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New Scheme of Dual Models and Exotic Peaks*

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A simple s-channel resonance model based on a new scheme of duality is proposed and is shown to explain successfully (i) the appearance of distinct exotic peaks with slopes and relative heights similar to those of the ordinary Regge exchanges, and (ii) the rapid falloff of exotic cross sections vs energy, which have been recently observed in K^-p and $\bar{p}p$ backward scattering. We also discuss the predictions of the model for the spherical-harmonic moments and the dip systematics of two-body hadronic amplitudes. In particular, the model predicts the persistence of the Odorico dips as well as the usual wrong-signature-nonsense-zero dips up to high energies if the increase of Imα_s is not faster than linear in s. Experimental data now available support our approach.

I. INTRODUCTION

The existence of distinct exotic peaks has been revealed by the recent measurements at CERN of 5-GeV/c K^-p and $\bar{p}p$ backward scattering[1] (see Fig. 1).

The prominent features of these experiments are the following:

(i) The differential cross sections of K^-p and $\bar{p}p$ scatterings, in spite of their exotic nature, have distinct backward peaks with slopes similar to those of the ordinary Regge exchanges, while the magnitudes of the differential cross sections in these exotic cases are greatly suppressed compared with the nonexotic ones, by two or three orders of magnitude at 5 GeV/c.

(ii) The K^-p and $\bar{p}p$ backward cross sections $d\sigma/du|_{u\sim 0}$ fall off very rapidly vs energy, varying as s^{-8} or s^{-9} up to 5 GeV/c.[2]

The interpretation of these features has usually been attempted either in terms of exotic cuts based on the double-Regge exchange mechanism[3] or by exotic trajectory exchanges such as Z^*. Exotic cuts, however, predict neither the correct magnitude nor the s dependence of the rapid falloff of exotic cross sections up to 5 GeV/c.[3] Further, the whole theoretical scheme of double-Regge cuts suffers from serious problems in its enumeration of intermediate states.[4] The interpretation due to the exotic trajectory exchange also seems improbable, since it requires an unusual value of $s_0 \sim 0.05$ GeV^2 in its Regge fit to K^-p backward scattering.[5]

In this paper we would like to show that both the existence of exotic peaks and the rapid falloff of exotic cross sections are quite naturally understood from the direct-channel-resonance point of view based on duality.

In Sec. II, we will propose a simple resonance model based on a new scheme of duality[6] which successfully explains the essential features of exotic amplitudes. The physical picture of this model will also be explained. In Sec. III, we will investigate various physical implications of our model. We first give a simple formula which relates the u- (or t-) channel exotic cross section to those of the line-reversed s-channel exotic process at all angles. It shows that the occurrence of exotic peaks is quite a universal phenomenon in two-body hadronic collisions and these exotic peaks must have slopes and relative heights similar to those of the allowed ordinary Regge exchanges. We also investigate the predictions of the model on the dip systematics and the s dependence of spherical-harmonic moments of two-body hadronic amplitudes. It will be shown that the so-called Odorico dips as well as the usual wrong-signature-nonsense-zero dips persist even up to high energies if the increase of Imα_s is not faster than linear in s. In Sec. IV we make a check of our understanding of exotic amplitudes with recourse to experimental data. By comparing the K^-p and K^+p differential cross sections we estimate Imα_s at 5 GeV/c. This, together with the low-energy resonance data, determines the rate of the falloff of

$$\frac{d\sigma}{du}(K^-p)\Big/\frac{d\sigma}{du}(K^+p)\bigg|_{u\sim 0}$$

up to 5 GeV/c to be $s^{-4.1}$, in agreement with $s^{-4.4\pm 0.5}$ obtained from experimental measurements. Finally, Sec. V is devoted to discussions and concluding remarks.

An abbreviated version of this paper has already been reported in Ref. 7.

II. THE GENERAL APPROACH

We begin by explaining the physical picture of our direct-channel resonance model based on a new scheme of duality.

Contrarily to the conventional view of exotic amplitudes,[8] we would like to consider in this paper the physical consequences of a new scheme of dual models with the boson system of Regge trajectories spaced by *two* units of angular momenta bootstrapping among themselves.

This possibility of dual models has been motivated[6] by the partial-wave projection of the Regge exchange amplitude. Since the physical amplitude would have the resonance poles in the second sheet of the s plane, we assume that it reduces to the narrow-width limit of the dual amplitude as $\mathrm{Im}\alpha_s \to 0$ and to its Regge asymptotic expansion as $\mathrm{Im}\alpha_s \to \infty$. Since the validity of the Regge behaviors on the positive real s axis at high energies is assumed, the resonance poles in the narrow-width limit of the dual amplitude must be shifted into the second sheet of the s plane in such a way as to produce the Regge behaviors on the positive real s axis. Therefore we consider that the above displaced resonance poles would appear as Argand-diagram circles in the partial-wave analysis of the Regge formula.

The recurrence of simple zeros with distance $\Delta\alpha_t = 1$ in the residue functions like $1/\Gamma(\alpha_t)$ makes the imaginary part of the individual s-channel partial-wave amplitudes oscillate with the interval $\Delta\alpha_s = 2$ and hence generates resonances on leading and even-daughter trajectories. Thus the resonance poles on the odd-daughter trajectory must be absent also in the narrow-width limit of dual models in order to have the same number of resonance poles as appear in the partial-wave projection of their Regge asymptotic expansions. Therefore, we are led to the conjecture[6] that the Regge trajectories spaced by two units of angular momentum bootstrap among themselves.[9] This possibility has also been advocated by the presumed linear propagation of amplitude zeros in meson-meson scattering.[10,11] Low-energy $\pi\pi$ and $K\pi$ phase-shift data also give several experimental supports to this scheme.[12]

As a basic illustration of the u-channel exotic amplitude, let us confine ourselves to the $\pi^+\pi^- \to \pi^+\pi^-$ scattering. Since odd-daugther trajectories are absent in this scheme, only even (or odd) angular momentum states are present in each resonance tower, which give a forward-backward symmetric (or antisymmetric) contribution to the scattering amplitude. As a consequence, backward (forward) peaks naturally arise even in the u- (or t-) channel exotic reactions at low energies. Experimentally, a strong backward peak is observed in $\pi^+\pi^- \to \pi^+\pi^-$ backward cross sections up to $W \sim 1.7$ GeV.[13] As the energy increases, the total widths of resonances begin to spread and the neighboring resonance towers tend to overlap. Since

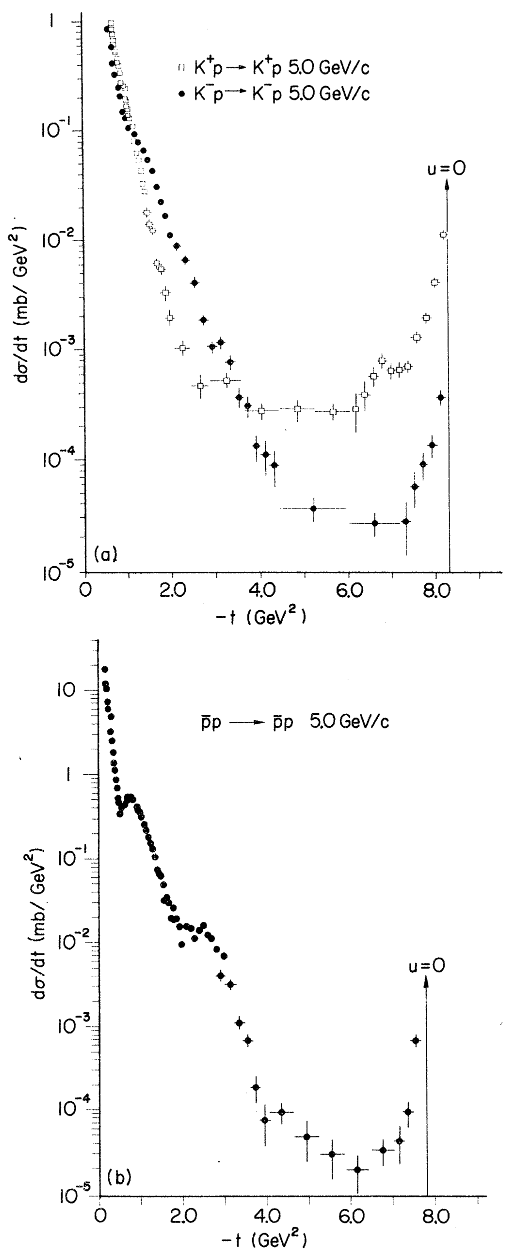

FIG. 1. (a) $K^\pm p \to K^\pm p$ elastic differential cross sections at $p_L = 5$ GeV/c. Data are from Ref. 1. (b) $\bar{p}p \to \bar{p}p$ elastic differential cross sections at $p_L = 5$ GeV/c. Data are from Ref. 1.

the neighboring resonance towers have alternate signs in the backward-scattering regions in u-channel exotic reactions, they make destructive interference between themselves and cause the over-all decrease of exotic cross sections with increasing energy.[14] However, the sharp backward (forward) peaks made up by the individual resonance towers still survive and appear as exotic peaks at high energy.

In order to give mathematical expressions to the above-explained mechanism of exotic amplitudes, we next propose a simple resonance model starting from considerations of dual models in the zero-width limit. As is easily recognized from the preceding discussions, the crucial points in our understanding of exotic amplitudes are that

(i) the angular dependence of each resonance contribution should be correctly evaluated, and

(ii) the overlapping-resonance effects caused by the finite hadronic widths constitute the essential ingredient of the exotic mechanism.

Dual models in their zero-width limit apparently violate the above requirement (ii), which is indispensable for obtaining the over-all decrease of exotic cross sections. In the conventional $\operatorname{Im}\alpha_s$ prescription to keep α_t (or α_u) fixed and then to replace α_s with $\alpha_s + i\operatorname{Im}\alpha_s$,[15] the requirement (i) is violated since $z=1+t/2q^2$ in the resonance expansion of the dual model

$$(s,t)\equiv\frac{\Gamma(1-\alpha_s)\Gamma(1-\alpha_t)}{\Gamma(1-\alpha_s-\alpha_t)}=-\sum_{N,l}\frac{\Gamma_{N,l}}{N-\alpha_s}P_l(z) \tag{1}$$

becomes complex by the above procedure, which invalidates the $t \leftrightarrow u$ crossing property of each resonance contribution.

Therefore in order to remedy these defects and at the same time to keep good properties of dual models in the zero-width approximation, we propose

(1) to preserve all the elastic widths $\Gamma_{N,l}$ and angular dependences of individual resonances as they are (in the zero-width limit), and then

(2) only to shift poles into the second sheet of the s plane.

Thus our new (s,t) term, which we denote as $(s,t)'$, should be expressed as

$$-\sum_{N,l}\frac{\Gamma_{N,l}}{N-\alpha_s-i\operatorname{Im}\alpha_s}P_l(z)\,.$$

Evidently the $t\leftrightarrow u$ crossing property and the finite-width effects of individual resonance contributions are properly taken into account by our prescription, and further, by preserving the elastic widths of resonances, the Regge behavior in the forward direction is also ensured through finite-energy sum rules (FESR). Thus our prescription is expected to give a reasonable description in both forward and backward scattering regions of the u- (t-) channel exotic amplitudes.

Instead of the complicated procedure of calculating $\Gamma_{N,l}$ in the zero-width limit and then summing up these resonances with finite total widths, we can use the following simple procedure: Keeping z fixed, replace α_s with $\alpha_s+i\operatorname{Im}\alpha_s$ in Eq. (1). In order to elucidate this procedure, let us rewrite the (s,t) term as

$$(s,t)=\frac{\sin\pi(\alpha_s+\alpha_t)}{\sin\pi\alpha_s}\,\frac{\Gamma(\alpha_s+\alpha_t)\Gamma(\alpha_s+\alpha_u)}{\Gamma(\alpha_s)} \tag{2}$$

using the condition $\alpha_s+\alpha_t+\alpha_u=1$, which eliminates odd daughters.

It is to be noted here that one should not take this condition $\alpha_s+\alpha_t+\alpha_u=1$ very seriously, since it reduces to $\alpha_\rho(0)\approx\frac{1}{3}$, which is somewhat unrealistic. The condition is used here only as a mathematical tool in order to explore the dynamical consequences of the model which has no odd daughter poles.

Expressing Eq. (2) in terms of α_s and z, and then replacing α_s with $\alpha_s+i\operatorname{Im}\alpha_s$ for fixed z, we obtain

$$\begin{aligned}(s,t)'=&\frac{\sin\pi[(\alpha_s+i\operatorname{Im}\alpha_s)\frac{1}{2}(1+z)+(\alpha_0-\frac{1}{2})z+\frac{1}{2}]}{\sin\pi(\alpha_s+i\operatorname{Im}\alpha_s)}\\&\times\frac{\Gamma((\alpha_s+i\operatorname{Im}\alpha_s)\frac{1}{2}(1+z)+(\alpha_0-\frac{1}{2})z+\frac{1}{2})\Gamma((\alpha_s+i\operatorname{Im}\alpha_s)\frac{1}{2}(1-z)-(\alpha_0-\frac{1}{2})z+\frac{1}{2})}{\Gamma(\alpha_s+i\operatorname{Im}\alpha_s)}\,,\end{aligned} \tag{3}$$

where $\operatorname{Im}\alpha_s=0$ below threshold.

It is worthwhile to note here that the $(s,t)'$ term in Eq. (3) can be explicitly expanded as

$$-\sum_{N,l}\frac{\lim_{\alpha_s+i\operatorname{Im}\alpha_s\to N}\tilde{\Gamma}_{N,l}(q^2)}{N-\alpha_s-i\operatorname{Im}\alpha_s}P_l(z)$$

and the resonance residues

$$\lim_{\alpha_s+i\operatorname{Im}\alpha_s\to N}\tilde{\Gamma}_{N,l}(q^2)$$

are the same as $\Gamma_{N,l}$, those of the zero-width limit (s,t), because of the property

$$\lim_{\alpha_s\to N}(\alpha_s-N)(s,t)=\lim_{\alpha_s+i\operatorname{Im}\alpha_s\to N}(\alpha_s+i\operatorname{Im}\alpha_s-N)(s,t)'.$$

Hence the $(s,t)'$ term in Eq. (3) is expanded as

$$(s,t)'=-\sum_{N,l}\frac{\Gamma_{N,l}}{N-\alpha_s-i\,\mathrm{Im}\,\alpha_s}P_l(z)\,. \tag{4}$$

Therefore, our amplitude $(s,t)'$ enjoys the following favorable properties as compared with the conventional $\mathrm{Im}\,\alpha_s$ prescription:

(i) Particle spectra and resonance residues remain unchanged even after the replacement $\alpha_s \to \alpha_s + i\,\mathrm{Im}\,\alpha_s$. In particular, odd daughters do not appear and also no ancestors arise in the above procedure.

(ii) The $t \leftrightarrow u$ crossing properties of the amplitude is preserved as in the zero-width limit.

(iii) Regge behaviors in the forward direction are also ensured through FESR by preserving $\Gamma_{N,l}$.

So far as the $s \leftrightarrow t$ crossing is concerned, it is broken as in the conventional $\mathrm{Im}\,\alpha_s$ prescription. However, the many desirable properties in the zero-width limit are maximally preserved in our amplitude $(s,t)'$.

In this way, our model amounts to an infinite sum of direct-channel Breit-Wigner resonances with their residues constrained by duality. All the resonances of the same mass are assumed to have the same total widths ($\sim\mathrm{Im}\,\alpha_s/\sqrt{s}$). Since duality determines only the relative strength of the resonance couplings to each two-body hadronic process and leaves the total widths of resonances entirely unspecified,[16] our resonance model here possesses the elastic widths of resonances $\Gamma_{N,l}$ specified by duality and has the total width of resonances ($\sim\mathrm{Im}\,\alpha_s/\sqrt{s}$) as an unspecified parameter to be determined from experimental data. We note here that our simplifying assumption of the same total widths for the equal-mass resonances may or may not be true in reality. We assume it for the sake of simplicity of the mathematical expressions of our model.

III. PHYSICAL IMPLICATIONS OF THE MODEL

A. Exotic Peaks

In order to investigate the physical implications of this model for exotic peaks, it is convenient to start from Eq. (3). With some trivial algebra, it is easily shown that our prescription of replacing α_s by $\alpha_s + i\,\mathrm{Im}\,\alpha_s$ keeping z fixed is equivalent to adding extra terms to α_s, α_t, and α_u such as $\alpha_s + i\,\mathrm{Im}\,\alpha_s$, $\alpha_t + i(\mathrm{Im}\,\alpha_s/4q^2)t$, and $\alpha_u + i(\mathrm{Im}\,\alpha_s/4q^2)u$, respectively. The condition $\alpha_s+\alpha_t+\alpha_u=1$ is still preserved after the above replacement. In terms of these variables, Eq. (3) is rewritten in the form

$$(s,t)'=\frac{\sin\pi[\alpha_s+i\,\mathrm{Im}\,\alpha_s+\alpha_t+i(\mathrm{Im}\,\alpha_s/4q^2)t]}{\sin\pi(\alpha_s+i\,\mathrm{Im}\,\alpha_s)}\;\frac{\Gamma(\alpha_s+i\,\mathrm{Im}\,\alpha_s+\alpha_t+i(\mathrm{Im}\,\alpha_s/4q^2)t)\Gamma(\alpha_s+i\,\mathrm{Im}\,\alpha_s+\alpha_u+i(\mathrm{Im}\,\alpha_s/4q^2)u)}{\Gamma(\alpha_s+i\,\mathrm{Im}\,\alpha_s)} \tag{5a}$$

$$=\frac{\pi}{\Gamma(\alpha_u+i(\mathrm{Im}\,\alpha_s/4q^2)u)}\;\frac{1}{\sin\pi(\alpha_s+i\,\mathrm{Im}\,\alpha_s)}\;\frac{\Gamma(\alpha_s+i\,\mathrm{Im}\,\alpha_s+\alpha_u+i(\mathrm{Im}\,\alpha_s/4q^2)u)}{\Gamma(\alpha_s+i\,\mathrm{Im}\,\alpha_s)}\,. \tag{5b}$$

Equation (5a) shows that our $(s,t)'$ term reduces to[17]

$$(s,t)'\sim e^{-i\pi\alpha_t}\Gamma(1-\alpha_t)\alpha_s{}^{\alpha_t} \tag{6}$$

for large values of s and $\mathrm{Im}\,\alpha_s$ at small $|t|$ ($|t|\ll 4q^2/\pi\,\mathrm{Im}\,\alpha_s$) and hence it in fact exhibits the Regge behavior in the forward direction. For a fixed u value, Eq. (5b) leads to[17]

$$(s,t)'\sim -2\pi i\,e^{-\pi\,\mathrm{Im}\,\alpha_s}e^{i\pi\alpha_s}\times\frac{1}{\Gamma(\alpha_u+i(\mathrm{Im}\,\alpha_s/4q^2)u)}\,\alpha_s{}^{\alpha_u}\,, \tag{7}$$

which shows the "exotic behavior," whose simpler form has previously been introduced in Ref. 12.

Therefore, our $(s,t)'$ term exhibits the Regge behavior at small $|t|$, starts to deviate from it as $|t|$ increases, and then changes smoothly into the "exotic behavior" at small $|u|$.

Let us see how Eq. (7) gives a nice quantitative expression of our foregoing discussions on exotic amplitudes.

(i) It exhibits the damped oscillation of exotic cross sections caused by the alternating signs and cancellation of neighboring resonance towers.[14]

(ii) There appear distinct backward peaks with slopes and relative heights similar to the allowed ones,[1] even though no Regge exchange is allowed in the u-channel.

(iii) It also shows the appearance of the backward fixed-u dips (the so-called Odorico zeros) which have been observed in low-energy $K^-p\to\bar{K}^0n$ cross sections[18] and also in low-energy $\pi\pi$ scattering amplitudes.[19] Thus our Eq. (7) concisely embodies our empirical knowledge of backward exotic amplitudes.

Let us next take the absolute squared values of Eq. (3) to obtain cross-section relations. Neglecting the dips in $(s,t)'$, we have for large values of s and $\mathrm{Im}\,\alpha_s$

$$|(s,t)'|^2\sim e^{-\pi\,\mathrm{Im}\,\alpha_s(1-z)}|(u,t)|^2 \tag{8}$$

(see Ref. 20). This is quite an interesting formula relating the u-channel exotic cross section to its line-reversed s-channel exotic cross section at all angles in a very simple fashion. All the effects of the overlap and destructive interference of s-channel resonances are simply factored into $e^{-\pi \operatorname{Im}\alpha_s(1-z)}$, and aside from this the two cross sections are identical. *The (s,t) term inherits the "allowed" backward peaks in the (u,t) term and exhibits them as "forbidden" peaks.* Therefore, the occurrence of these forbidden peaks must be a universal phenomenon in two-body hadron collisions, and these peaks must have slopes and relative heights similar to the allowed ones. We make an experimental check of the above formula in Sec. IV.

B. Systematics of the Properties of Dips

Apart from its implications on exotic peaks, our amplitude predicts an interesting systematics of the properties of dips in two-body hadronic amplitudes.

As we have already noted in the previous section, the $(s,t)'$ term predicts the existence of fixed-u dips as

$$(s,t)' \propto \frac{1}{\Gamma(\alpha_u + i(\operatorname{Im}\alpha_s/4q^2)u)} . \qquad (9)$$

Therefore, if the increase of $\operatorname{Im}\alpha_s$ is not faster than linear in s (which in fact is required in order to have Regge behaviors[17]), distinct fixed-u dips persist even up to high energies. Especially when $\operatorname{Im}\alpha_s$ is proportional to s, the depths of the fixed-u dips do not change along the line $\alpha_u =$ integer in the Mandelstam plane. On the other hand, the depths of the dips become shallower as we go from the first dip ($\alpha_u = 0$) to the second ($\alpha_u = -1$), third ($\alpha_u = -2$), and so on at fixed energy, since the imaginary part of the argument of Eq. (9) increases with $|u|$.

We show in Fig. 2(a) the typical predictions of our model for the differential cross sections of backward exotic amplitude at various energies. The dips in fact fade away rapidly as we go to larger values of $|\alpha_u|$.

All of the above remarks on the fixed-u dips are equally applicable to the other Odorico zeros as well as the usual wrong-signature nonsense zeros that appear in various combinations of Veneziano terms such as $(s,t)' \pm (u,t)$, $(s,t)' \pm (u,t) \pm (s,u)'$. Therefore, one of the crucial predictions of our resonance model is that *the dips in the angular distributions of two-body hadronic amplitudes*[21] *will persist even at high energy, while the widths of resonances continue to spread indefinitely.*

C. Normalized Spherical-Harmonic Moments

The arguments of the previous subsection show that in the case of u-channel exotic process we have successive entrances of fixed-u dips into the physical regions through the forward boundary as

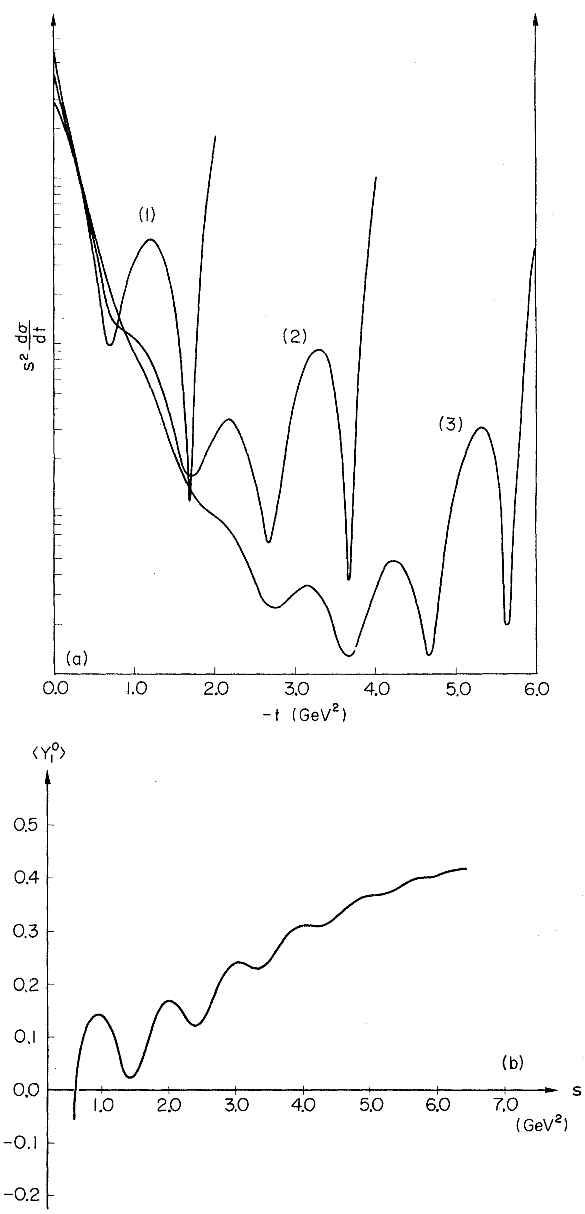

FIG. 2. (a) Typical prediction of our model for differential cross sections of u-channel exotic processes. For simple illustration we show calculations in $\pi^+\pi^-$ elastic scattering. We take $\alpha' = 1$, $\alpha_0 = \frac{1}{3}$ and neglect pion mass. $\operatorname{Im}\alpha_s$ is taken to be linear in s as $\operatorname{Im}\alpha_s = 0.09s + 0.04$, which reproduces the ρ and f widths. (1) is for $s = 2$ GeV^2, (2) is for 4 GeV^2, and (3) is for 6 GeV^2. The units of the vertical axis are arbitrary. (b) Calculation of $\langle Y_1^0(s) \rangle$ for the same process.

we go to higher and higher energies. Since the normalized spherical-harmonic moments

$$\langle Y_l^0(s)\rangle \equiv \int d\Omega \frac{d\sigma}{d\Omega} Y_l^0(z) \Big/ \int d\Omega \frac{d\sigma}{d\Omega}$$

with odd l are sensitive indicators of the passing of amplitude zeros through the forward- or backward-peak regions,[22] it is also interesting to consider the s dependence of $\langle Y_{\text{odd}}^0(s)\rangle$ predicted by our $(s,t)'$ term.

As the entrance of fixed-u dips through the forward physical-region boundary accompanies the sharp drops in $\langle Y_{\text{odd}}^0(s)\rangle$,[22,19] and also the over-all decrease of backward cross sections raises $\langle Y_{\text{odd}}^0(s)\rangle$ to large positive values at high energies, the harmonic moments $\langle Y_{\text{odd}}^0(s)\rangle$ of the u-channel exotic amplitude would at first show sharp drops periodically $(\Delta\alpha_s = 1)$ at low energies and then make a gradual rise to large positive values, with their periodical falls growing less and less significant at higher energies. Exactly such a behavior of $\langle Y_1^0(s)\rangle$ is predicted in Fig. 2(b). As we see in the figure, an interesting correlation exists in our model between the rate of the over-all decrease of exotic cross sections and the depths of fixed-u dips, which are both controlled by $\text{Im}\,\alpha_s$. Readers should compare Fig. 2(b) with the zero-width calculation in Ref. 12 to appreciate the finite-width effects in exotic amplitudes. $\langle Y_{\text{odd}}^0(s)\rangle$ oscillates around zero even at high energies in the zero-width calculation, and in particular $\langle Y_{\text{odd}}^0(s)\rangle = 0$ at α_s = integer due to the absence of odd daughters. Similar behaviors as $\langle Y_1^0(s)\rangle$ in Fig. 2(b) are predicted by our model for the other odd-l moments, and also the periodical falls in $\langle Y_{\text{odd}}^0(s)\rangle$ occur simultaneously for all odd l, as observed experimentally in $\pi\pi$ scattering.[19]

D. Comparison with Conventional $\text{Im}\,\alpha_s$ Prescriptions

The advantages of our prescription of keeping z fixed over the conventional ones of keeping α_t or α_u fixed becomes quite apparent if we compare the predictions of both prescriptions at all scattering angles. Denoting the conventional amplitude with the prescription of keeping α_t fixed as $(s,t)_t$, we have for large values of s and $\text{Im}\,\alpha_s$

$$(s,t)_t = \frac{\Gamma(1-\alpha_s - i\,\text{Im}\,\alpha_s)\Gamma(1-\alpha_t)}{\Gamma(1-\alpha_s-\alpha_t - i\,\text{Im}\,\alpha_s)}$$

$$\sim e^{-i\pi\alpha_t}\Gamma(1-\alpha_t)\alpha_s^{\alpha_t} \qquad (10)$$

at all scattering angles. Therefore $|(s,t)_t| \sim |(u,t)|$ and there occurs no over-all decrease of exotic cross sections. Also the fixed-u zeros disappear with increasing $\text{Im}\,\alpha_s$, since

$$(s,t)_t = \frac{\Gamma(1-\alpha_s - i\,\text{Im}\,\alpha_s)\Gamma(1-\alpha_t)}{\Gamma(\alpha_u - i\,\text{Im}\,\alpha_s)} \ . \qquad (11)$$

On the other hand, with the α_u-fixed prescription, the $(s,t)_u$ term behaves as

$$(s,t)_u = \frac{\Gamma(1-\alpha_s - i\,\text{Im}\,\alpha_s)\Gamma(\alpha_s + \alpha_u + i\,\text{Im}\,\alpha_s)}{\Gamma(\alpha_u)}$$

$$\sim -2\pi i\, e^{i\pi\alpha_s} e^{-\pi\,\text{Im}\,\alpha_s} \frac{1}{\Gamma(\alpha_u)}\, \alpha_s^{\alpha_u} \qquad (12)$$

at all angles. Hence the $(s,t)_u$ term is suppressed even in the forward direction, and also the fixed-u zeros remain the exact zeros.

As we have pointed out in Sec. II, all these difficulties arise from making z complex in the resonance expansion of the dual model in Eq. (1) and the subsequent violation of $t \leftrightarrow u$ crossing properties of each resonance contribution to the amplitude. In other words, the α_t-fixed (α_u-fixed) prescription makes the resonance residues $\Gamma'_{N,l}$ ($\Gamma''_{N,l}$) defined by

$$\sum_l \Gamma_{N,l} P_l\left(1 + \frac{t}{2q^2 + i\,\text{Im}\,\alpha_s/\alpha'}\right) = \sum_l \Gamma'_{N,l} P_l\left(1 + \frac{t}{2q^2}\right) \qquad (13a)$$

$$\left(\sum_l \Gamma_{N,l} P_l\left(-1 - \frac{u}{2q^2 + i\,\text{Im}\,\alpha_s/\alpha'}\right) = \sum_l \Gamma''_{N,l} P_l\left(-1 - \frac{u}{2q^2}\right)\right) \qquad (13b)$$

complex numbers, and hence the $(s,t)_t$ ($(s,t)_u$) term

$$(s,t)_t = -\sum_{N,l} \frac{\Gamma'_{N,l}}{N - \alpha_s - i\,\text{Im}\,\alpha_s} P_l(z) \qquad (14a)$$

$$\left((s,t)_u = -\sum_{N,l} \frac{\Gamma''_{N,l}}{N - \alpha_s - i\,\text{Im}\,\alpha_s} P_l(z)\right) \qquad (14b)$$

is no longer a sum of Breit-Wigner resonances.

Our prescription of keeping z fixed resembles technically to the α_t-fixed (α_u-fixed) one in the forward (backward) region, and therefore our amplitude interpolates the Regge behavior and the "exotic behavior." In this way our resonance model is a successful construction of dual amplitudes having second-sheet poles with their residues constrained by duality *both* in the forward- and backward-scattering regions.

IV. COMPARISON WITH EXPERIMENTS

As we have shown in the previous section, our resonance model gives nice predictions on exotic peaks, on systematics of dips, and also on the s dependence of harmonic moments.

In this section we focus our attention on an experimental check of our formula at large angles and exotic peaks. Firstly let us use Eq. (8) to determine $\mathrm{Im}\,\alpha_s$ at high energy and see if it lies on a reasonable extrapolation from the low-energy resonance data. One way to do this is to compare the s-channel exotic cross sections with their line-reversed u-channel exotic cross sections, such as $d\sigma/dt\,(K^{\pm}p)$:

$$\ln \frac{d\sigma}{du}(K^-p)\Big/\frac{d\sigma}{du}(K^+p) \sim \ln[\,|(s,t)'|^2/|(u,t)|^2] \sim -\pi\,\mathrm{Im}\,\alpha_s(1-z)\,. \quad (15\text{a})$$

In particular, the energy dependence at $z\sim -1$ is given by

$$\ln \frac{d\sigma}{du}(K^-p)\Big/\frac{d\sigma}{du}(K^+p)\Big|_{z\sim -1} \sim -2\pi\,\mathrm{Im}\,\alpha_s\,. \quad (15\text{b})$$

In making a realistic fit to experimental data, spin and other complications arise.[23] However, we can show that the crucial exponential dependence on z always appears in our damping factor $e^{-\pi\,\mathrm{Im}\,\alpha_s(1-z)}$, and therefore we *assume* that the estimation of $\mathrm{Im}\,\alpha_s$ through the damping factors in Eqs. (15a), (15b) is legitimate even in the presence of spin.

Determination of $\mathrm{Im}\,\alpha_s$ from 5-GeV/c data is shown in Fig. 3. The logarithm of the ratio of $d\sigma/du\,(K^-p)$ and $d\sigma/du\,(K^+p)$ turns out to be approximately linear in z for $u \gtrsim -4$ GeV^2, in accordance with our formula. The resulting $\mathrm{Im}\,\alpha_s$ is plotted in Fig. 4 along with the low-energy Y_0^* and Y_1^* resonances taken from Ref. 24. It is remarkable that it lies on a linear extrapolation of $\mathrm{Im}\,\alpha_s$ from the low-energy region. A fit by eye gives approximately $\mathrm{Im}\,\alpha_s = 0.09s - 0.16$, which, when approximated by $\mathrm{Im}\,\alpha_s \approx 0.65\ln s - 0.76$ in the 2–5-GeV/c range, gives

$$\frac{d\sigma}{du}(K^-p)\Big/\frac{d\sigma}{du}(K^+p)\Big|_{z\sim -1} \sim s^{-4.1}\,,$$

in agreement with the experimental value $s^{-4.4\pm 0.5}$. Therefore the rapid falloff of exotic cross sections varying as s^{-8} or s^{-9} is semiquantitatively understood.

Though these figures cannot be taken very seriously owing to both theoretical and experimental uncertainty, the nice consistency of the data analysis seems to support our understanding of the exotic mechanism.

V. DISCUSSIONS AND CONCLUDING REMARKS

Finally, let us compare our understanding of exotic amplitudes with other interpretations.

The conventional view of exotic amplitudes would be to consider the local parent-daughter cancellations as the cause of the over-all suppressions of exotic cross sections. Such a view has been motivated[8] by the conventional $\pi\pi$ Veneziano model,[25] where each resonance tower has opposite-parity states (even daughters and odd daughters). It is easy to show, however, that such a local cancellation does not lead to essential differences between exotic and nonexotic amplitudes.[26] Contrary to the

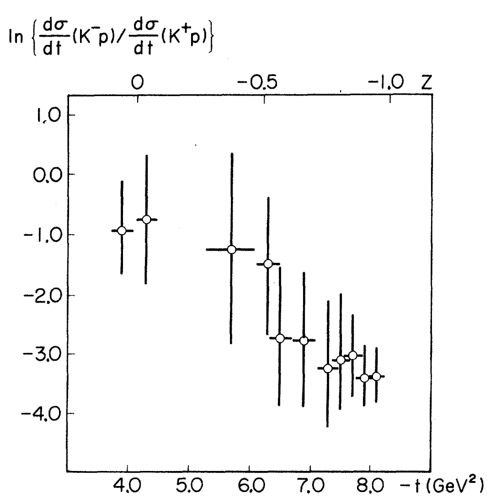

FIG. 3. Plot of the logarithm of the ratio of K^-p and K^+p large-angle cross sections at 5 GeV/c. Data for $|t|<3$ GeV^2 are excluded on account of Pomeron contributions. Data are from Ref. 1.

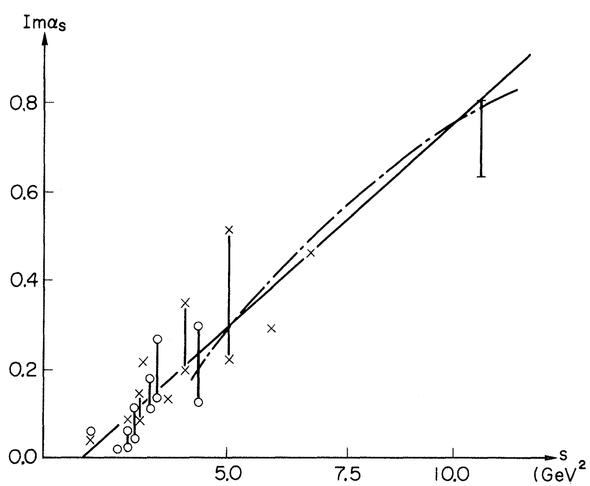

FIG. 4. Plot of $\mathrm{Im}\,\alpha_s$ of Y_0^*, Y_1^* resonances. $\mathrm{Im}\,\alpha_s$ determined at 5 GeV/c is plotted with $\alpha' m\Gamma^{\text{tot}}$ of low-energy Y_0^*, Y_1^* resonances (α' is taken to be 1 GeV^{-2}). The solid line $\mathrm{Im}\,\alpha_s = 0.09s - 0.16$ is a fit by eye to low-energy data, and the dash-dotted line $\mathrm{Im}\,\alpha_s = 0.65 \ln s - 0.76$ is its approximation in the 2–5-GeV/c region. Resonance data (○ for Y_0^*'s, × for Y_1^*'s) are from Ref. 24.

conventional view, our resonance model has only the states with the same parity in each resonance tower and embodies the cancellation mechanism between neighboring resonance towers. It naturally leads to the existence of strong backward peaks at low energies which persist and appear as exotic peaks at high energies.

On the other hand, the usual folklore of exotic peaks has been due to double-Regge cuts.[27] However, the contributions of exotic cuts below 5 GeV/c turn out to be two or three orders of magnitude smaller than experimental cross sections, and hence they cannot explain the rapid falloff of exotic cross sections up to 5 GeV/c.[3] Also, the slopes of exotic peaks are predicted to be typically half the slopes of the allowed ones. These predictions seem to be inconsistent with experiments. However, the most crucial predictions of exotic cuts will be the flattening of the s dependence of exotic cross sections above 5 GeV/c. On the other hand, our predictions are quite contrary to this. $\mathrm{Im}\alpha_s$ linear in s will give the exponential s dependence, and even the logarithmically increasing $\mathrm{Im}\alpha_s$ gives a strong power s dependence (typically s^{-8} or s^{-9}). Until now only upper bounds have been reported in K^-p backward scattering above 5 GeV/c. Thus the future measurement of the s dependence of the decrease of exotic cross sections above several GeV would make a crucial test discriminating among theoretical models.

In conclusion, we have shown how the simple s-channel resonance model based on the new scheme of duality can successfully explain the essential features of exotic amplitudes, i.e., the rapid falloff of exotic cross sections and the appearance of exotic peaks. Our approach to exotic amplitudes also gives a novel way to estimate the averaged hadronic total widths ($\sim\mathrm{Im}\alpha_s/\sqrt{s}$) in high-energy regions. Experimental data now available give support to our approach.

It has also been pointed out that our resonance model gives an interesting predictions on dips, i.e., the persistence of Odorico dips as well as the usual wrong-signature nonsense-zero dips up to high energies.

Future experimental checks of these theoretical predictions would be extremely interesting.

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$$\left|\frac{\Gamma\left(\alpha_s+i\,\mathrm{Im}\alpha_s+\alpha_t+i\,\frac{\mathrm{Im}\alpha_s}{4q^2}\,t\right)\Gamma\left(\alpha_s+i\,\mathrm{Im}\alpha_s+\alpha_u+i\,\frac{\mathrm{Im}\alpha_s}{4q^2}\,u\right)}{\Gamma(\alpha_s+i\,\mathrm{Im}\alpha_s)}\right|$$

$$=\left[1+\left(\frac{\mathrm{Im}\alpha_s}{\alpha_s}\right)^2\right]^{1/2}\left|\frac{\Gamma(\alpha_s+\alpha_t)\Gamma(\alpha_s+\alpha_u)}{\Gamma(\alpha_s)}\right|$$

$$=\left[1+\left(\frac{\mathrm{Im}\alpha_s}{\alpha_s}\right)^2\right]^{1/2}|(u,t)|^2$$

and neglected $(\mathrm{Im}\alpha_s/\alpha_s)^2$ since it is of order 10^{-2} in reality. See Sec. IV.
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Matrix Padé Approximants for the $^1S_0$ and $^3P_0$ Partial Waves in Nucleon-Nucleon Scattering*

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In order to correct the threshold behavior of scalar Padé approximants in *NN* scattering, "matrix Padé approximants," which take into account the various positive- and negative-energy states, have been considered by several authors. In a recent paper by Bessis, Turchetti, and Wortman, truncated matrices based on an incomplete set of basis states were used and a qualitative description of the energy dependence of the $^3P_0$ phase shift was obtained. In this work it is shown that this result is not obtained when a complete set of basis states is used. The main effect of matrix Padé approximants using a complete set of basis states is to introduce an additional attraction at higher energies. Our analysis of the fourth-order graphs is done in a way which allows the external momenta to be completely off shell so that the irreducible graphs can be used as kernels in the Bethe-Salpeter equation. This will make possible the calculation of several higher-order graphs by iterating the Bethe-Salpeter equation.

I. INTRODUCTION

Nucleon-nucleon scattering, in particular for partial waves $L \geq 1$ (L being the orbital angular momentum) and low energies, is an appropriate case in which to study whether Padé approximants can be successfully applied to the summation of the perturbation series of a strong-interaction Lagrangian. The reason is that the phase shifts do not exhibit a resonance behavior and that therefore the perturbation theory can be assumed not to be too drastically divergent. Earlier calculations[1] show that the Born term alone describes higher partial waves reasonably well, and one can therefore hope that P waves and higher waves can be described by a manageable higher-order calculation.

In this paper we investigate a fourth-order calculation in the Yukawa model with the interaction Lagrangian

$$\mathcal{L}_{\mathrm{int}} = -ig\bar{\psi}\gamma_5\vec{\tau}\cdot\vec{\phi}\psi \tag{1}$$

of pseudoscalar pion-nucleon interaction.

A deficiency of low-order Padé approximants has been that the Born term in many partial waves $[{}^1S_0, {}^3P_2, {}^3D_3, \ldots$ (see Ref. 2)$]$ has an anomalous threshold behavior. For the $^1S_0$ wave, e.g., the Born term behaves like a P wave at threshold ($\sim p^3$, p being the modulus of the c.m. momentum). As the fourth-order term has a normal threshold behavior ($\sim p$), the $[1/1]$ Padé approximant behaves like a D wave, and also sharp resonances at low energies occur.[3] This deficiency has been cured by Bessis, Turchetti, and Wortman[4] and before that by Barlow and Bergère,[5] following Bessis's suggestion. These authors considered "matrix Padé approximants" by taking matrix elements between the various positive- and negative-energy states. The elements of these matrices have in general a normal threshold behavior, and therefore forming Padé approximants in this space does in fact correct the threshold behavior. Furthermore it is hoped that the use of these matrix Padé approximants will improve the convergence because the whole matrices contain more information than just the physical element.

The authors of Ref. 4 did not perform a calculation of the complete matrices. They used the same set of basis states used by Barlow and Bergère, which, although sufficient at threshold,