(m(y^2)^{1/2})}{2(y^2)^{1/2}}\right\},\quad (7a)$$

$$\Delta'(y,m^2)=\frac{1}{8\pi}\epsilon(y_0)\theta(y^2)\left\{1-m^2y^2\left[\frac{1-J_0(m(y^2)^{1/2})}{m^2y^2}\right]\right\}.\quad (7b)$$

We note that $\Delta'(y,m^2)=-d/dm^2\,\Delta(y,m^2)$, and that the second terms in brackets in Eq. (7a) and Eq. (7b) (where the mass m appears) are less singular than the first ones and do *not* contribute to the scaling limit as defined in Eqs. (5), in agreement with the analysis of Jackiw, Van Royen, and West.[8]

In the following, we display the coordinate-space representation of the matrix element of the current commutator in the case of generalized vector dominance for the diffractive region: $\nu\to\infty$ any $Q^2\geq 0$ such that $x\to 0$. For the sake of simplicity, we only consider the transverse part[9] of the virtual photon-nucleon cross section $\sigma_T(q^2,\nu)$ which can be written (See Appendix) in the *diffractive* region as

$$\lim_{\substack{\nu\to\infty\\ Q^2\ll 2M\nu}}\frac{\sigma_T(q^2,\nu)}{4\pi^2\alpha}=\sum_V\frac{A_V}{(1+Q^2/m_V{}^2)^2}+\frac{A_c}{1+Q^2/m_c{}^2},\quad (8)$$

where the V's denote vector-meson resonances like ρ,ω,ϕ and m_c is the threshold mass of the vector-state continuum. The term proportional to A_c in Eq. (8) represents the collective effect of the vector hadronic states which couple to the photon in a nearly continuous way and cannot be taken into account by a finite discrete set of terms proportional to the A_V's. The numerical values of the constants A_V and A_c are determined from photoproduction data. See Sakurai and Schildknecht in Ref. 1. They take $m_c=1.4$ GeV.

With Eq. (8), we can write the expression of νW_2 in the diffractive region as

$$\lim_{\substack{|\nu|\to\infty\\ Q^2\ll|2M\nu|}}\nu W_2(\nu,q^2)=Q^2\left[\sum_V A_V\frac{1}{(1+Q^2/m_V{}^2)^2}+A_c\frac{1}{1+Q^2/m_c{}^2}\right].\quad (9)$$

Observe that Eqs. (5b) and (9) imply that

$$F_2(x=0)=A_c\,m_c{}^2.\quad (10)$$

After introducing Eq. (9) into Eq. (2b), we expand in Taylor series with respect to $m_V{}^2/Q^2$ and $m_c{}^2/Q^2$:

$$\lim_{\substack{|\nu|\to\infty\\ Q^2\ll|2M\nu|}}C_2(q^2,\nu)=\frac{1}{Q^2M\nu}\left\{\sum_V A_Vm_V{}^2\left[\sum_{n=1}^\infty(-1)^{n+1}n\left(\frac{m_V{}^2}{Q^2}\right)^n\right]+A_cm_c{}^2\left[\sum_{n=0}^\infty(-1)^n\left(\frac{m_c{}^2}{Q^2}\right)^n\right]\right\}$$

$$\equiv\sum_V\sum_{n=1}^\infty C_V^{(n)}(q^2,\nu)+\sum_{n=0}^\infty C_c^{(n)}(q^2,\nu).\quad (11)$$

Using the relation[9]

$$\int d^4y\,e^{ik\cdot y}(y^2)^n\theta(y^2)\epsilon(y_0)=\pi^2 i\,2^{(2n+4)}n!\,\epsilon(k_0)\left(\frac{d}{dk^2}\right)^{n+1}\delta(k^2),\quad (12)$$

it can be shown[10] that $C_{V,c}^{(n)}(q^2,\nu)$, as defined in Eq. (11), is given in the diffractive region by

$$C_{V,c}^{(n)}(q^2,\nu)=\frac{1}{4\pi^2 i}m_{V,c}{}^2\,c_{V,c}^{(n)}\,A_{V,c}\int d^4y\,e^{iq\cdot y}\left(\frac{m_{V,c}{}^2}{4}y^2\right)^n\theta(y^2)\epsilon(y_0)\int_0^1\frac{d\eta}{\eta(y\cdot P)}\sin\eta(y\cdot P),\quad (13)$$

with

$$c_V^{(n)} = (-)^{n+1}\frac{n}{(n!)^2},$$
$$c_c^{(n)} = (-)^n\frac{1}{(n!)^2}.$$

Note that

$$\int_0^1 \frac{d\eta}{\eta(y\cdot P)}\sin\eta(y\cdot P) = \frac{\mathrm{Si}(y\cdot P)}{y\cdot P}, \tag{14}$$

with $\mathrm{Si}(y\cdot P)$ varying between zero when $y\cdot P=0$, and $\frac{1}{2}\pi$ when $y\cdot P=\infty$, with slow oscillations. Knowing that

$$J_0(m(y^2)^{1/2}) = \sum_{n=0}^{\infty}\frac{(-1)^n}{(n!)^2}\left(\frac{m^2}{4}y^2\right)^n, \tag{15a}$$

$$-\tfrac{1}{2}(m(y^2)^{1/2})\,J_0'(m\,(y^2)^{1/2}) = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(n!)^2}\,n\left(\frac{m^2}{4}y^2\right)^n, \tag{15b}$$

and using Eqs. (3), (7b), (11), and (13), we get the space-time representation of $C_2(q^2,\nu)$ (up to terms whose Fourier transforms vanish in the *diffractive* limit):

$$C_2(y^2, y\cdot P) = \frac{1}{2\pi i}\,4\left\{\Delta'(y, m_c^2)\,{}_0\!\int^1\frac{d\eta}{\eta(y\cdot P)}\sin\eta(y\cdot P)m_c^2A_c - \sum_V\frac{d}{dm_V^2}\,\Delta'(y, m_V^2)\,{}_0\!\int^1\frac{d\eta}{\eta(y\cdot P)}\sin\eta(y\cdot P)m_V^4A_V\right\}. \tag{16}$$

Comparison between Eqs. (6b) and (16)—using Eq. (10)—gives us the announced result: The parton model and the generalized vector-dominance model have the same analytical structure when we are dealing with the deep diffractive region: $Q^2\to\infty$, $x\to 0$, i.e., the intersection between the *scaling* region ($Q^2\to\infty$, $0<|x|<1$), where the parton model is strictly applicable and the *diffractive* region ($\nu\to\infty$, any $Q^2\geq 0$ such that $x\to 0$), where the generalized vector-dominance model is applicable independently of additional assumptions[1] on Regge parametrizations, final-state distributions, etc. Furthermore, comparison between Eqs. (6b) and (16) shows that, at least in the diffractive limit, the contributions of the vector hadronic-state *continuum* in the generalized vector-dominance model, after making the "diagonal approximation" $V=V'$ (Fig. 2), have an analytical structure identical to the four-point functions of the parton model in which the parton propagates freely between the ingoing and outgoing photons (Fig. 1).

In the present analysis we only considered the transverse part of virtual photon-nucleon structure functions.

We have checked that if we include a longitudinal cross section $\sigma_L(q^2,\nu)$ compatible with Bjorken scaling, such as

$$\lim_{\substack{\nu\to\infty\\ Q^2\ll 2M\nu}}\frac{\sigma_L(q^2,\nu)}{4\pi^2\alpha} = \sum_V B_V\frac{Q^2}{m_V^2}\left(1+\frac{Q^2}{m_V^2}\right)^{-2}, \tag{17}$$

the conclusion of our above analysis is unchanged: We get the same space-time configuration representation for the parton model (with spin-0 partons of *different* masses m_V) and the generalized vector-dominance model in the deep diffractive limit.

Finally, two difficulties could arise with our comparison: (i) The arbitrary distinction between low-lying vector states and higher states and (ii) the rather high value of m_c (=1.4 GeV) in the analysis of Ref. 1. On the other hand, direct computation of $\nu W_2(q^2,\nu)$ from Eq. (6b), using Eqs. (2b), (3), and (4) gives

$$\lim_{\substack{|\nu|\to\infty\\ Q^2\to\infty}}\nu W_2(q^2,\nu) = \frac{1}{1+m^2/Q^2}\int_0^1 d\eta\,F_2(\eta)\times[\delta(\eta-\tilde{x})+\delta(\eta+\tilde{x})]$$
$$= \frac{1}{1+m^2/Q^2}F_2(\tilde{x}), \tag{18}$$

with

$$\tilde{x} = \left[\frac{-\nu+\epsilon(\nu)(\nu^2+Q^2+m^2)^{1/2}}{M}\right]\xrightarrow[|x|\ll 1]{}\frac{Q^2+m^2}{2M\nu}.$$

Equation (18) provides a natural extension of Eq. (9) into the scaling region with the scaling variable $\tilde{x}=[-\nu+\epsilon(\nu)(\nu^2+Q^2+m_c^2)^{1/2}]/M$. But because scaling appears already satisfied with $Q^2\approx 1\ \mathrm{GeV}^2$ in terms of x, the Bjorken variable, we expect that $m_c^2\ll 1\ \mathrm{GeV}^2$.

An equation similar to Eq. (18), with $\tilde{x}$ replaced by $(Q^2+m^2)/2M\nu$, has been used in Ref. 11 to fit the photoproduction data as well as the electroproduction data for $\tilde{x}\ll 1$. In these analyses $m \leq 0.5$ GeV and *no* distinction is made between the low-lying vector states ρ,ω,ϕ and the higher-mass vector states or the nonresonating vector hadronic-state contributions. This means in practice that the A_V's are taken equal to zero in Eq. (8). It is interesting to compare $\tilde{x}$ with x', the Bloom-Gilman variable[12] and x^W, the Rittenberg-Rubinstein variable[13] for large ν and x fixed:

$$\tilde{x}=\frac{Q^2+m_c^{\,2}}{M((\nu^2+Q^2+m_c^{\,2})^{1/2}+\nu)}$$

$$\simeq x\left(1-x\frac{M^2}{2M\nu}\right)+\frac{m_c^{\,2}}{2M\nu}+O\left(\frac{1}{\nu^2}\right),$$

$$x'=\frac{Q^2}{2M\nu+M^2}\simeq x\left(1-\frac{M^2}{2M\nu}\right)+O\left(\frac{1}{\nu^2}\right),$$

$$x^W=\frac{Q^2+\mu^2}{2M\nu+M_W^{\,2}}\simeq x\left(1-\frac{M_W^{\,2}}{2M\nu}\right)+\frac{\mu^2}{2M\nu}+O\left(\frac{1}{\nu^2}\right),$$

with $M_W^{\,2}\simeq 1.5\ \text{GeV}^2$ and $\mu^2\simeq 0.4\ \text{GeV}^2$. The three variables $\tilde{x}$, x', and x^W will be nearly equivalent when $x\lesssim 1$ if $m_c^{\,2}\ll\mu^2$. Observe that m_c, the threshold mass of the vector-state continuum which couples to the photon, has to satisfy the obvious lower bound $m_c\geq 2m_\pi=0.279$ GeV.
For these reasons, we are inclined to believe that $m_c^{\,2}=4m_\pi^{\,2}\simeq 0.08\ \text{GeV}^2$. Furthermore, we can note from Eq. (10) that the smaller m_c would be, the smaller the diffractive part of $F_2(x)$ would be too, a a situation which is not unwelcome[14] if we want to satisfy the Adler sum rule.

In conclusion, we would like to make the following observations:

(a) The realization of Bjorken scaling, in the generalized vector-dominance model is obtained from the dominant singularities of the current commutator on the light cone and *not* from those inside it.[15]

(b) The second term in brackets in Eq. (7b) (where the mass m appears), which does not contribute to the scaling limit, is present in the two models: In the spin-$\frac{1}{2}$ parton model, m is the parton mass and, in the generalized vector-dominance model, with $\sigma_L=0$, m is just the threshold mass (m_c) of the vector-state continuum. Generalizations to the cases where partons have different masses and $\sigma_L\neq 0$ are straightforward if we introduce spin-0 as well as spin-$\frac{1}{2}$ partons.

(c) In view of satisfying $Q^2\sigma_T\neq 0$ when $Q^2\to\infty$, the generalized vector-dominance model cannot be reduced to a finite sum of isolated vector-meson contributions [this is the case if $A_c=0$ in Eq. (8)].

(d) Then the nonvanishing contributions to $Q^2\sigma_T$ when $Q^2\to\infty$, come exclusively from the vector-state continuum, i.e., $A_c\neq 0$ in Eq. (8).

(e) The vector-state continuum contribution has certainly, in addition to the term $A_c(1+Q^2/m_c^{\,2})^{-1}$ in Eq. (8), other terms which vanish in the scaling limit (see Appendix). Therefore, the parameter A_c is rather arbitrary.

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APPENDIX

Let us derive Eq. (8). We express the transverse part of the virtual-photon–nucleon cross section as a double-spectral representation[1]:

$$\frac{\sigma_T(q^2,\nu)}{4\pi^2\alpha}=\int_{4m_\pi^{\,2}}^{\infty}dm^2\int_{4m_\pi^{\,2}}^{\infty}dm'^2\,\frac{\tilde{\rho}_T(m^2,m'^2,W^2)}{(m^2-q^2)(m'^2-q^2)},\tag{A1}$$

where m and m' are the masses of the hadronic states V and V', respectively (Fig. 2) and $(p+q)^2=W^2$. In the *diffractive* limit we can expect that $V+N\to V'+N$, with $V'\neq V$, has a negligible cross section compared with $V+N\to V+N$, and we can therefore make the diagonal approximation:

$$\lim_{W\to\infty}\tilde{\rho}_T(m^2,m'^2,W^2)=\delta(m^2-m'^2)\rho_T(m^2,W^2).\tag{A2}$$

Introducing Eq. (A2) into Eq. (A1), we get

$$\lim_{\substack{\nu\to\infty\\ Q^2\ll 2M\nu}}\frac{\sigma_T(q^2,\nu)}{4\pi^2\alpha}=\int_{4m_\pi^{\,2}}^{\infty}\frac{\rho_T(m^2,W^2)}{(m^2+Q^2)^2}\,dm^2.\tag{A3}$$

$\rho_T(m^2,W^2)$ can contain a finite discrete set of terms corresponding to the isolated vector-meson resonances such as ρ,ω,ϕ and other terms which represent the collective effect of the vector hadronic states which couple to the photon in a nearly continuous way. Then we use for $\rho_T(m^2,W^2)$ the following form, *compatible with Bjorken scaling:*

$$\rho_T(m^2,W^2)=\sum_V A_V m_V^{\,4}\,\delta(m^2-m_V^{\,2})+A_c m_c^{\,2} f(m^{-2})\theta(m^2-m_c^{\,2}),\tag{A4}$$

where $f(m^{-2})$ is a function such as

$$f(m^{-2})=1+O(1/m^2),\quad f(0)\equiv 1$$

and $m_c^{\,2}\geq 4m_\pi^{\,2}$. Introducing Eq. (A4) into Eq. (A3) we get

$$\lim_{\substack{\nu\to\infty\\ Q^2\ll 2M\nu}}\frac{\sigma_T(q^2,\nu)}{4\pi^2\alpha}=\sum_V\frac{A_V}{(1+Q^2/M_V^{\,2})^2}+\frac{A_c}{(1+Q^2/m_c^{\,2})}\times\left[1+O\left(\frac{m_c^{\,2}}{Q^2}\ln\frac{m_c^{\,2}}{Q^2}\right)\right].$$

This last equation is identical to Eq. (8) except for the terms of the vector-state continuum which vanish in the scaling limit.

Note added in proof. As we have argued at the end of this article, because scaling appears already satisfied with $Q^2 \simeq 1$ GeV^2, *no* distinction can be made between the low-lying vector mesons ρ, ω, ϕ and the higher-mass vector mesons *or* the nonresonating vector-state continuum which couple to the photon. Therefore the A_V's can be taken equal to zero in Eq. (A4) while the fine structure of the low-lying vector states can be included in the function $[f(m^{-2})-1]$, which gives a vanishing contribution in the scaling limit.

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Bilocal Currents and Scaling in Generalized Bjorken Limits

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We investigate the processes $eN \to ehX$, $(\nu,\bar{\nu})N \to \mu hX$ $(h=\pi,\rho,\gamma)$ in three different kinematic regions and study the behavior of the relevant integrated structure functions in generalized Bjorken limits. The technique used is a generalization of the bilocal current algebra put forward by Fritzsch and Gell-Mann. As a consequence of the hypothesis of light-cone dominance, we obtain scaling in the Bjorken limits of all the three kinematic regions. We also find that the scaling functions are expressible as a linear combination of those for the ordinary inclusive reactions and that their explicit dependence on the scale parameters is uniquely predicted once the scaling functions for the processes $eN \to eX$ and $(\nu,\bar{\nu})N \to \mu X$ are known.

I. INTRODUCTION

The recognition that several features of the MIT-SLAC experiments, as viewed in the context of Bjorken scaling, resemble the behavior of free-field theory (rather than perturbation theory) in the vicinity of the light cone led to the proposal of light-cone algebra of bilocal operators put forward by Fritzsch and Gell-Mann.[1] They abstracted the algebra from free-quark field theory. Subsequently Gross and Treiman[2] showed that the same algebraic structure remains even in the presence