Nonleptonic Interaction and Its Consequences in a Renormalizable Theory of Weak Interactions

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The nonleptonic interaction and its consequences in a particular renormalizable theory of weak interactions mediated by spin-zero bosons are discussed. It is pointed out that the $\Delta S = 0$ weak nuclear processes and the deviations from octet dominance in the $\Delta S = 1$ nonleptonic decays $B \to B' + \pi$ can serve to distinguish it from the usual $V - A$ theory.

It was shown by Kummer and Segrè[1] that the usual $V-A$ structure of the known leptonic and semileptonic processes can be obtained from a fourth-order weak interaction mediated by heavy spin-zero bosons. Such a theory is renormalizable and needs a charged and a neutral scalar boson and two new leptons, one for the electron, L_e, and one for the muon, L_μ. The earlier difficulties[2] with such models were overcome in a theory proposed recently.[3] In this theory, L_e and L_μ are charged and massive (~10 GeV) like the single charged scalar intermediate boson $W^\pm$. The role of the neutral intermediate boson is played by the usual pseudoscalar mesons, π^0, η, and X^0, thus requiring a minimum of new particles. Further, the scalar (S) and pseudoscalar (P) hadronic currents entering into the theory were assumed to satisfy the standard quark-model equal-time commutation rules. This, together with the massiveness of $L_l^\pm$ ($l=e,\mu$) and $W^\pm$, gives the usual $V-A$ structure to the leading contribution to the leptonic and semileptonic decays.[3,4]

The confrontation of the theory with experiment is possible (a) with improved limits on neutral lepton currents[3]; (b) by direct production of $L_l^\pm$ and $W^\pm$ (Ref. 3); and (c) by looking at high-energy quasielastic neutrino processes,[5] e.g., $\nu_\mu + n \to \mu^- + p$. The form factors involved have only a dependence on the square of the momentum transfer in the usual current $\times$ current $V-A$ theory. However, in a scalar theory there is in addition a dependence on the incoming energy. This can be used as a test[5] of our theory.[3]

Earlier it was shown[3] that our theory was consistent with all the known results for the leptonic, semileptonic, and nonleptonic weak processes. In particular, the nonleptonic processes were merely shown to be of the right order of magnitude. The purpose of this note is to comment on the general properties of the nonleptonic interaction and its consequences in this theory, which is referred to below as the scalar theory, as opposed to the usual $V-A$ theory.

Nonleptonic interaction. The interaction Lagrangian density describing the nonleptonic weak processes in our scalar theory[3] is

$$\mathcal{L}_{\rm NL} = g_+ W^+[\cos\theta\,(S_2^1 - iP_2^1) + \sin\theta\,(S_3^1 - iP_3^1)] + \text{H.c.}, \tag{1}$$

where W^+ is the heavy scalar intermediate boson, S_β^α and P_β^α are the scalar and pseudoscalar hadron currents, and $\alpha,\beta = 1,2,3$ are the SU(3) indices. The angle θ was identified with the Cabibbo angle, and the coupling constant g_+ was such that $g_+{}^2$ was of the order of G, the usual weak coupling constant. For the usual $V-A$ theory the nonleptonic interaction is $\mathcal{L}_{\rm NL}(VA) = g_W W_\mu^+ J_\mu^h + \text{H.c.}$, where

$$J_\mu^h = \cos\theta\,(V_2^1 - A_2^1)_\mu + \sin\theta\,(V_3^1 - A_3^1)_\mu. \tag{2}$$

We have written it with an intermediate vector boson W_μ^+ for the sake of comparison with (1); also, $g_W{}^2$ is of order G.

The first-class V and A currents which enter (2) have $\mathcal{C} = -1$ and $+1$, respectively. For definition of $\mathcal{C}$, see Gell-Mann[6] and Dothan.[7] In the scalar theory the pseudoscalar currents P were taken to be the source of the pseudoscalar mesons π^0, etc., so their $\mathcal{C} = +1$. For the scalar currents S, $\mathcal{C} = +1$ is fixed by the quark-model commutator of S and P which gives A. Throughout our discussion we assume CP invariance. In (1), the hadronic current is

$$j^h = \cos\theta j_0 + \sin\theta j_1, \tag{3}$$

where $j_0 = S_2^1 - iP_2^1$ and $j_1 = S_3^1 - iP_3^1$ are the strangeness-conserving $\Delta S = 0$ and strangeness-changing $\Delta S = 1$ currents. The weak nonleptonic processes arise from the effective interaction $\{j^h, j^{h\dagger}\}$, corresponding to $\{J^h, J^{h\dagger}\}$ in the $V-A$ case. Denote by H(p.c.) and H(p.v.) the parity-conserving and parity-violating parts of the effective interaction in the scalar theory. Further, in each case, $H = H_0 + H_1$, where H_0 and H_1 are the $\Delta S = 0$ and $\Delta S = 1$ parts.

H(p.c.) has $\mathcal{C}=+1$ and contains the symmetric SU(3) representations $\underline{1}$, $\underline{8}_S$, and $\underline{27}$, the singlet occurring in H_0(p.c.) only. The transformation properties of H(p.c.) are exactly the same as that for the $V-A$ theory. However, since S and P currents have opposite CP, unlike the V and A currents, H(p.v.) is antisymmetric in S and P due to CP invariance. Consequently, H(p.v.) contains the antisymmetric representations $\underline{8}_A$, $\underline{10}$, and $\underline{10}^*$, and has $\mathcal{C}=-1$. In fact, only the combination $\underline{10}$-$\underline{10}^*$ occurs. This is to be contrasted with the $V-A$ theory, in which the parity-violating part has $\mathcal{C}=-1$ and the SU(3) properties are the same as for the parity-conserving part. We discuss the consequences of these differences below.

1. $\Delta S=0$ *weak nuclear processes.* The parity-violating amplitude arising from H_0(p.v.), giving $|\Delta\vec{I}|=0$, has a factor $\sin^2\theta$, while that giving $|\Delta\vec{I}|=1$ has a factor $\cos^2\theta$. Thus the parity-violating effects with $|\Delta\vec{I}|=1$ are expected to be larger, in contrast with the $V-A$ theory, where $|\Delta\vec{I}|=0$ is expected to predominate, because of the smallness of θ. Further, in the scalar theory there is no $|\Delta\vec{I}|=2$, $\Delta S=0$ parity-violating amplitude, in contrast with the $V-A$ case. These differences between the two theories arise because H_0(p.v.) is the antisymmetric combination of two octets in the scalar theory, in contrast with the $V-A$ case.

2. $\Delta S=1$ *nonleptonic decays.* Each of the seven decays of the type $B\to B'+\pi$ is given in terms of an s-wave and a p-wave amplitude arising from H_1(p.v.) and H_1(p.c.), respectively. The scalar and $V-A$ theories differ only in the non-octet part of H_1(p.v.). Consequently, as far as octet dominance holds, the two theories will be indistinguishable. However, differences will show up in the deviations from octet dominance [viz., the $|\Delta\vec{I}|=\frac{1}{2}$ rule and the Lee-Sugawara (LS) relation[8]] for the s-wave amplitudes alone. Explicitly, the SU(3) content in the $V-A$ case of H_1(p.v.) gives

$$H_1(\text{p.v.})\sim\sqrt{5}\,X(27,\tfrac{3}{2})+X(27,\tfrac{1}{2})+3X(8,\tfrac{1}{2}),\qquad(4)$$

where $X(N,I)$ transforms like the isospin-I part of the SU(3) representation N. In the scalar case,

$$H_1(\text{p.v.})\sim X(10^*,\tfrac{3}{2})-X(10,\tfrac{3}{2})+X(10,\tfrac{1}{2})-X(10^*,\tfrac{1}{2})+X(8,\tfrac{1}{2}).\qquad(5)$$

Incidentally, (4) also gives the SU(3) content of H_1(p.c.) in either theory. Denote the deviations from octet dominance for the s-wave amplitudes by

$$S(\Delta\Lambda)\equiv S(\Lambda^0_-)-\sqrt{2}\,S(\Lambda^0_0),$$

$$S(\Delta\Xi)\equiv S(\Xi^-_-)-\sqrt{2}\,S(\Xi^0_0),$$

$$\sqrt{2}\,S(\Delta\Sigma)\equiv S(\Sigma^-_-)+\sqrt{2}\,S(\Sigma^+_0)-S(\Sigma^+_+),$$

and

$$S(\Delta(\text{LS}))=S(\Lambda^0_-)-2S(\Xi^-_-)+\sqrt{3}\,S(\Sigma^+_0),$$

where Λ^0_- represents the decay $\Lambda^0\to p+\pi^-$, etc. For the $V-A$ case a straightforward SU(3) analysis gives[9]

$$S(\Delta\Lambda)=-S(\Delta\Xi),\qquad(6)$$

$$\sqrt{3}\,S(\Delta\Sigma)+S(\Delta\Lambda)=2S(\Delta(\text{LS})).\qquad(7)$$

In obtaining (7) one needs the value of the relative coefficient of $X(27,\frac{3}{2})$ and $X(27,\frac{1}{2})$ as given by (4). In the scalar theory one obtains[10], using (5),

$$S(\Delta\Lambda)=S(\Delta\Xi),\qquad(8)$$

$$\sqrt{3}\,S(\Delta\Sigma)+29S(\Delta\Lambda)=-6S(\Delta(\text{LS})),\qquad(9)$$

In principle, (7) and (9) can be used to distinguish the two theories. However, the present data verify the predictions of octet dominance rather well. As a result, within errors one cannot say anything definite at present.

3. $\Delta S=1$ *radiative decays.* There are six decays of the type $B\to B'+\gamma$. The amplitude can be written as

$$\epsilon^\mu K^\nu\,\overline{U}_{B'}(a+ib\gamma_5)\sigma_{\mu\nu}U_B\,,\qquad(10)$$

where ϵ^μ is the photon polarization vector and K^ν its four-momentum. The amplitudes a and b arise from H_1(p.c.) and H_1(p.v.), respectively. Clearly, the results for a amplitudes coincide in the two theories. Since, H_1(p.v.) in the $V-A$ theory has $\mathcal{C}=-1$, one finds

$$b(\Sigma^+\to p\gamma)=b(\Xi^-\to\Sigma^-\gamma)=0,\qquad(11a)$$

$$b(\Lambda\to n\gamma)=-b(\Xi^0\to\Lambda\gamma),\qquad(11b)$$

$$b(\Xi^0\to\Sigma^0\gamma)=-b(\Sigma^0\to n\gamma).\qquad(11c)$$

Equivalently, one can use U-spin arguments[11]. If one keeps the octet part and neglects the $\underline{27}$ part then one has the additional relation

$$\sqrt{3}\,b(\Lambda\to n\gamma)=b(\Sigma^0\to n\gamma).\qquad(11d)$$

For an H_1(p.v.) which is a general mixture of $\underline{10}$, $\underline{10}^*$, and $\underline{8}$ but has $\mathcal{C}=-1$, there are four sum rules which are different. However, the particular combination in (5) of $\underline{10}$-$\underline{10}^*$ just does not contribute to the radiative decays. Consequently, for both the scalar and $V-A$ theories, (11a)–(11d) are expected to hold.

A consequence of (11a) is that the asymmetry parameter $\alpha\sim\text{Re}(a^*b)$ for the two decays is zero. Experimentally,[12] $\alpha=1.03^{+0.52}_{-0.42}$ for $\Sigma^+\to p\gamma$. However, the value of α depends crucially on the dynamics. For example, the use of the baryon pole model according to Graham and Pakvasa[13] would give a small $\alpha\sim0.1$ for the scalar theory too. On the other hand, looking at the short-distance effects using operator-product expansions[14] can change the picture.[15,16] By writing the amplitudes a and b as the sum of a short-distance part and nonshort-distance part and taking the latter as given by the $V-A$ theory itself, i.e., by (11a)–(11d) for the b amplitudes, it has been argued[16] that the asymmetry parameter in $\Sigma^+ \to p\gamma$ could be large. The inclusion of the short-distance contributions modifies the sum rules (11a)–(11d) to

$$6b(\Lambda \to n\gamma) + 6b(\Xi^0 \to \Lambda\gamma) = -\sqrt{6}\,[b(\Sigma^+ \to p\gamma) + b(\Xi^- \to \Sigma^-\gamma)], \quad (12a)$$

$$b(\Xi^0 \to \Sigma^0\gamma) - \sqrt{3}\,b(\Xi^0 \to \Lambda\gamma) = \sqrt{2}\,[b(\Sigma^+ \to p\gamma) - b(\Xi^- \to \Sigma^-\gamma)], \quad (12b)$$

$$\sqrt{2}\,[b(\Sigma^0 \to n\gamma) + b(\Xi^0 \to \Sigma^0\gamma)] = -[b(\Sigma^+ \to p\gamma) + b(\Xi^- - \Sigma^-\gamma)]. \quad (12c)$$

The argument in the $V-A$ case[15] for the nature of the short-distance contribution goes through, *mutatis mutandis,* for the scalar theory. The short-distance contribution involves the product of three currents $j_0^\dagger(x) j_1(y) V^{em}(0)$ when $x \to y \to 0$. The leading contribution will again come from an odd-rank tensor, which gauge invariance limits to be the divergence of a tensor (or a pseudotensor) which is an SU(3) octet but behaves like $(3, 3^*) \oplus (3^*, 3)$ under $SU(3) \otimes SU(3)$. Since j_1 is $(3^*, 3)$ and $j_0^\dagger$ is $(3, 3^*)$, their product with V^{em}, which is $(1, 8) \oplus (8, 1)$, will not yield a $(3, 3^*)$ or a $(3^*, 3)$. Thus, scale and $SU(3) \otimes SU(3)$-invariance-breaking terms of the same order are needed, as in the $V-A$ case. Thus the nature of the short-distance contributions is the same in the two theories. To summarize, the radiative decays do not afford a distinction between the two theories.

We have seen that on the whole the scalar theory[3] discussed here and the $V-A$ theory have the same predictions, though deviations from octet dominance in the nonleptonic decays can in principle distinguish between the two theories. A clearer test, though experimentally difficult, is provided by the $\Delta S = 0$ parity-violating weak nuclear processes.

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[10]To make use of the $\mathcal{C}$ properties it is convenient to treat H_1(p.v.) [or H_1(p.c.)] as a spurion and look at the reaction $B\bar{B}' \to \pi + H_1$(p.v.), etc. We have used this technique for the radiative decays also.

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