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[14]S. L. Adler, Phys. Rev. 143, 1144 (1966).

[15]The $I=0$ $\pi\pi$ phase shift given recently by S. D. Protopopescu *et al*. [see *Experimental Meson Spectroscopy–1972*, edited by A. H. Rosenfeld and K. Lai (American Institute of Physics, New York, 1972)] is not easy to interpret in terms of resonance behavior. In this paper, we shall assume the existence of a broad resonance, $\epsilon(700)$, to characterize the significant pion-pion interaction for energies $0.4 \le W \le 1.0$ (GeV). See also P. Carruthers, Phys. Rev. D 3, 959 (1971), for previous work on this subject.

[16]This is just the axial-vector-current analog to the derivation of the Cabibbo-Radicati sum rule [N. Cabibbo and L. A. Radicati, Phys. Letters 19, 697 (1966)].

[17]The spin-parity of D, E are, of course, as yet not known with certainty. If they are found to be 1^+ they must be included in our analysis. Nevertheless their contribution to the sum rule can be shown to be small.

[18]The ρ contribution is that obtained for $\delta=0$, and is not changed significantly if $|\delta| \le 1$.

[19]S. Fubini, G. Furlan, and C. Rossetti, Nuovo Cimento 40, 1171 (1965).

[20]This is particularly true when $|k^2|$ exceeds the values of momentum transfer for which the model was designed to be accurate, $|k^2| \lesssim 1$ GeV^2.

[21]By dominant, we mean that the sum rule $\int W_2(\nu, k^2)d\nu = constant$ is well estimated for $k^2 \cong 0$ in terms of the resonance contributions. In view of this, a question which naturally arises is: What can one infer from the sum rule about experimental studies of inclusive processes? The only unambiguous statement we can make is that resonance production is the major component in the "neutrino + pion → lepton + everything" cross section, $d^2\sigma/(d|k^2|d\nu)$, for $k^2 \cong 0$, $\nu \lesssim \frac{1}{2}\nu_{\max}$.

[22]R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).

[23]That is, the vector-current analog of $N_{ab}(q, p)$ vanishes.

[24]I.e.,

$$m_\rho^2 r_\pi^2 = 6m_\rho^2 \frac{d}{dq^2} \ln F_\pi(q^2)\bigg|_{q^2=0} = -6m_\rho^2 \frac{d}{dq^2} \ln\left(1 - \frac{q^2}{m_\rho^2}\right)\bigg|_{q^2=0} \cong 6.$$

In obtaining numerical estimates for certain of the terms in Eq. (51) we referred to the work of V. S. Mathur and L. K. Pandit, Phys. Letters 19, 523 (1965).

[25]M. A. Keppel-Jones, Phys. Rev. D 6, 1130 (1972), has questioned the value of the constant appearing in the Adler sum rule, while J. Bjorken and S. F. Tuan [Comments Nucl. Part. Phys. 5, 71 (1972)] and J. J. Sakurai, H. B. Thacker, and S. F. Tuan [Nucl. Phys. B48, 353 (1972)] have questioned the sum rule's convergence for large k^2. However, our model is not inconsistent with the conventional sum rules for $k^2 \cong 0$.

[26]E. Golowich, Phys. Rev. 184, 1815 (1969).

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Multiperipheral Theory of Massive-Muon Pair Production in Hadron Collisions*

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Massive-lepton pair production in high-energy hadron collisions is studied in the ABFST (Amati-Bertocchi-Fubini-Stanghellini-Tonin) multiperipheral model. The cross section for point electromagnetic couplings is given by $d\sigma/dQ^2 = Q^{-4}f(s/Q^2)$ when $Q^2 \gg M^2$, where $\sqrt{s}$ is the center-of-mass energy of the colliding protons, Q the mass of the lepton pair, and M the nucleon mass. The scaling function f is expressed in terms of πN and NN off-shell forward absorptive amplitudes. When $s \gg Q^2$, the function f behaves like $a\ln(s/Q^2)+b$, Pomeron dominance being assumed. Gauge invariance of the model is discussed.

I. INTRODUCTION

The interesting SLAC-MIT experiments on deep-inelastic electron scattering probe the electromagnetic structure of hadrons when the current carries a spacelike momentum. The BNL-Columbia experiment[1] extends the probe to timelike momentum by studying the reaction

$$\text{proton} + \text{proton} \to \mu^+ + \mu^- + \text{anything}. \quad (1)$$

If unpolarized protons of momenta p_1 and p_2, energies E_1 and E_2, and mass M collide to produce a muon pair of momentum q in addition to anything else (Fig. 1), then, summing over muon polarizations and momentum variables except $q^2 \equiv Q^2$, the cross section neglecting muon mass is given by

$$\frac{d\sigma}{dQ^2} = \frac{4\alpha^2}{3\pi^3} \frac{1}{[s(s-4M^2)]^{1/2}} W(Q^2, s), \quad (2)$$

where $s=(p_1+p_2)^2$ and $W(Q^2; s) = \tilde{W}_\mu{}^\mu(Q^2, s)$, with

$$\frac{1}{(2\pi)^6}\frac{1}{4E_1E_2}\tilde{W}_{\mu\nu}(Q^2,s)=-\int d^4q\ \delta_+(q^2-Q^2)\sum_n\langle p_1p_2\ \text{in}|J_\mu^{\text{em}}|n\ \text{out}\rangle\langle n\ \text{out}|J_\nu^{\text{em}}|p_1p_2\ \text{in}\rangle(2\pi)^4\delta^4(q+p_n-p_1-p_2). \tag{3}$$

An average over proton spins is understood. The above reaction has been investigated theoretically in several ways.[2] The purpose of this paper is to study the reaction in a multiperipheral model. The ABFST (Amati-Bertocchi-Fubini-Stanghellini-Tonin) model[3] rather than a multi-Regge model is used since the former has a firmer physical basis.[4] In Sec. II the model is used to express $W(Q^2, s)$ in terms of πN and NN off-shell (but not far off) forward absorptive amplitudes with no adjustable parameters; the scaling behavior of $W(Q^2, s)$ is studied. In Sec. III a discussion of the results is given. The generality of the model, in the sense of not being affected by some of the weaknesses of the ABFST model, is pointed out. In the Appendix, gauge invariance of the model is studied.

II. THE MODEL

In reaction (1) the final state consists, in addition to the detected muon pair, of undetected hadrons which for simplicity may be taken to be two nucleons and the rest, pions. The virtual timelike massive photon and the hadrons are supposed to form a multiperipheral chain. First consider emission of the photon from the middle of the chain as in Fig. 2; emission from the ends will be considered later. Where the photon comes from in the chain depends on its momentum. The chain is arranged such that the pion propagators in Fig. 2 all have small masses. Section III has further discussion of this point; we will simply note here that the photon may be looked upon as replacing several neighboring pairs of the usual multiperipheral chain. The amplitude corresponding to Fig. 2 is given by

$$M_{p\pi}(p_1,l_1;K_1,k_1)\frac{1}{l_1^2-m^2}M_{\pi\pi}(-l_1,l_1';k_1',k_1'')\frac{1}{l_1'^2-m^2}$$
$$\times\cdots\frac{1}{q_1^2-m^2}e(-q_1+q_2)_\mu\frac{1}{q_2^2-m^2}$$
$$\times\cdots M_{p\pi}(p_2,l_2;K_2,k_2)\ , \tag{4}$$

where $M_{p\pi}(p_a,p_b;p_c,p_d)$ is the amplitude for the process: proton(p_a)+pion$(p_b)\to$ nucleon(p_c)+pion(p_d), with the pions not necessarily on shell; similarly for $M_{\pi\pi}$. Pion mass is denoted by m. In writing Eq. (4) we have ignored proton spin and also isospin; the photon is, of course, attached only to a charged pion. Furthermore, we have assumed a point coupling at the $\gamma\pi\pi$ vertex; a form factor leads to an additional factor $F(Q^2,q_1^2,q_2^2)$. The question of gauge invariance is taken up in the Appendix. Substituting Eq. (4) into Eq. (3), summing over the hadrons and using the optical theorem, we get

$$W(Q^2,s)=-\frac{1}{4\pi^4}\int d^4q_1\,d^4q_2\,\delta_+((-q_1-q_2)^2-Q^2)A_{p\pi}(p_1,q_1)\frac{1}{(q_1^2-m^2)^2}(q_1-q_2)^2\frac{1}{(q_2^2-m^2)^2}A_{p\pi}(p_2,q_2)\ , \tag{5}$$

where $A_{p\pi}(p_i,q_i)$ is the forward absorptive amplitude for the process: proton(p_i) +pion$(q_i)\to$proton(p_i) +pion(q_i). Had we taken into account proton spin, then writing the $p\pi$ elastic amplitude as $M_1(p_a,p_b;p_c,p_d)+\frac{1}{2}(\not{p}_b+\not{p}_d)M_2(p_a,p_b,p_c,p_d)$ we would get Eq. (5) with $A_{p\pi}(p,q)$ representing the absorptive part of $M_1(p,q;p,q)+(p\cdot q/M)M_2(p,q;p,q)$. Multiperipheralism requires $-q_1^2$, $-q_2^2$ to be restricted to small values of the order of M^2 *even when the photon is massive*, i.e., $Q^2\gg M^2$. Indeed, as noted before, the photon can be looked upon as replacing several consecutive pion pairs of a usual multiperipheral chain consisting of hadrons only, for which the smallness of the masses of the pion links is the motivation for the model. We will therefore replace $(q_1-q_2)^2$ by $-Q^2$. The structure of Eq. (5) is shown in Fig. 3. This structure is clearly more general than the specific amplitude (4) that led to it. We can expect the equation to be simplified enormously by the repeated application of the well-known partial diagonalization.[5] We will state here a general result which can be proved[6] by successive partial diagonalizations:

$$\int d^4q_1\,d^4q_2A_1(p_1,q_1)A_2(q_1,q_2)A_3(q_2,p_2)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}dl\,\frac{e^{(l+1)\theta}}{\sinh\theta}\left(\frac{\pi}{l+1}\right)^2\int_{-\infty}^{0}du_1(-u_1)du_2(-u_2)A_1(l,u,u_1)$$
$$\times A_2(l,u_1,u_2)A_3(l,v,u_2)\ , \tag{6}$$

where the following kinematic restrictions have been imposed: $(p_1+q_1)^2\geq(M+m)^2$, $(q_1+q_2)^2\geq Q^2$ [in our application we only need the particular case of $A_2=\delta_+((-q_1-q_2)^2-Q^2)$], $(q_2+p_2)^2\geq(M+m)^2$, and energy components of p_1+q_1, $-q_1-q_2$, and q_2+p_2 are positive. Also $p_1^2=u$, $p_2^2=v$, $q_i^2=u_i<0$, $\cosh\theta=(s-u-v)/2(uv)^{1/2}$.

$$A_1(l,u,u_1)=\int_{\phi_1}^{\infty}d\phi\, e^{-(l+1)\phi}\sinh\phi\, A_1(p_1,q_1)\ ,$$

where $\cosh\phi=[(p_1+q_1)^2-u-u_1]/2(uu_1)^{1/2}$ (we have taken $u<0$; when $u>0$, we only have to change u to $-u$ and interchange cosh and sinh) and $\cosh\phi_1=[(M+m)^2-u-u_1]/2(uu_1)^{1/2}$. Similarly, $A_2(l,u_1,u_2)$ and $A_3(l,v,u_2)$ are defined. Let us put $\cosh\xi=(Q^2-u_1-u_2)/2(u_1u_2)^{1/2}$ and $\cosh\phi_2=[(M+m)^2-u_2-v]/2(u_2v)^{1/2}$. Also, c is chosen such that the integrand is analytic for $\mathrm{Re}\,l\geq c$. Generalization to the case when there are several integrations $d^4q_1\cdots d^4q_n$ is straightforward but not needed here. We can now write Eq. (5) as

$$W(Q^2,s)=\frac{1}{4\pi^2}\frac{Q^2}{s}\int_{-\infty}^{0}\frac{du_1\,du_2\,(uvu_1u_2)^{1/2}}{(u_1-m^2)^2(u_2-m^2)^2}\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}dl\frac{e^{(l+1)(\theta-\xi)}}{(l+1)^2}A(l,u,u_1)A(l,v,u_2)\ . \tag{7}$$

Since only small values of u_1 and u_2 are important, we reach the significant conclusion (the same conclusion is reached in the parton model of Drell and Yan, Ref. 2)

$$W(Q^2,s)\xrightarrow[s/Q^2\ \mathrm{fixed}]{s,Q^2\to\infty}f(s/Q^2). \tag{8}$$

A form factor at the $\gamma\pi\pi$ vertex as discussed before would give in the same limit

$$W(Q^2,s)\sim[F(Q^2)]^2f(s/Q^2)\ . \tag{9}$$

If $A(l,u,u_1)$ consists only of poles, Eq. (7) can be further simplified. In particular, for $A(l,u,u_1)=a(u,u_1)\exp[-(l+1)\phi_1]/(l-\alpha)$,

$$W(Q^2,s)=\frac{1}{4\pi^2}\frac{Q^2}{s}\int_{-\infty}^{0}\frac{du_1\,du_2(uvu_1u_2)^{1/2}}{(u_1-m^2)^2(u_2-m^2)^2}a(u,u_1)\,a(v,u_2)\,\Theta(\theta-\xi-\varphi_1-\varphi_2)$$
$$\times\left[\frac{e^{(1+\alpha)(\theta-\xi-\varphi_1-\varphi_2)}+1}{(1+\alpha)^2}(\theta-\xi-\varphi_1-\varphi_2)-\frac{2e^{(1+\alpha)(\theta-\xi-\varphi_1-\varphi_2)}-2}{(1+\alpha)^3}\right], \tag{10}$$

where Θ is the step function. The step function is important and leads to a complicated dependence on s/Q^2. When $Q^2\ll s$, any increase in Q^2 reduces the region of integration corresponding to small values of u_1 and u_2 leading to a rapid fall in the cross section $d\sigma/dQ^2$ faster than $1/Q^4$. However, when $Q^2/s\ll 1$, the s/Q^2 dependence coming from the step function can be neglected to get, with $\alpha=1$ corresponding to the Pomeranchuk pole,

$$\frac{d\sigma}{dQ^2}=\frac{1}{Q^4}\left[a\ln\frac{s}{Q^2}+b\right]. \tag{11}$$

The constants a and b in Eq. (11), or more generally $W(Q^2,s)$ in Eq. (7), are determined completely by the off-shell πN forward absorptive amplitude. One does not have to go far off the shell since only small values of u_1 and u_2 are important. The $\ln(s/Q^2)$ factor in Eq. (11) arises from the double pole at $l=\alpha$. One can expect such a factor for the following reason: Even though for any given event the position of the timelike photon along the chain is determined by the requirement of small momentum transfers, when we sum over the momenta of the photon, keeping its mass fixed, there will be events when the photon comes out after various number of rungs of the chain and the mean number of rungs behaves like $\ln(s/Q^2)$.

Now consider emission of the virtual photon from the ends of the chain (Fig. 4). Averaging over proton spins and assuming $Q^2\gg M^2$, we get

$$W(Q^2,s)=4Q^2\int d^4p_1'\,\delta_+((p_1-p_1')^2-Q^2)\times\frac{1}{(p_1'^2-M^2)^2}\,\mathcal{a}(p_1',p_2), \tag{12}$$

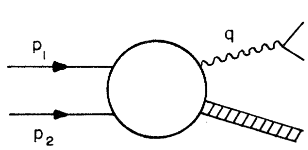

FIG. 1. Proton + proton $\to\mu^+ + \mu^-$ + anything.

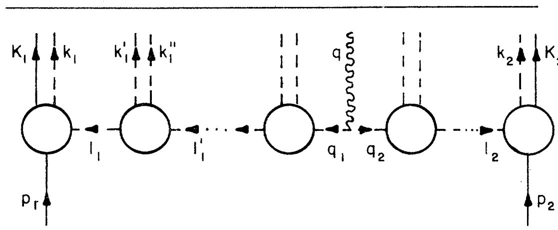

FIG. 2. Diagram for the multiperipheral amplitude with the photon coming from the middle. Dashed lines correspond to pions, wavy line to the photon.

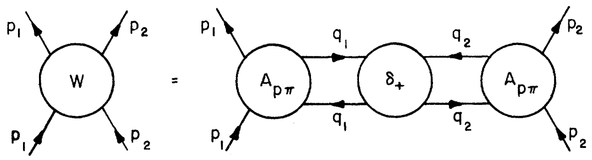

FIG. 3. Structure of Eq. (5).

where $\mathcal{A}$ is some combination of the invariant functions characterizing NN forward absorptive amplitude. Diagonalization of Eq. (12) gives

$$W(Q^2, s) = \frac{4Q^2}{s} \int_{-\infty}^{0} \frac{du'\,(u'v)^{1/2}}{(u'-M^2)^2} \frac{1}{2\pi i} \times \int_{c-i\infty}^{c+i\infty} \frac{dl}{l+1} e^{(l+1)(\theta-\theta_0)} \mathcal{A}_l(u', v), \tag{13}$$

where $\cosh\theta_0 = (Q^2 - u' - u)/2(uu')^{1/2}$. The scaling behavior is the same as before. If we take

$$\mathcal{A}(l, u', v) = \beta(u', v) \frac{e^{-(l+1)\varphi_0}}{l-\alpha},$$

with $\cosh\varphi_0 = [(2M)^2 - u' - v]/2(u'v)^{1/2}$, we get

$$W(Q^2, s) = \frac{4Q^2}{s} \int_{-\infty}^{0} \frac{du'(u'v)^{1/2}}{(u'-M^2)^2} \beta(u', v) \times \left[\frac{e^{(1+\alpha)(\theta-\theta_0-\varphi_0)} - 1}{1+\alpha}\right] \Theta(\theta - \theta_0 - \varphi_0). \tag{14}$$

As before, when $Q^2/s \ll 1$, we can neglect the dependence of s/Q^2 coming from the step function to get, with $\alpha = 1$,

$$\frac{d\sigma}{dQ^2} = \text{const} \times \frac{1}{Q^4}. \tag{15}$$

Exactly similar results follow for the emission of the photon from the other end of the chain.

III. DISCUSSION

(a) The cross section in the model presented above is completely determined by πN and NN forward absorptive amplitudes with no other parameters. When the subenergies are large, the use of Regge form should be valid.

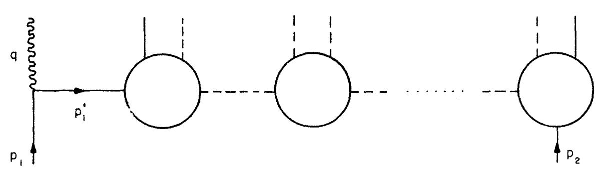

FIG. 4. Diagram for the multiperipheral amplitude with the photon coming from the end.

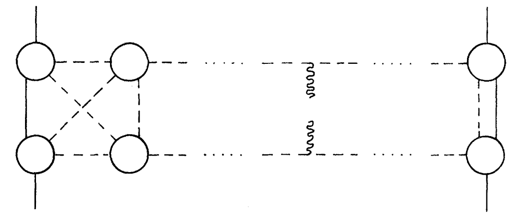

FIG. 5. Interference terms taken into account.

(b) A photon of given value of Q^2 can come either from the middle or from the ends of the chain, depending on its momentum; interference of the amplitudes for the different possibilities is small.

(c) We have neglected the exchange of particles other than pions along the chain. Inclusion of, say, kaon exchange gives an additional term on the right-hand side of Eq. (7) corresponding to KN amplitude.

(d) Point electromagnetic coupling leads to the scaling behavior (8); introduction of form factors changes it to (9), where f is the known function containing all s dependence. Exchanging different particles as in (c) with different form factors is complicated.

(e) Cross terms of the type of Fig. 5 are automatically included when we sum over the final hadrons to get the forward absorptive part, whereas terms corresponding to Fig. 6 are negligible (remembering that the photon can be looked upon as corresponding to several blobs). In the usual ABFST model, the interference terms are all ignored.

(f) A comparison of the model given here with the experiment[1] requires suitably chosen off-shell amplitudes. The experimenters required the muon-pair lab momentum to exceed 12 GeV/c. Since the incident proton energy was 29.5 GeV, the virtual photon has to come from that end of the chain corresponding to the incident proton; creation of even an $N\pi$ state with an appreciable probability before emitting the photon would take away a fraction of incident energy about equal to the elasticity, which is about 0.7,[7] leaving insufficient energy for the photon to satisfy the experimental constraint.

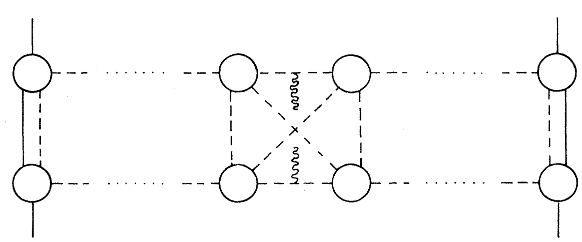

FIG. 6. Interference terms neglected but small.

IV. CONCLUSIONS

Even though the model is presented in the ABFST language, it is more general. The cross section is determined by πN and NN off-shell forward absorptive amplitudes. The scaling behavior has been determined. Even though point electromagnetic coupling is used, inclusion of form factors is easy. Gauge-noninvariant terms are small in the limit of massive-muon pairs.

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APPENDIX

Gauge invariance of the model proposed in the main text is studied here. First of all, since one end of the virtual photon is attached to a real lepton pair, whatever gauge we choose for the photon propagator, the cross section after summing over lepton polarizations and momenta, keeping q fixed, depends only on $(g_{\mu\nu} - q_\mu q_\nu/q^2)\, W^{\mu\nu}(q, p_1, p_2)$, where $W^{\mu\nu}$ is given by Eq. (3) by replacing $\overline{W}^{\mu\nu}$ with $W^{\mu\nu}$ and dropping q integration and the factor $\delta_+(q^2 - Q^2)$. Considering the ladder model now, it is easy to see that for large q^2 one can approximate $(g_{\mu\nu} - q_\mu q_\nu/q^2) W^{\mu\nu}$ by $g_{\mu\nu} W^{\mu\nu}_{(a)}$ where the subscript a denotes contribution from Fig. 7(a). Equation (5) or Eq. (12) essentially involves the quantity $g_{\mu\nu} W^{\mu\nu}_{(a)}$. Contributions from figures of the type in Fig. 7(b) are small and so are those from the diagrams obtained by displacing the photon by one or more rungs of the ladder; the easiest way to see this is to imagine the massive virtual photon as corresponding to several rungs of the ladder, and displacing the virtual photon should result in large propagators which damp these contributions. For the model considered in the main text, $W_{\mu\nu}$ is given, for the case of the photon coming from the middle of the chain, by

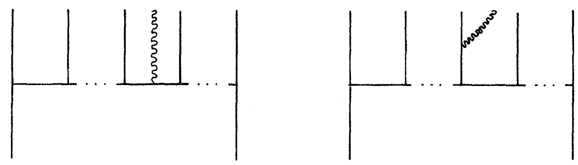

FIG. 7. Multiperipheral diagrams in a ladder model.

$$W_{\mu\nu} = -\frac{1}{4\pi^4}\int d^4q_1 d^4q_2\, \delta(q_1+q_2+q)\, \frac{A_{p\pi}(p_1, q_1)}{(q_1^2 - M^2)^2} \times \frac{A_{p\pi}(p_2, q_2)}{(q_2^2 - M^2)^2}(q_1 - q_2)_\mu (q_1 - q_2)_\nu . \quad \text{(A1)}$$

One can force current conservation, i.e., force $q^\mu W_{\mu\nu} = 0$, in a rather artificial way by multiplying the integrand of (A1) by $m^2\delta(q_1^2 - q_2^2)$, i.e., by requiring the mass of the virtual pion before and after emission of the photon to be the same. Even otherwise, it is easy to see that for $W_{\mu\nu}$ given by (A1), $g^{\mu\nu} W_{\mu\nu} \gg (q^\mu q^\nu/q^2)\, W_{\mu\nu}$ when $Q^2 \equiv q^2 \gg M^2$, since $-q_1^2,\ -q_2^2 \lesssim M^2$. Indeed, $W_{\mu\nu}$ has the general form

$$\begin{aligned} W_{\mu\nu} = {} & \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1 + \left(P_\mu - \frac{q\cdot P}{q^2} q_\mu\right)\left(P_\nu - \frac{q\cdot P}{q^2} q_\nu\right) W_2 \\ & + \left[\left(P_\mu - \frac{q\cdot P}{q^2} q_\mu\right)\left(\Delta_\nu - \frac{q\cdot\Delta}{q^2} q_\nu\right) + \left(\Delta_\mu - \frac{q\cdot\Delta}{q^2} q_\mu\right)\left(P_\nu - \frac{q\cdot P}{q^2} q_\nu\right)\right] W_3 \\ & + \left(\Delta_\mu - \frac{q\cdot\Delta}{q^2} q_\mu\right)\left(\Delta_\nu - \frac{q\cdot\Delta}{q^2} q_\nu\right) W_4 + q_\mu q_\nu W_5 + (q_\mu P_\nu + q_\nu P_\mu) W_6 + (q_\mu \Delta_\nu + q_\nu \Delta_\mu) W_7 , \end{aligned} \quad \text{(A2)}$$

where $P = p_1 + p_2$, $\Delta = p_1 - p_2$. The last three terms do not conserve current. Multiplying (A2) by $q^\mu q^\nu$, $q^\mu P^\nu$, $q^\mu \Delta^\nu$, $g^{\mu\nu}$, $P^\mu P^\nu$, $P^\mu \Delta^\nu$, and $\Delta^\mu \Delta^\nu$ successively, we get seven equations for the seven unknowns. Solving the first three for W_5, W_6, and W_7 and substituting into the last four, one can show that the terms in the last four equations involving W_5, W_6, and W_7 can be neglected if $q^2/M^2 \gg (s/q^2)^2$. Furthermore, the W_6 and W_7 terms appearing in $(g_{\mu\nu} - q_\mu q_\nu/q^2) W^{\mu\nu}$ can also be neglected. The case of the photon coming from the ends of the chain can be treated similarly.

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$$\int_{-\infty}^{0} du_1(-u_1)du_2(-u_2)\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dl\,\frac{\pi}{l+1}e^{(l+1)(\varphi'-\xi-\varphi_2)} \times\tilde{A}_2(l,u_1,u_2)\tilde{A}_3(l,v,u_2) \times\frac{1}{2\pi i}\int_{c'-i\infty}^{c'+i\infty} d\lambda\,\frac{\pi}{\lambda+1}\,\frac{e^{(\lambda+1)(\theta-\varphi_1-\varphi')}}{(\lambda-l)\sinh\theta}\,\tilde{A}_1(\lambda,u,u_1),$$

where $\tilde{A}_1(\lambda,u,u_1)=e^{(\lambda+1)\varphi_1}A_1(\lambda,u,u_1)$ so that $|\tilde{A}_1(\lambda,u,u_1)|\to 0$ as $|\lambda|\to\infty$, and similarly for A_2 and A_3. [A_1, A_2, A_3, φ_1, and φ_2 are defined following Eq. (6).] Also, $\cosh\varphi'=[(Q+M+m)^2-u_1-v]/2(u_1v)^{1/2}$. The number c (c') is chosen such that the integrand is analytic in l (λ) for $\mathrm{Re}\,l\geq c$ ($\mathrm{Re}\,\lambda\geq c'$). While doing the λ integration, singularities other than the pole at $\lambda=l$ are inconsequential; the reason is as follows: These other singularities will have $e^{(l+1)(\varphi'-\xi-\varphi_2)}$ in the integrand and the easily verifiable condition $\varphi'-\xi-\varphi_2\leq 0$ allows us to close the contour of l integration on the right where there are no singularities, thus giving zero.

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Off-Mass-Shell Parity-Violating Pion-Nucleon Interaction*

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In the usual theoretical description of parity violation, off-mass-shell effects of the weak pion-nucleon interaction are neglected. In the Cabibbo theory, the weak parity-violating pion-nucleon vertex is then proportional to $\sin^2\theta\approx 0.05$ as it arises only from strangeness-changing weak currents. If off-mass-shell effects are taken into account, then a vertex proportional to $\cos^2\theta$ is possible. In this work, we derive an effective $\cos^2\theta$ pion-nucleon interaction by means of the renormalizable σ model. We obtain an effective dimensionless coupling estimate which is roughly given by $Gg\,\vec{\tau}\cdot\hat{\phi}\,\Delta M/M$ if the nucleon is timelike and half off its mass shell by ΔM. Thus neutral as well as charged weak pion-nucleon couplings of sizable strengths ($\sim 10^{-6}$) are predicted by this model even though no neutral weak currents are invoked.

I. INTRODUCTION

The presence of parity-violating effects in nuclei is now well established. However, with perhaps the exception of the parity-violating α decay from a 2^- level in ^{16}O to the ground state of ^{12}C,[1] there has been some difficulty[2] in accounting for the data with the simple Cabibbo theory[3] of the weak interactions. Numerous variations of the Cabibbo theory, incorporating neutral weak currents,[2] have been proposed in the literature. However, such currents have not been observed. Since the Cabibbo theory remains the simplest framework for the treatment of low-energy weak interactions and has the fewest parameters, it is our philosophy that it should be examined in greater detail before introducing further variations of the theory.

The usual method of computing the parity-violat-